

Amplitude analysis for exotic states

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APS-DNP, Pittsburgh, October 25th, 2017

 **Jefferson Lab**

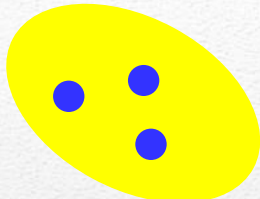


Hadron Spectroscopy

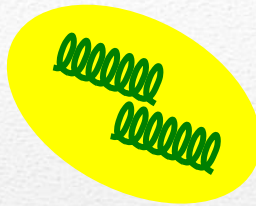
Meson



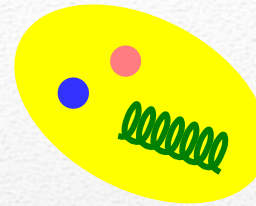
Baryon



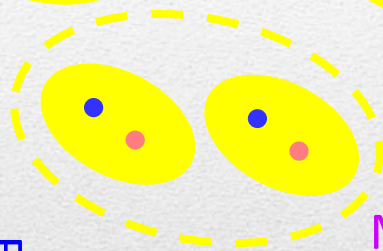
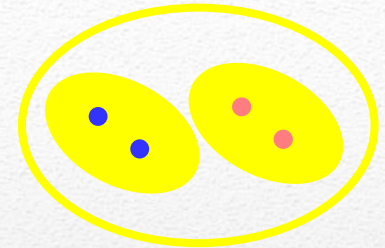
Glueball



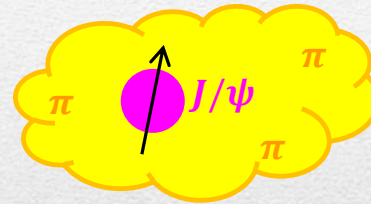
Hybrids



Tetraquark



Molecule



Hadroquarkonium



Experiment

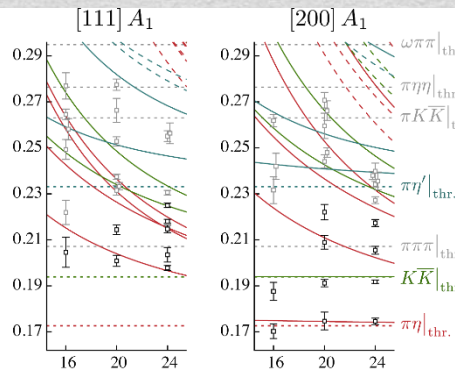
Data



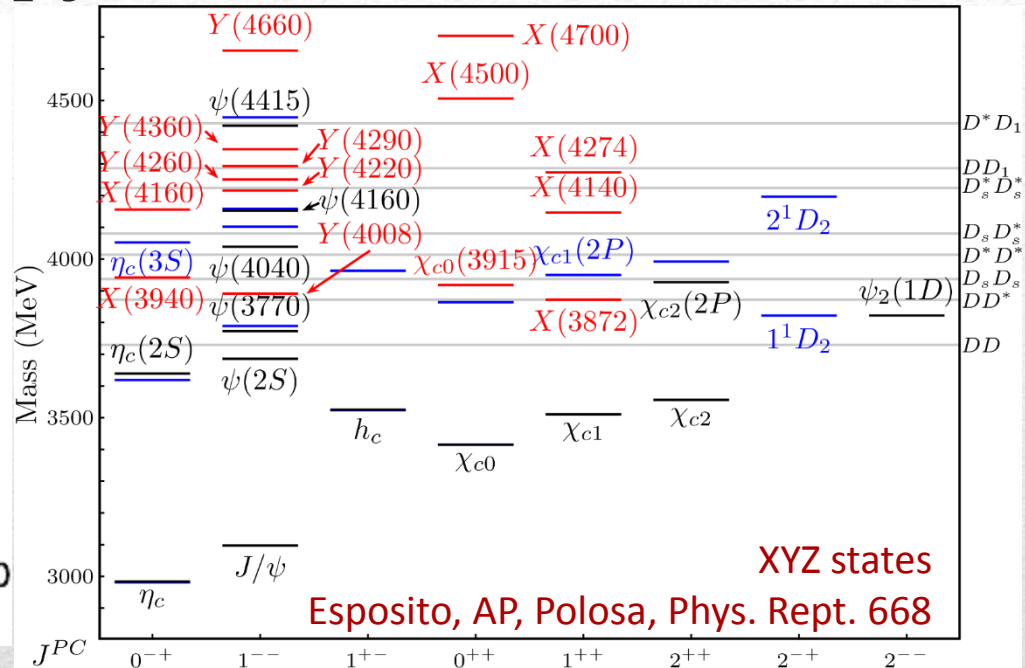
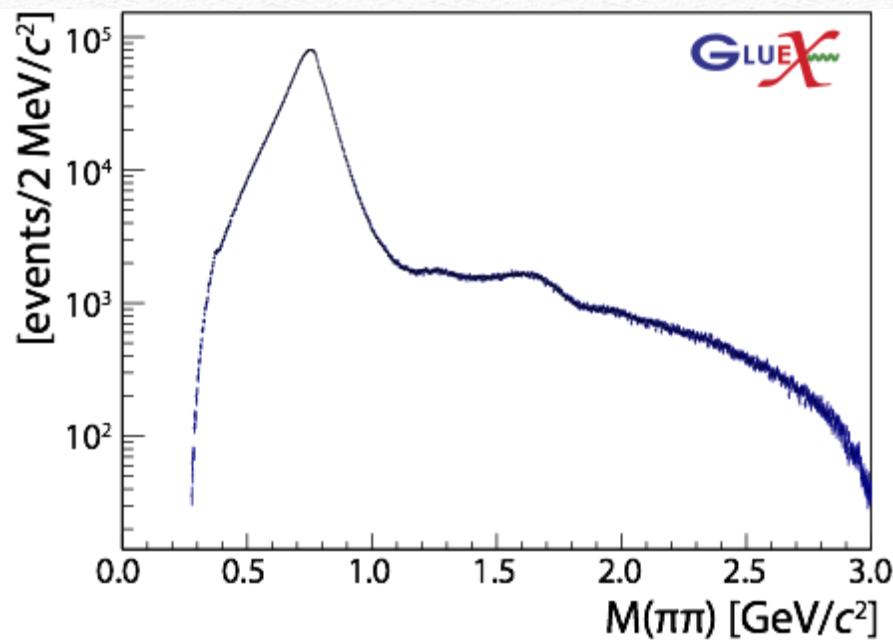
Fundamental properties,
Model building

Lattice QCD

Interpretations on the spectrum leads to understanding fundamental laws of nature



Hadron Spectroscopy

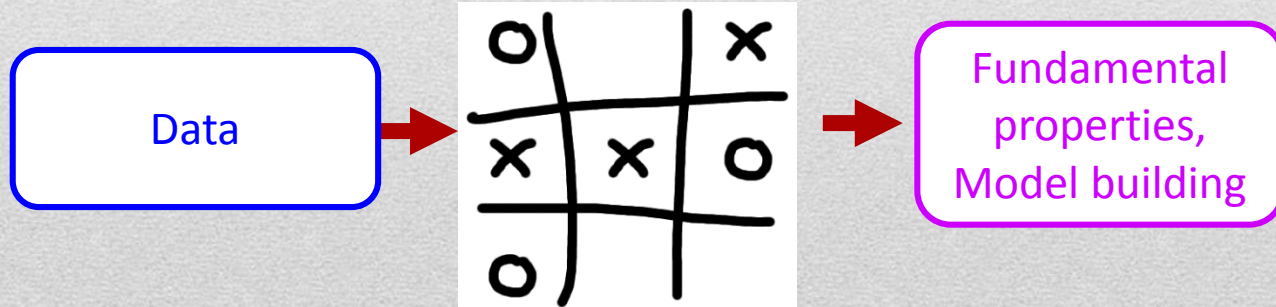
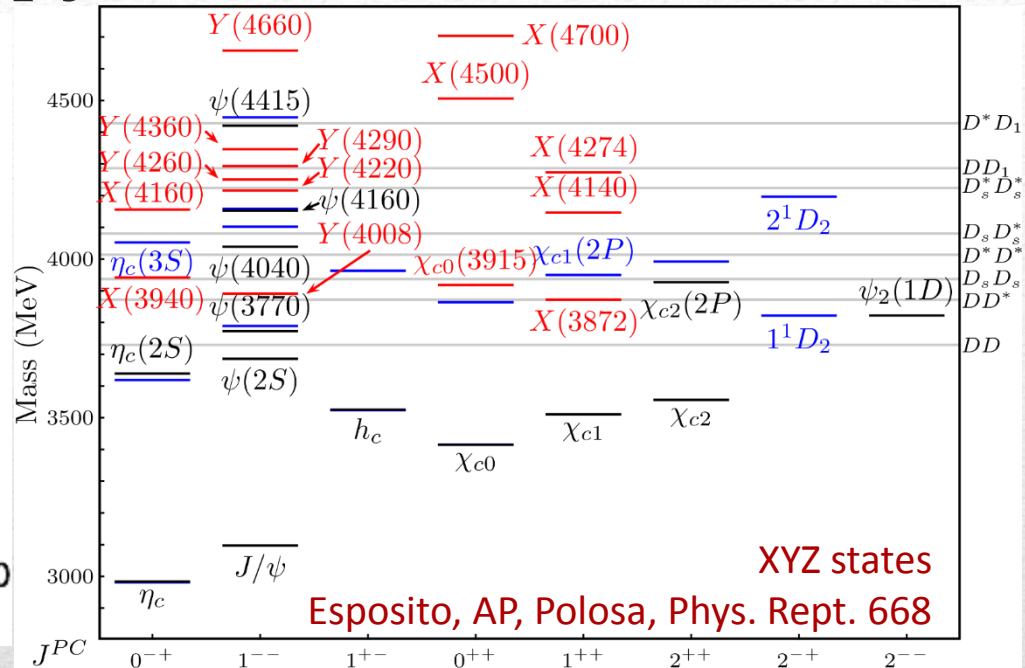
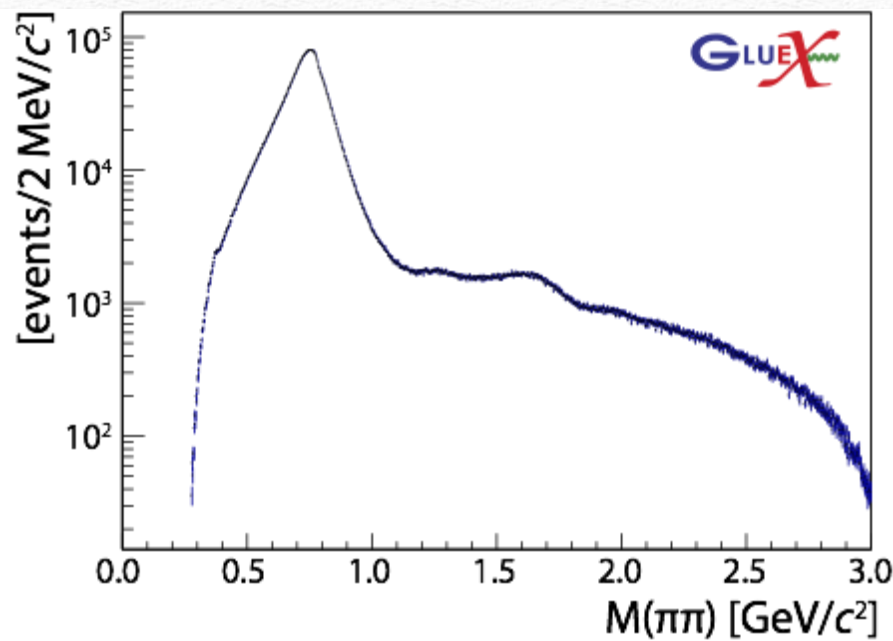


Data

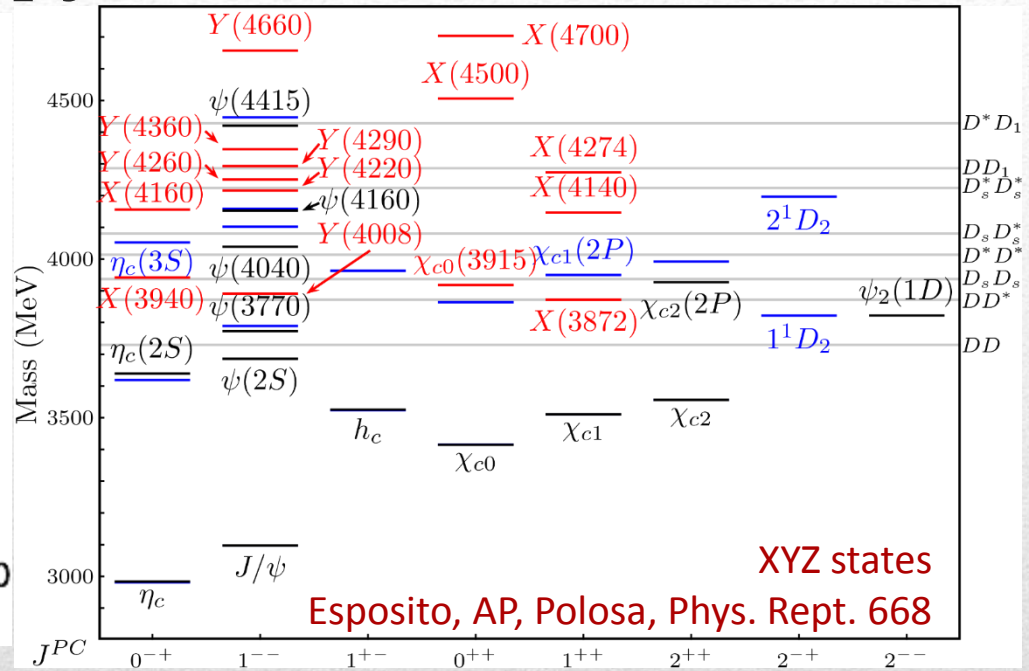
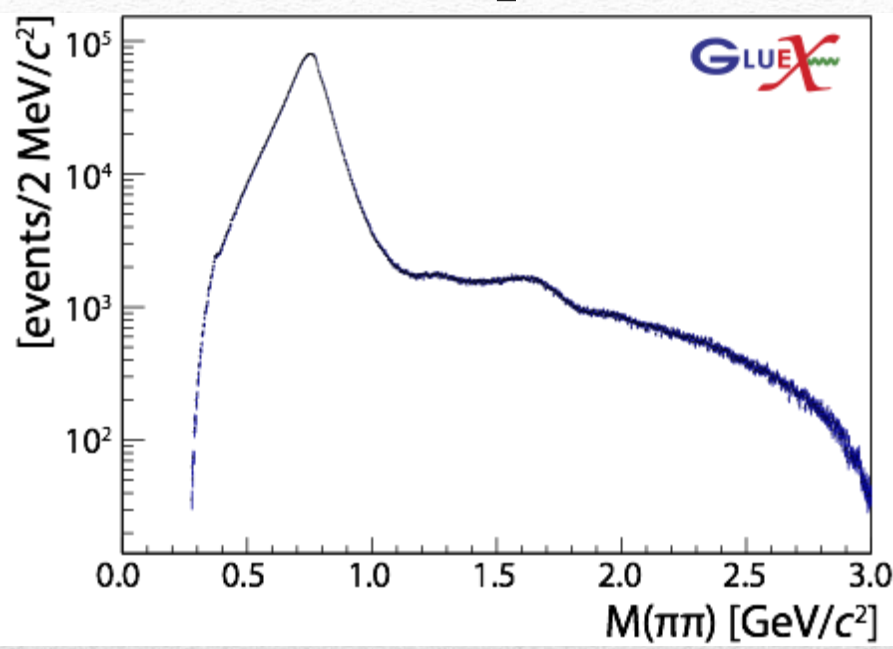


Fundamental properties, Model building

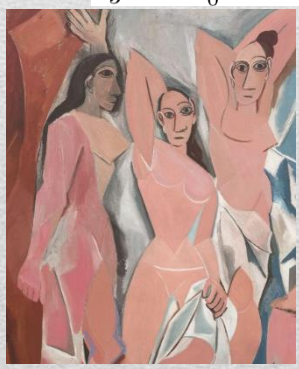
Hadron Spectroscopy



Hadron Spectroscopy



Data

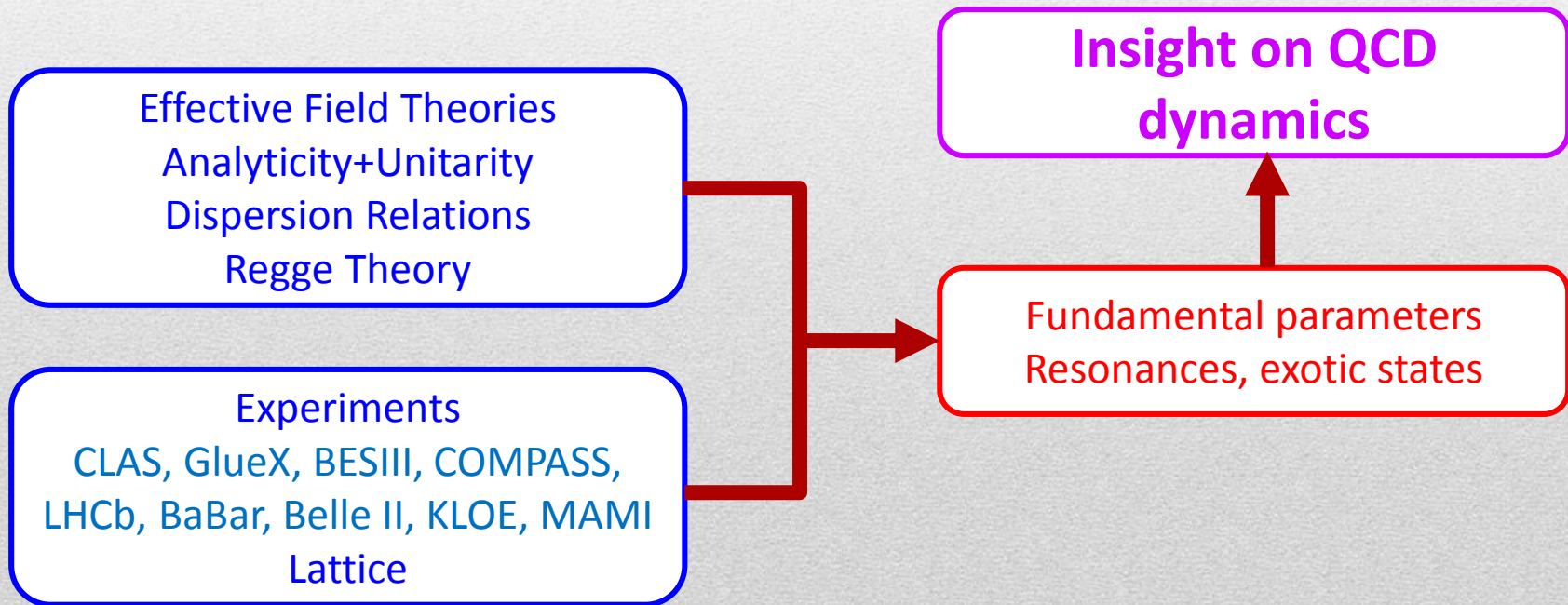


Fundamental properties, Model building

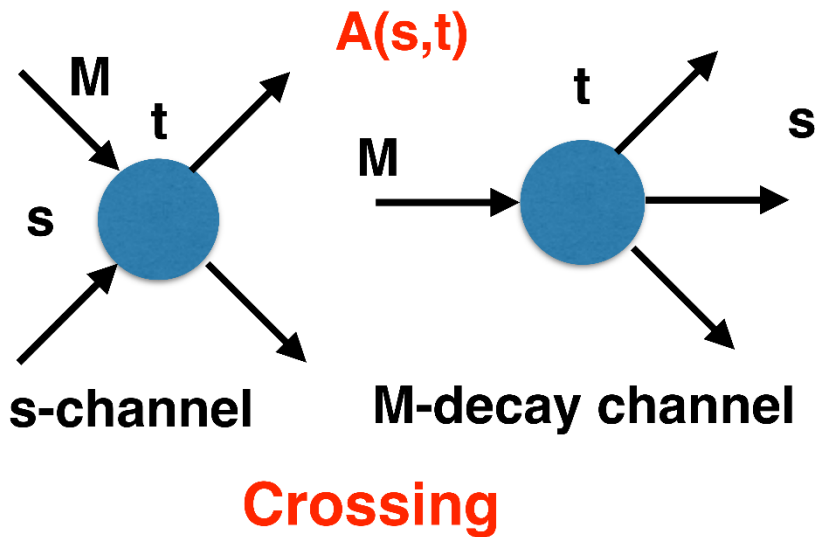
Improvement needed! With great statistics comes great responsibility!

Joint Physics Analysis Center

- **Joint effort** between **theorists** and **experimentalists** to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in **education** of further generations of hadron physics practitioners



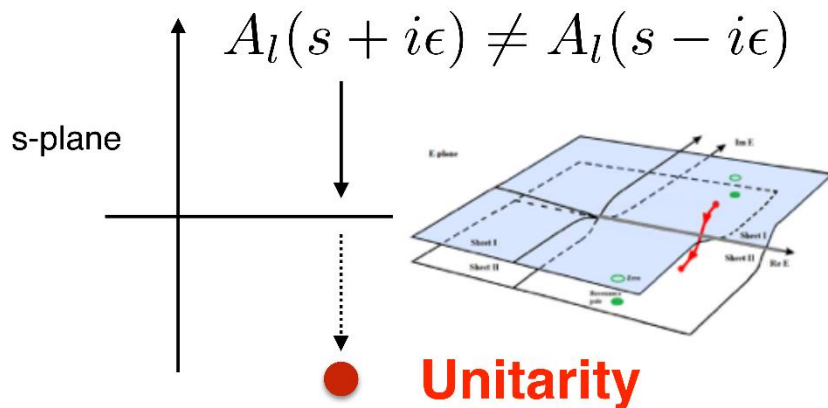
S-Matrix principles



$$A(s, t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \rightarrow 0} A_l(s + i\epsilon)$$

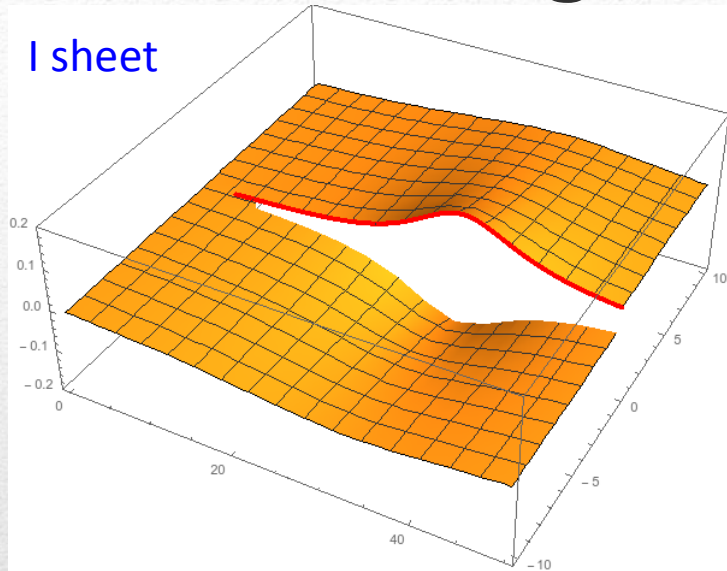


These are constraints the amplitudes have to satisfy, but do not fix the dynamics

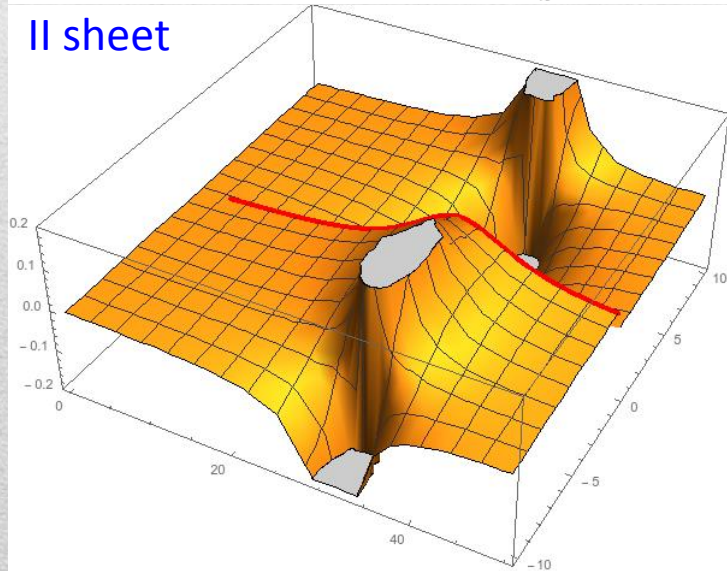
Resonances (QCD states) are poles in the unphysical Riemann sheets

Pole hunting

I sheet

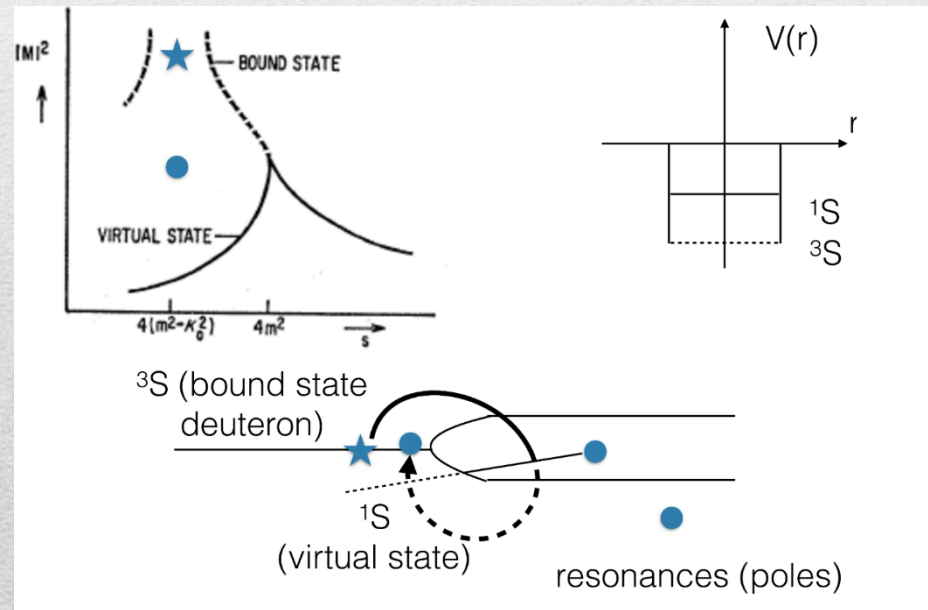


II sheet

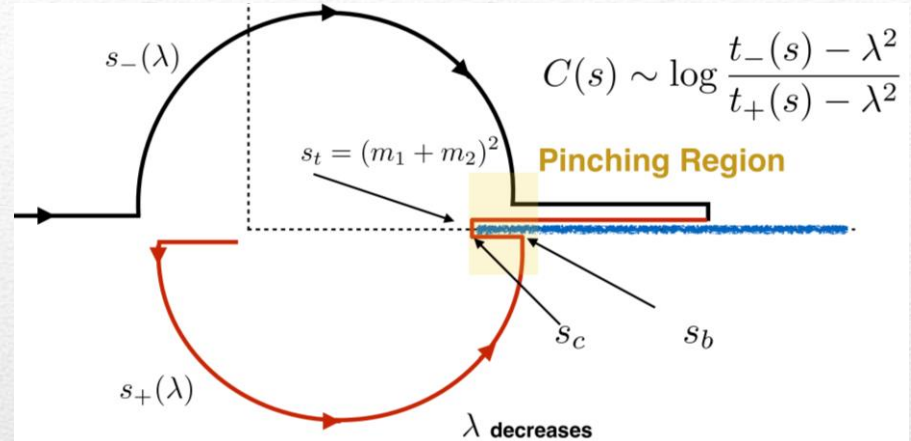
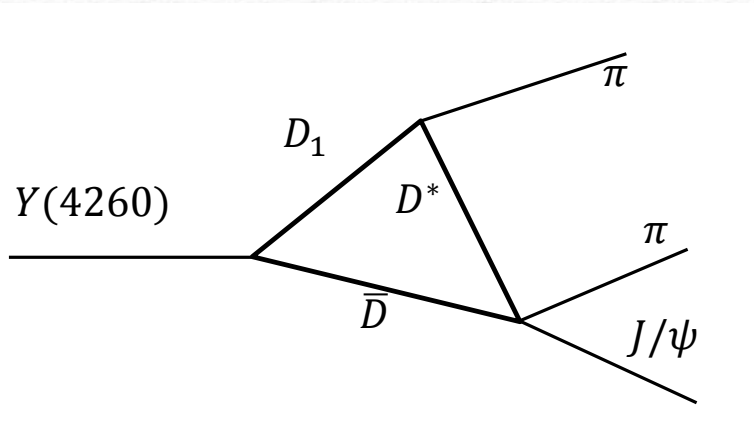


Bound states on the real axis 1st sheet

Not-so-bound (virtual) states on the real axis 2nd sheet



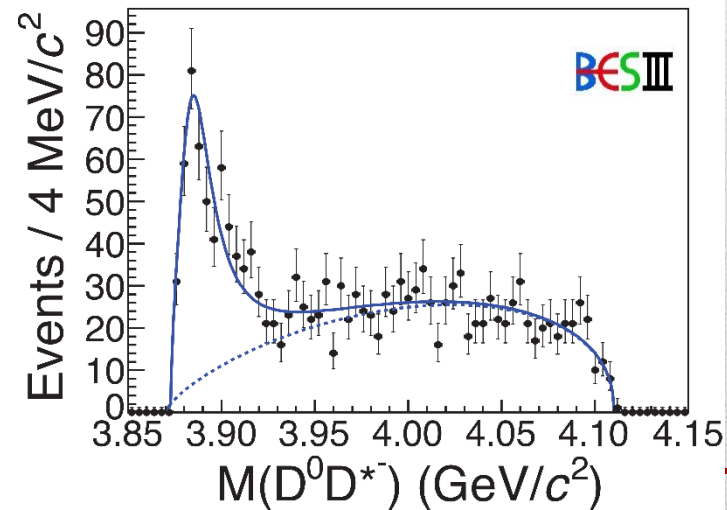
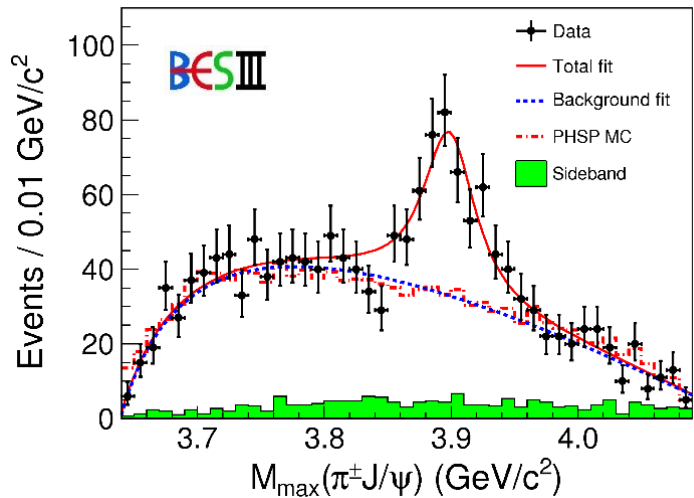
Triangle singularity



- Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in **very special kinematical conditions** (Coleman and Norton, *Nuovo Cim.* 38, 438)
- However, this effects **cancel in Dalitz projections, no peaks** (Schmid, *Phys.Rev.* 154, 1363)
- But the cancellation can be spread in different channels, you might still see peaks in other channels!

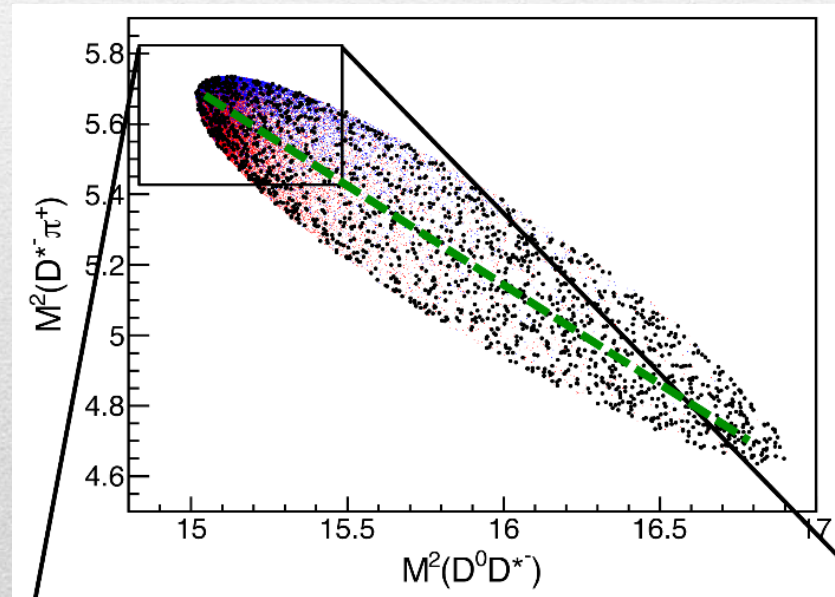
Example: The charged $Z_c(3900)$

A **charged charmonium-like** resonance has been claimed by BESIII in 2013.



$$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \pi^+\pi^- \text{ and } \rightarrow (DD^*)^+\pi^-$$

$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$

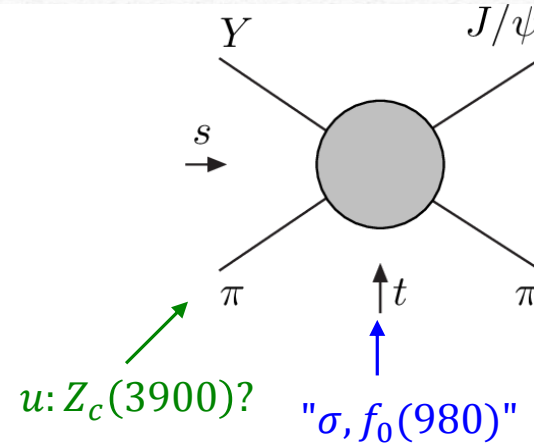
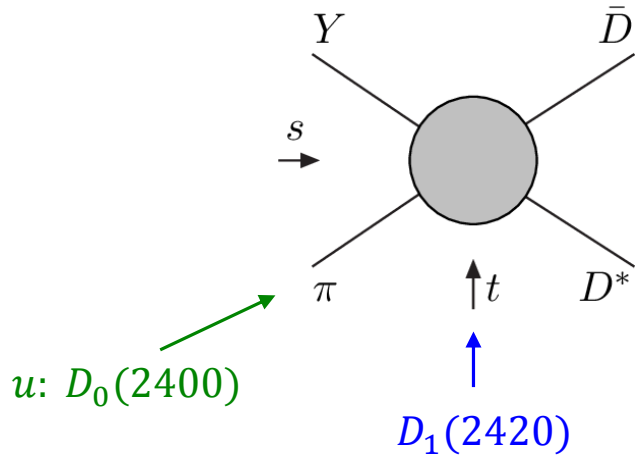


Such a state would require a **minimal 4q content** and would be manifestly exotic

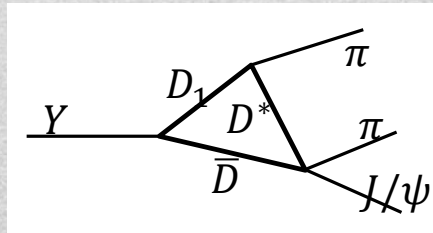
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to **different singularities** \rightarrow **different natures**

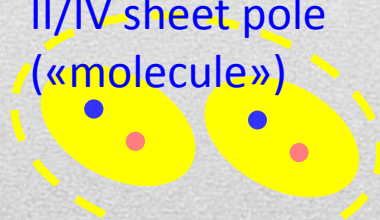
AP *et al.* (JPAC), PLB772, 200



Triangle rescattering,
logarithmic branching point

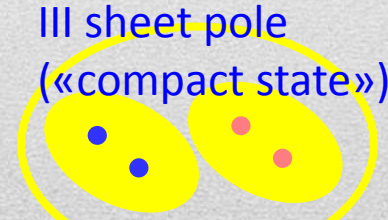


(anti)bound state,
II/IV sheet pole
(«molecule»)



Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Resonance,
III sheet pole
(«compact state»)



Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Szczepaniak, PLB747, 410

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

$$f_i(s, t, u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right],$$

The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i \rho_i \delta_{ij}$

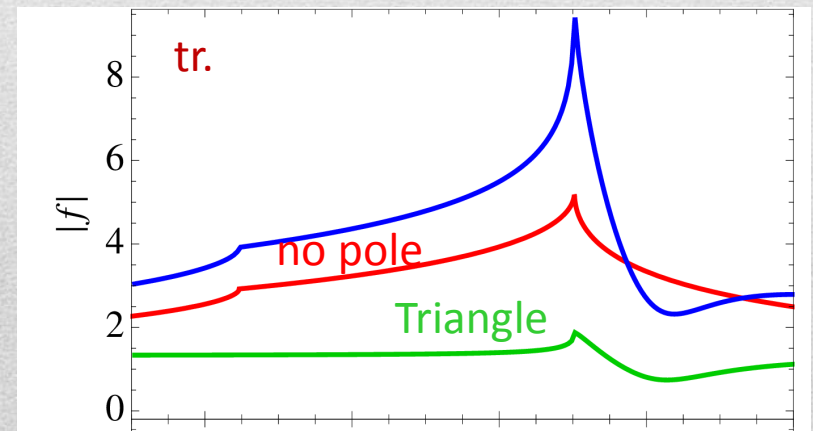
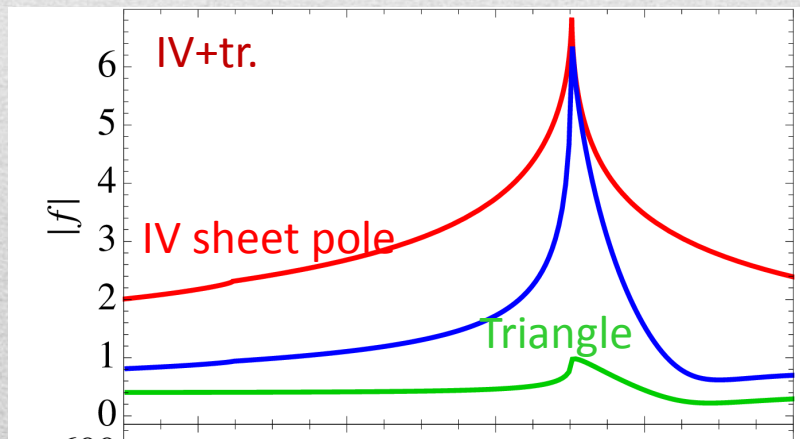
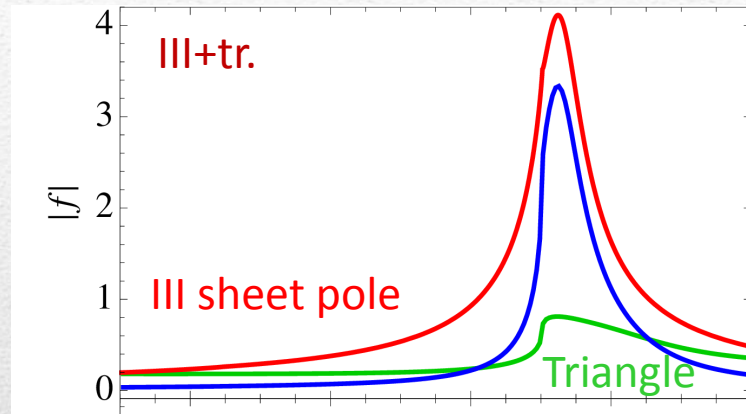
Four different scenarios considered:

- «III»: the K matrix is $\frac{g_i g_j}{M^2 - s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

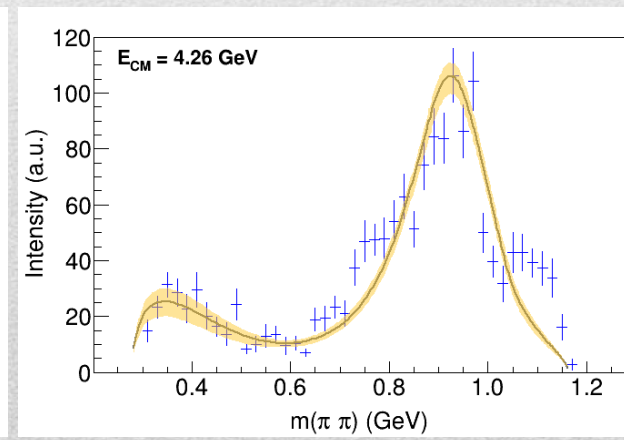
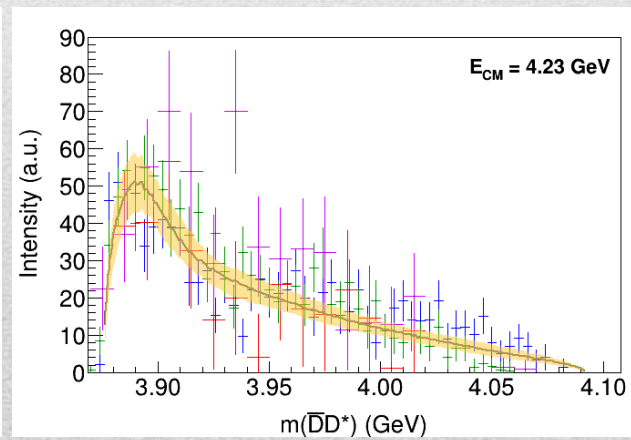
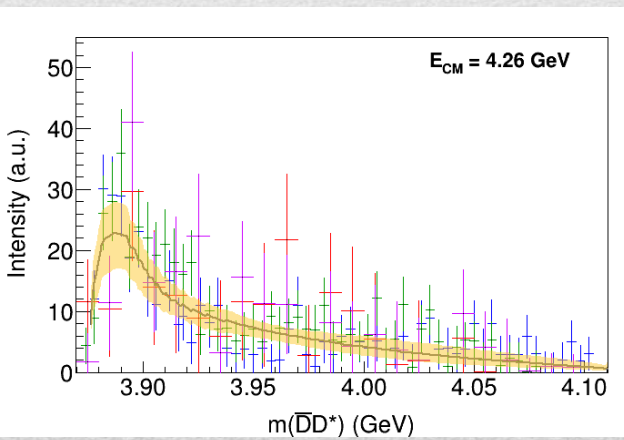
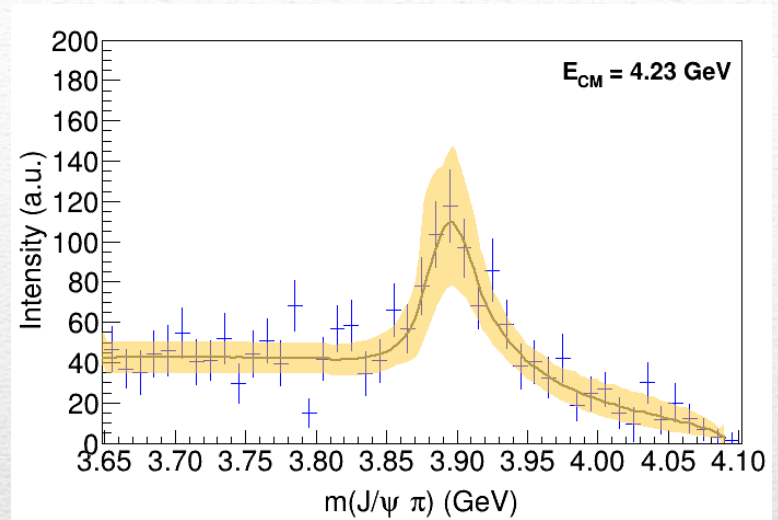
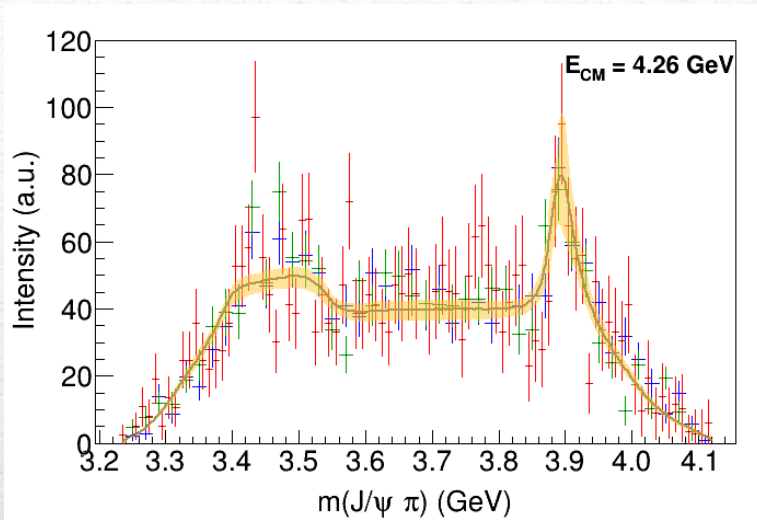
Singularities and lineshapes

Different lineshapes according to different singularities

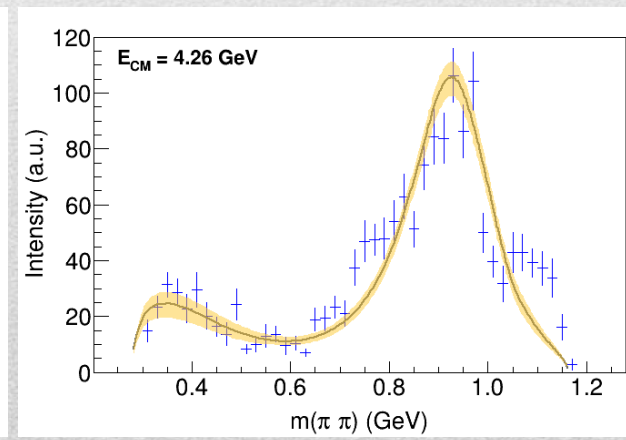
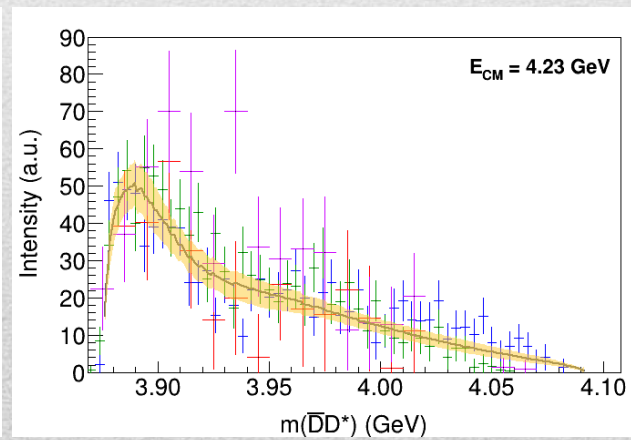
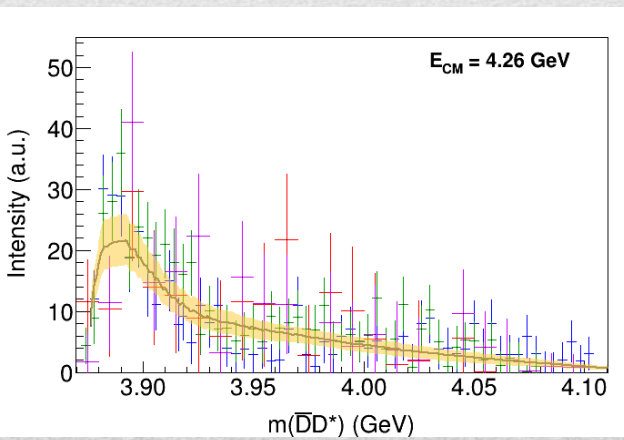
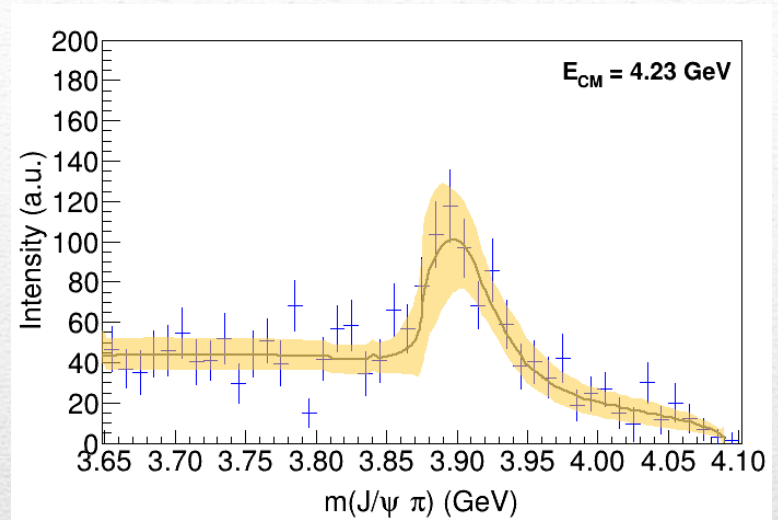
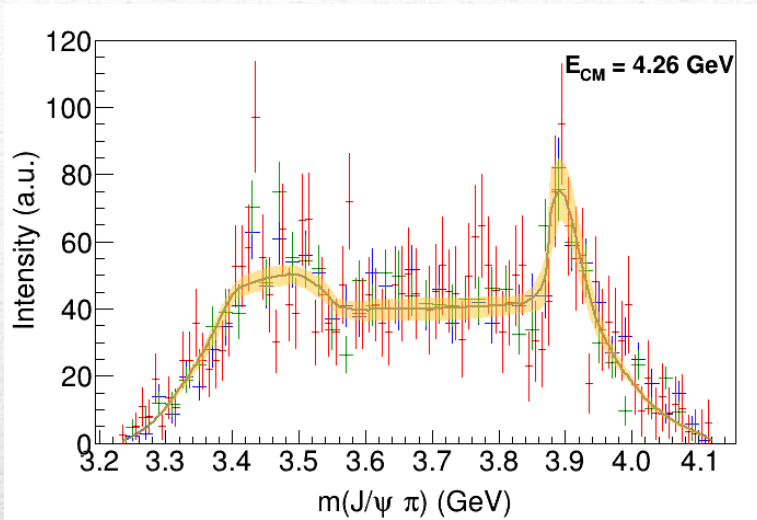
— Triangle
— t matrix
— Full



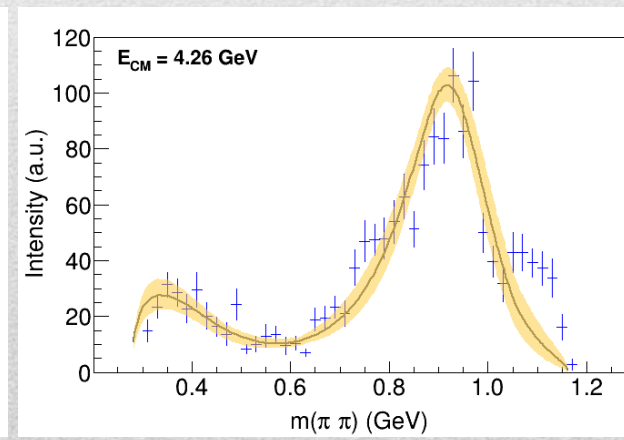
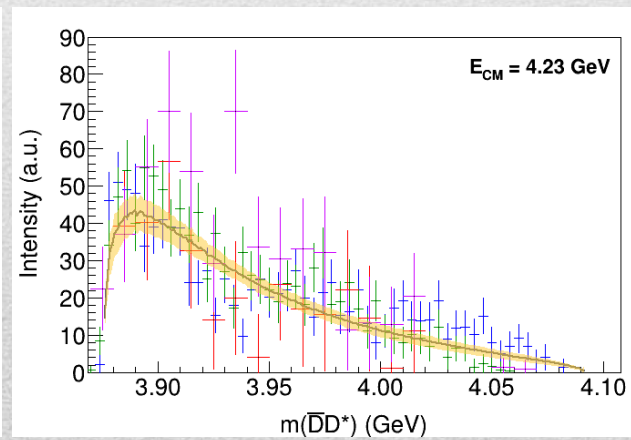
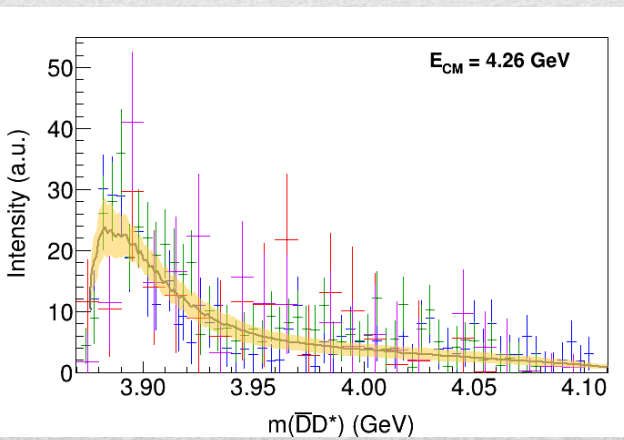
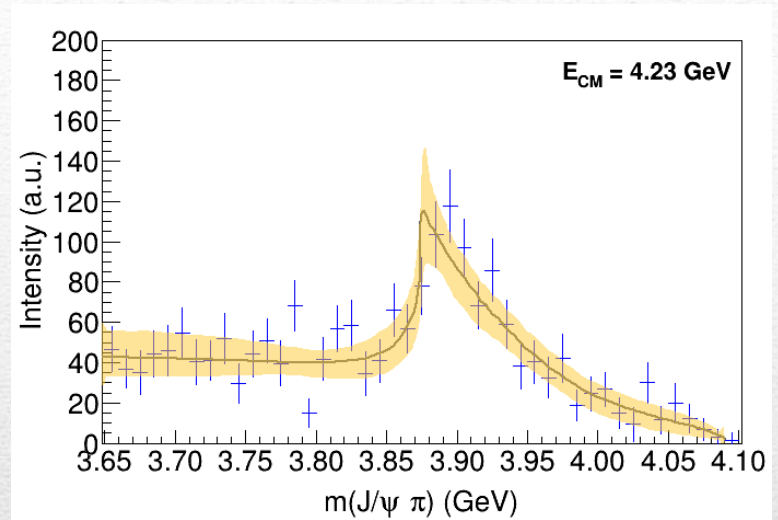
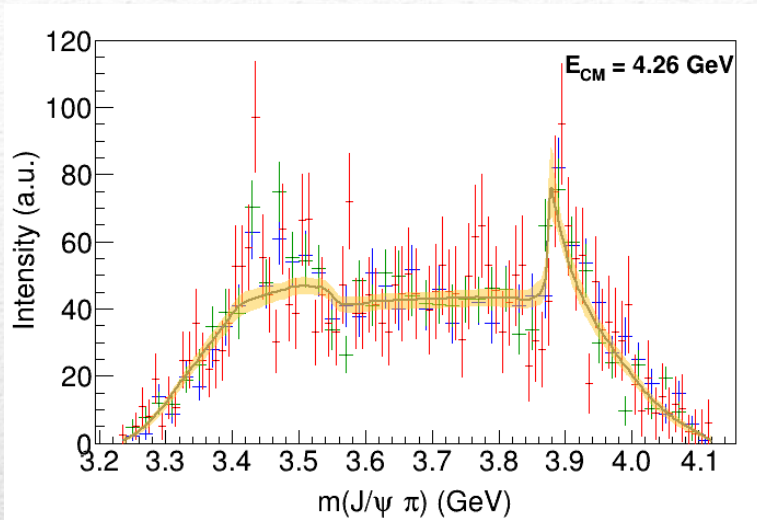
Fit: III



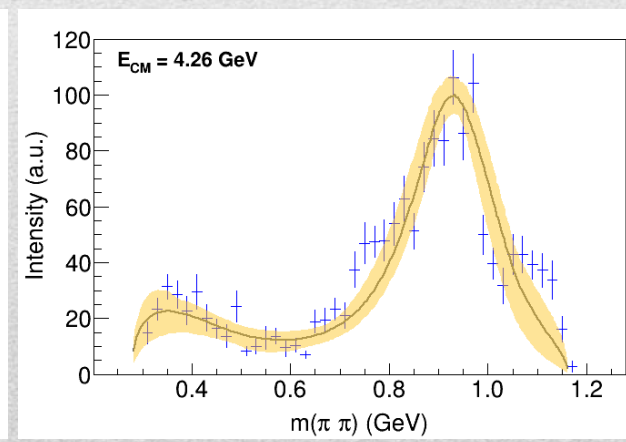
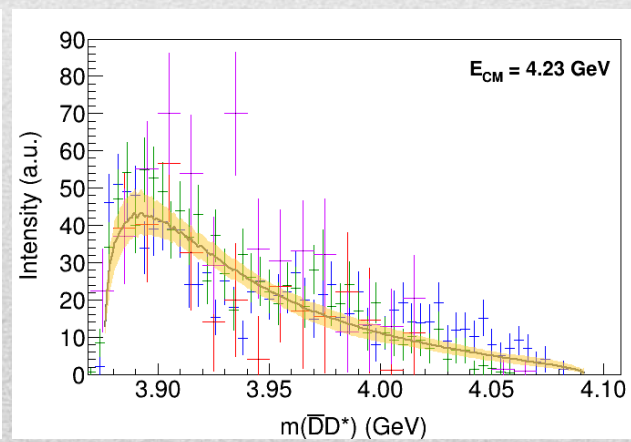
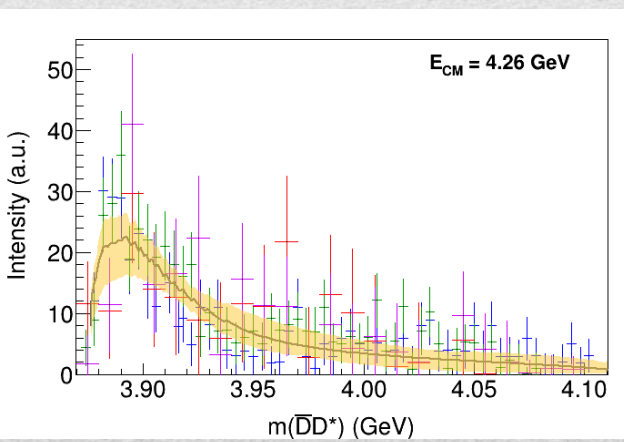
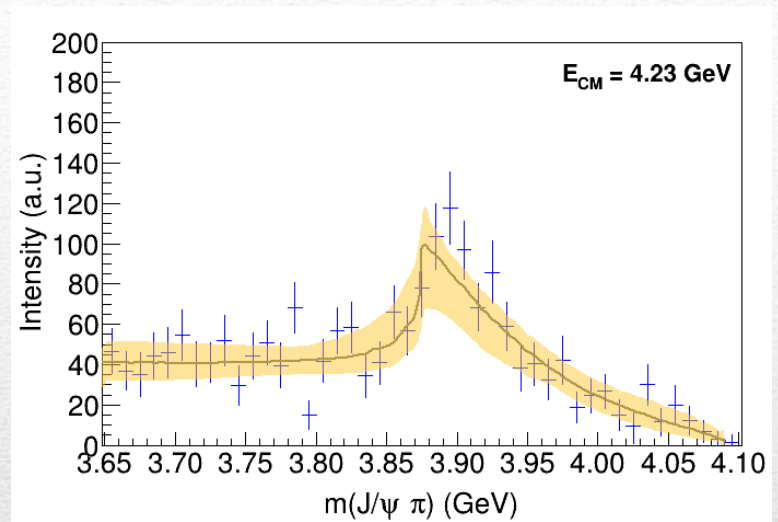
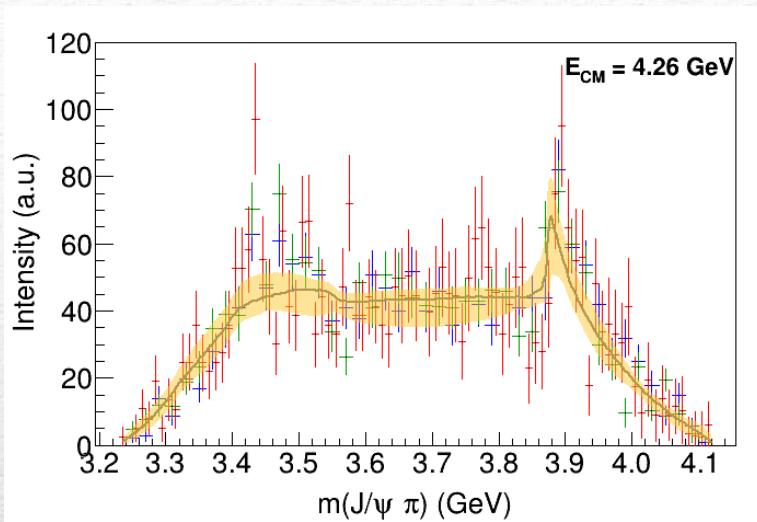
Fit: III+tr.



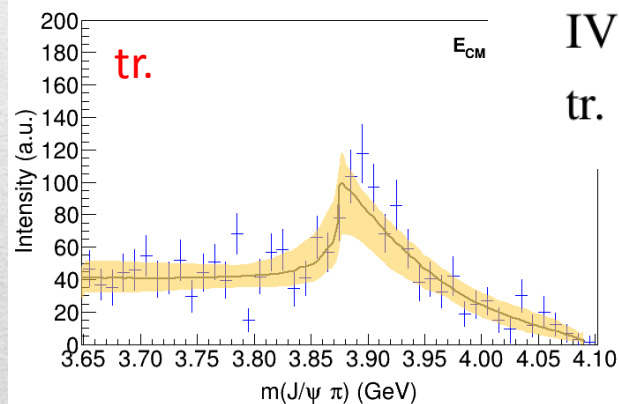
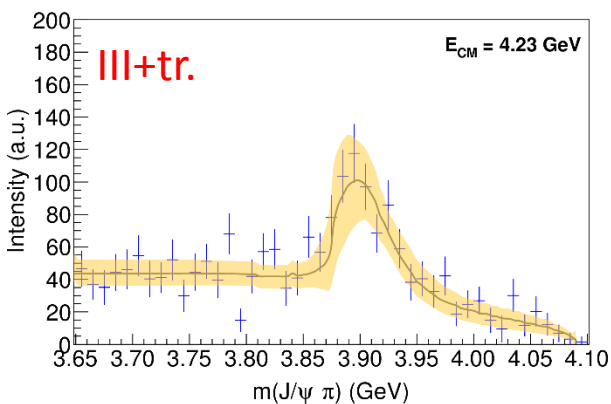
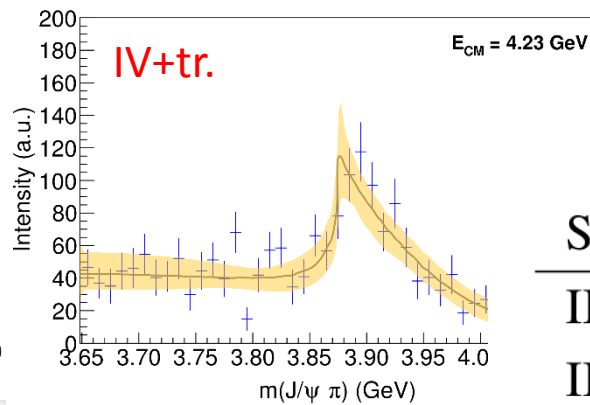
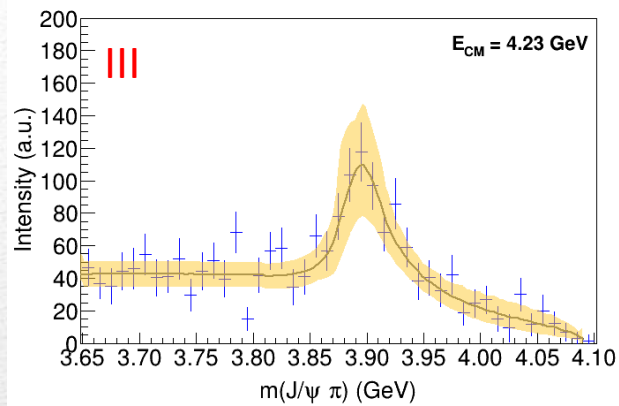
Fit: IV+tr.



Fit: tr.



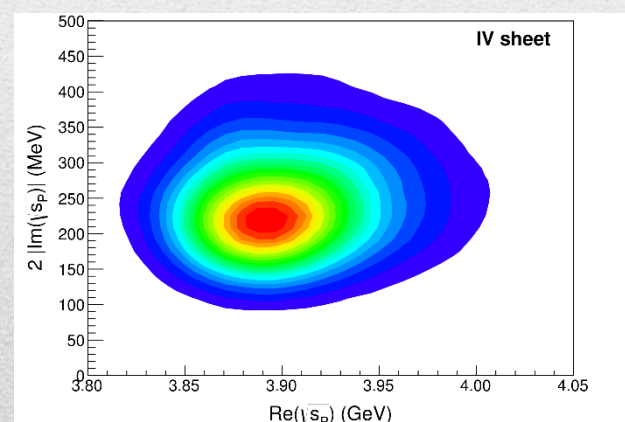
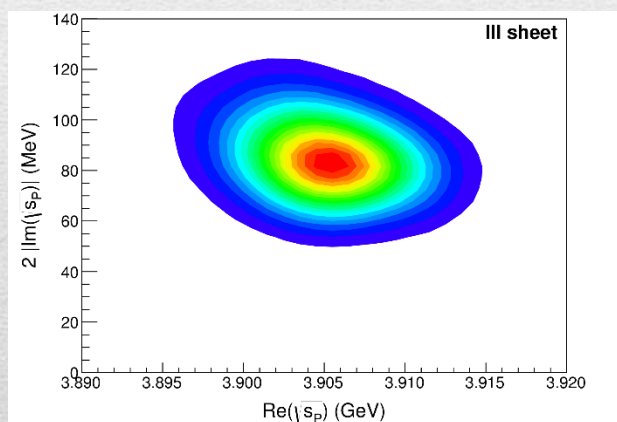
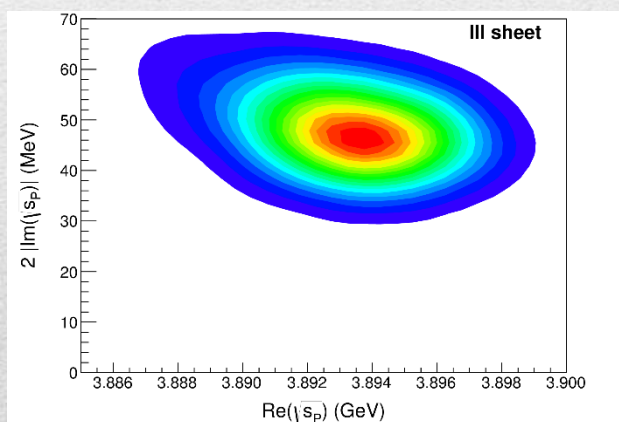
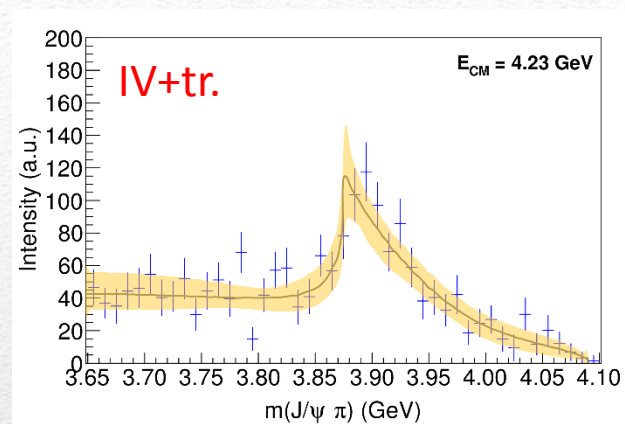
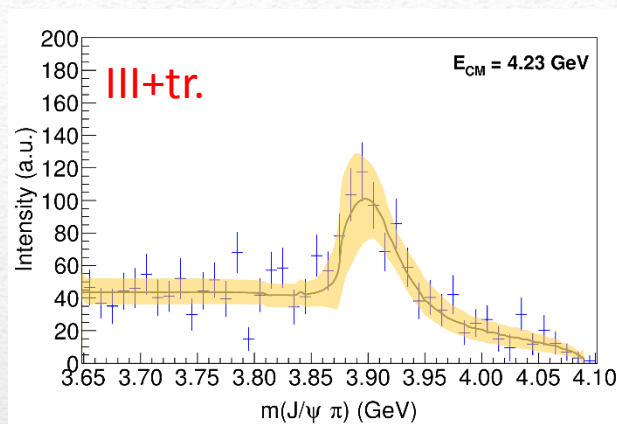
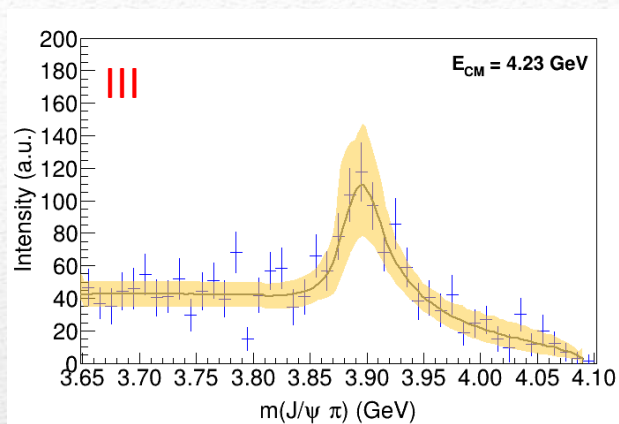
Fit summary



Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test

Pole extraction



Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	“ 2.4σ ” (“ 1.4σ ”)
III+tr.	–	1.5σ (3.1σ)	“ 2.6σ ” (“ 1.3σ ”)
IV+tr.	–	–	“ 2.1σ ” (“ 0.9σ ”)

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

Not conclusive at this stage

Higher energies: Regge exchange

Resonances are poles in s for fixed l
dominate low energy region

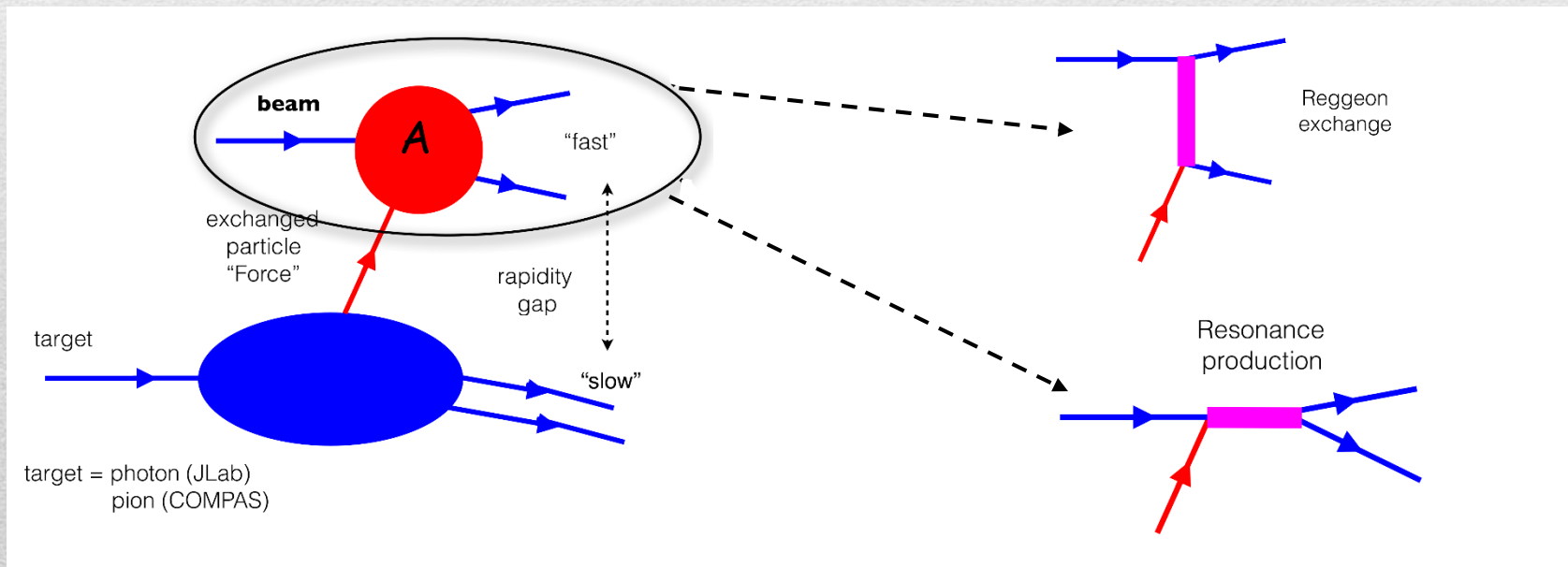


$$A_l \sim \frac{g_1 g_2}{s_p - s}$$

Reggeons are poles in l for fixed s
dominate high energy region

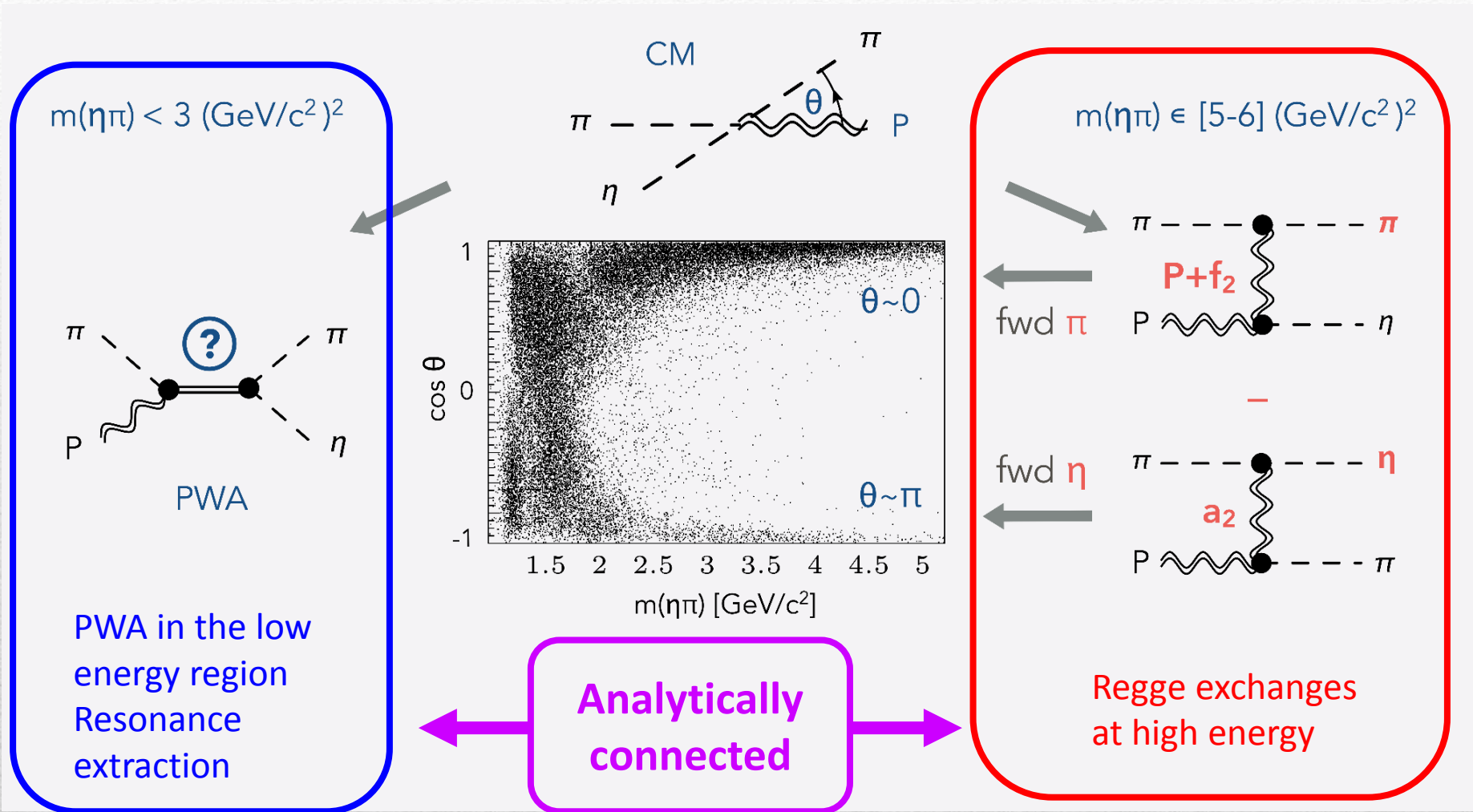


$$A \sim \sum s^l \sim \beta(t) s^{\alpha(t)}$$



Finite energy sum rules

See J. Nys and V. Mathieu talk on friday



Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting **hybrid mesons**
- We build the partial waves amplitude according to the **N/D method**
- We test against the D -wave data, where the a_2 and the a'_2 show up

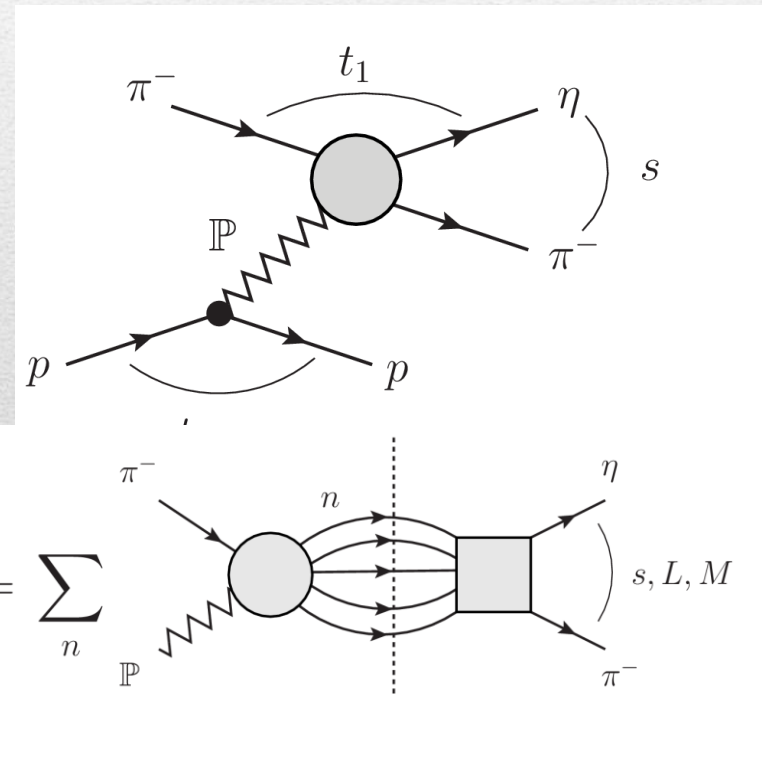
A. Jackura, *et al.* (JPAC & COMPASS), 1707.02848

see talk on friday

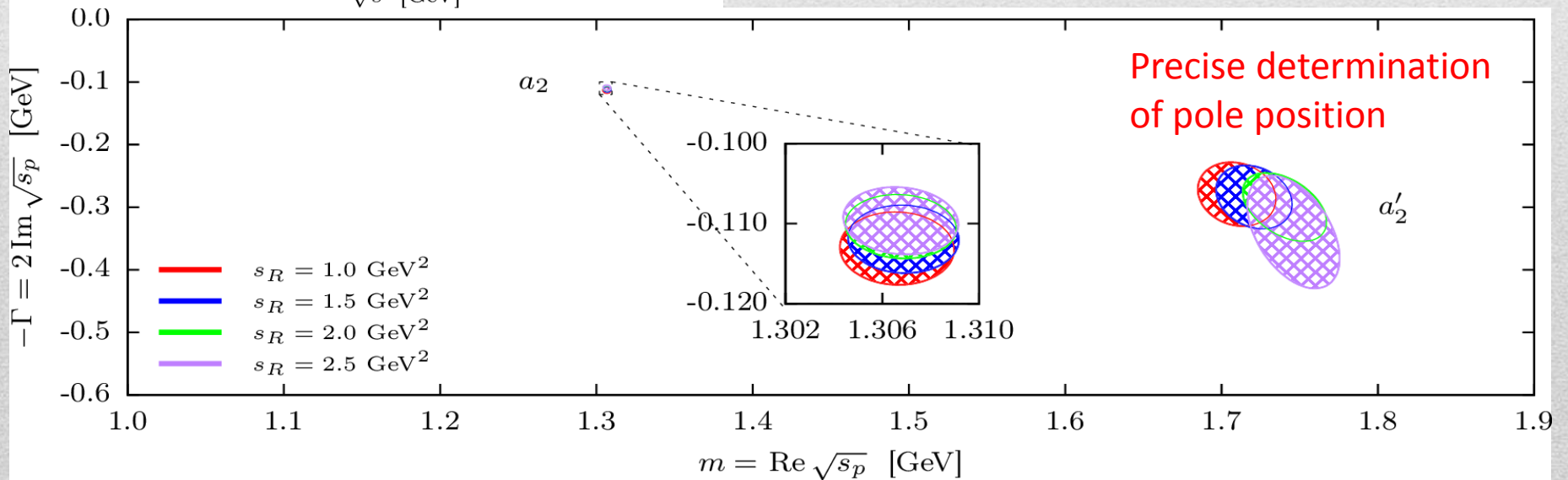
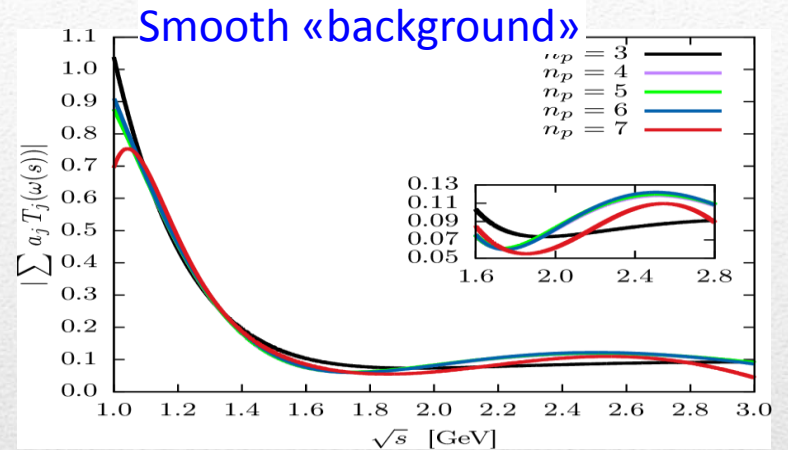
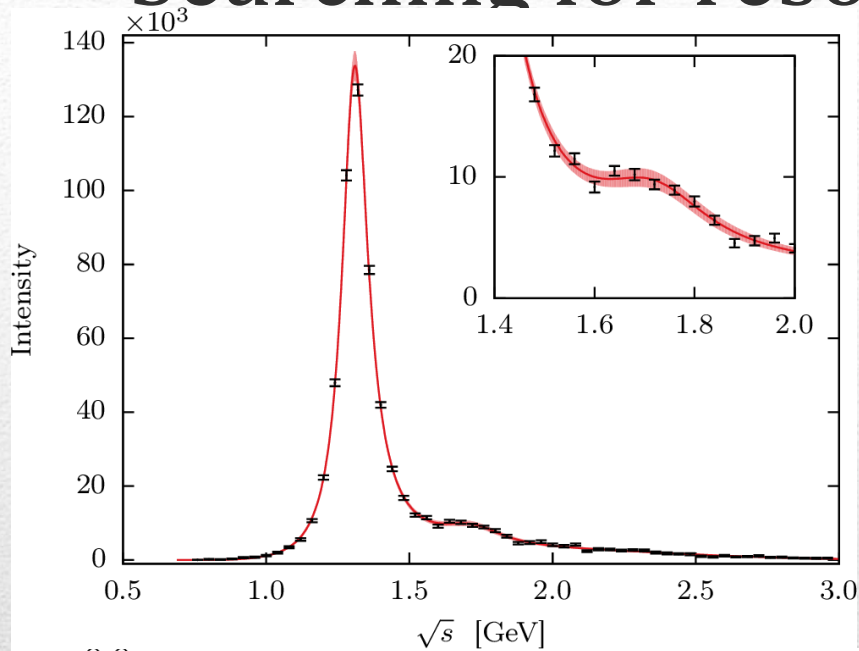
Resonant content

$$D(s) = c_0 - c_1 s - \frac{c_2}{c_3 - s} - \frac{s}{\pi} \int_{sth}^{\infty} ds' \frac{\rho(s') N(s')}{s'(s' - s)}$$

The denominator $D(s)$ contains all the Final State Interactions constrained by unitarity \rightarrow **universal**



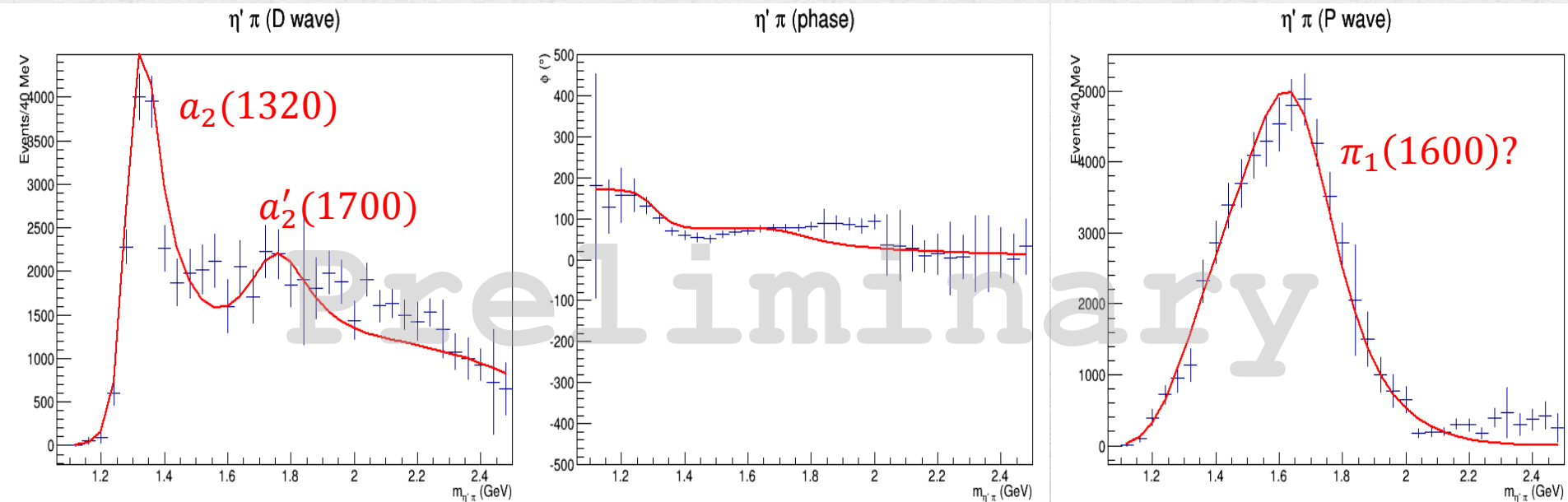
Searching for resonances in $\eta\pi$



Searching for resonances in $\eta\pi$

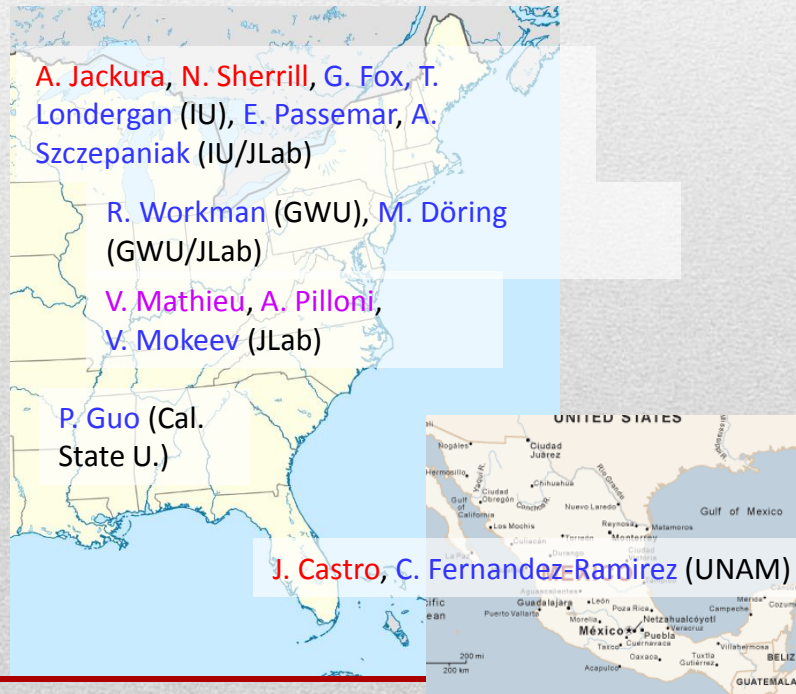
$$m(a_2) = (1307 \pm 1 \pm 6) \text{ MeV} \quad m(a'_2) = (1720 \pm 10 \pm 60) \text{ MeV}$$
$$\Gamma(a_2) = (112 \pm 1 \pm 8) \text{ MeV} \quad \Gamma(a'_2) = (280 \pm 10 \pm 70) \text{ MeV}$$

- The **coupled channel analysis** involving the **exotic P -wave** is **ongoing**, as well as the extension to the GlueX production mechanism and kinematics

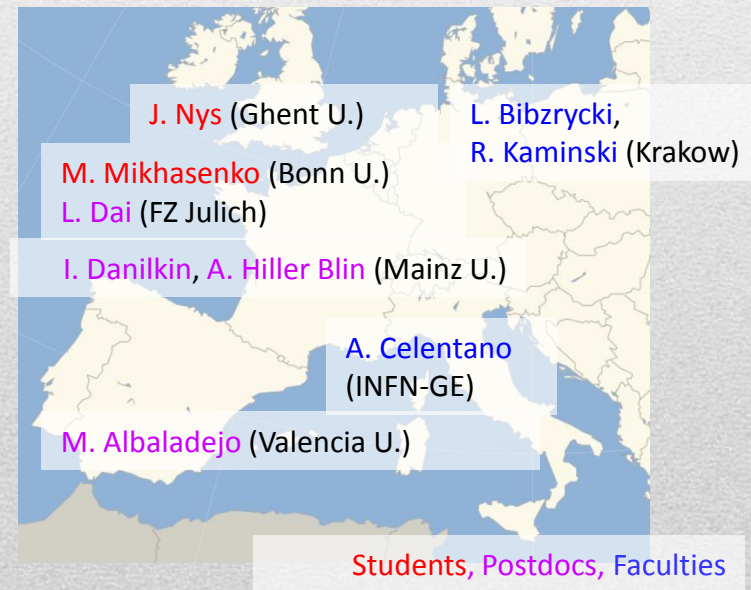


Conclusions & prospects

- We aim at developing **new theoretical tools**, to get insight on QCD using **first principles of QFT** (unitarity, analyticity, crossing symmetry, low and high energy constraints,...) to extract the physics out of the data
- Many other **ongoing projects** (both for meson and baryon spectroscopy, and for high energy observables), with a particular attention to producing complete reaction models for the **golden channels in exotic meson searches**



Joint Physics Analysis Center



BACKUP

Production

- > 40 Research Papers (Phys.Rev., Phys.Lett, Eur.J. Phys.)
- ~120 Invited Talks and Seminars
- $O(10)$ ongoing analyses
- Summer Schools on Reaction Theory (IU, 2015 and 2017)
- Workshop “Future Directions in Hadron Spectroscopy” (JLab, 2014 and UNAM 2017)

FESR	V. Mathieu <i>et al.</i> ,	arXiv:1708.07779
$\pi N \rightarrow \eta \pi N$	A. Jackura <i>et al.</i> ,	arXiv:1707.02848
$\gamma N \rightarrow \eta N$ vs. $\rightarrow \eta' N$	V. Mathieu <i>et al.</i> ,	arXiv:1704.07684
$Z_c(3900)$	A. Pilloni <i>et al.</i> ,	PLB772, 200
$\gamma N \rightarrow \eta N$	J. Nys <i>et al.</i> ,	PRD95, 034014
$\gamma p \rightarrow J/\psi p$	A. Blin <i>et al.</i> ,	PRD94, 034002
$K N \rightarrow K N$	C. Fernandez-Ramirez <i>et al.</i> ,	PRD93, 034029; PRD93, 074015
$\gamma p \rightarrow \pi^0 p$	V. Mathieu <i>et al.</i> ,	PRD92, 074013
$\pi N \rightarrow \pi N$	V. Mathieu <i>et al.</i> ,	PRD92, 074004
$\eta \rightarrow \pi^+ \pi^- \pi^0$	P. Guo <i>et al.</i> ,	PRD92, 054016; PLB771, 497
$\omega, \phi \rightarrow \pi^+ \pi^- \pi^0$	I. Danilkin <i>et al.</i> ,	PRD91, 094029
$\gamma p \rightarrow K^+ K^- p$	M. Shi <i>et al.</i> ,	PRD91, 034007

Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

<http://www.indiana.edu/~jpac/>

Joint Physics Analysis Center

HOME PROJECTS PUBLICATIONS LINKS



This project is supported by NSF

$$\pi N \rightarrow \pi N$$

Formalism

The pion-nucleon scattering is a function of 2 variables. The first is the beam momentum in the laboratory frame p_{lab} (in GeV) or the total energy squared $s = W^2$ (in GeV^2). The second is the cosine of



Resources

- Publications: [Mat15a] and [Wor12a]
- SAID partial waves: compressed zip file
- C/C++: C/C++ file
- Input file: param.txt
- Output files: output0.txt, output1.txt, SigTot.txt, Observables0.txt, Observables1.txt
- Contact person: Vincent Mathieu
- Last update: June 2016

The SAID partial waves are in the format provided online on the SAID webpage :

p_{lab} δ $\epsilon(\delta)$ $1 - \eta^2$ $\epsilon(1 - \eta^2)$ Re PW Im PW SGT SGR

δ and η are the phase-shift and the inelasticity. $\epsilon(x)$ is the error on x . SGT is the total cross section and SGR is the total reaction cross section.

Format of the input and output files: [show/hide]
Description of the C/C++ code: [show/hide]

Simulation

Range of the running variable:

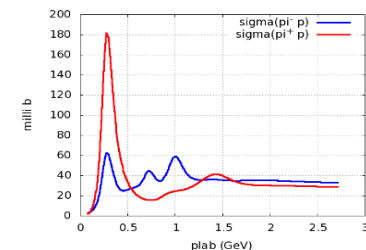
s in GeV^2 (min max step)	1,2	:	6	:	0,01
p_{lab} in GeV (min max step)	0,1	:	4	:	0,01
ν in GeV (min max step)	0,3	:	4	:	0,01
t in GeV^2 (min max step)	-1	:	0	:	0,01

The fixed variable:

t in GeV^2

p_{lab} in GeV

Results



Three-Body Unitarity

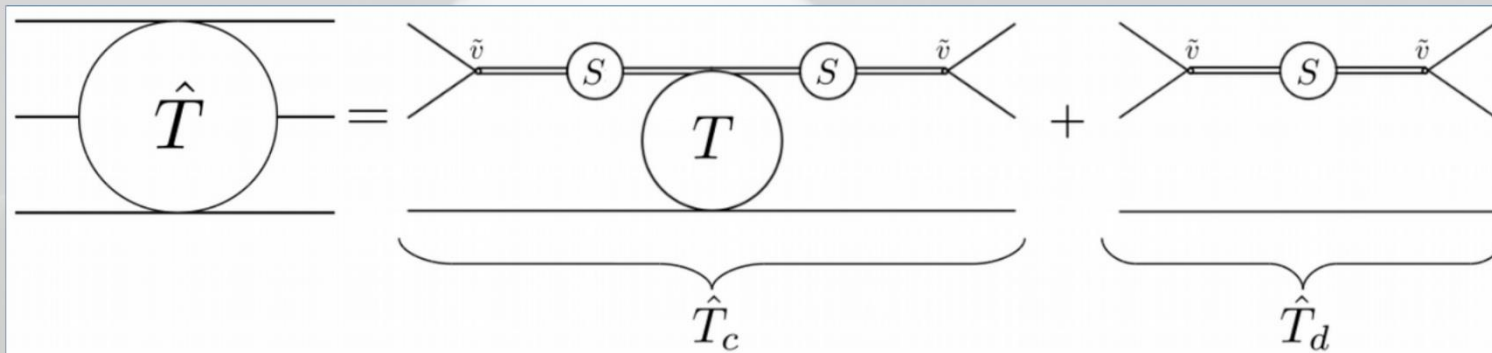
Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177

Original study by Amado/Aaron/Young

AAY(1968)

- 3-dimensional integral equation from unitarity constraint & BSE ansatz
- valid below break-up energies ($E < 3m$)
- analyticity constraints unclear

One has to begin with asymptotic states



- v a general but cut-free (in the phys. region) function
- two-body interaction is parametrized by an “isobar”

= has definite QN and correct r.h.-singularities w.r.t invariant mass

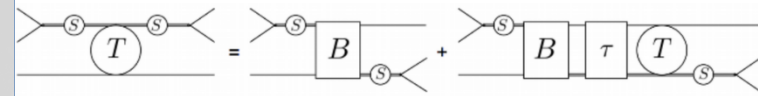
- S and T are yet unknown functions

M. Mai

Three-Body Unitarity

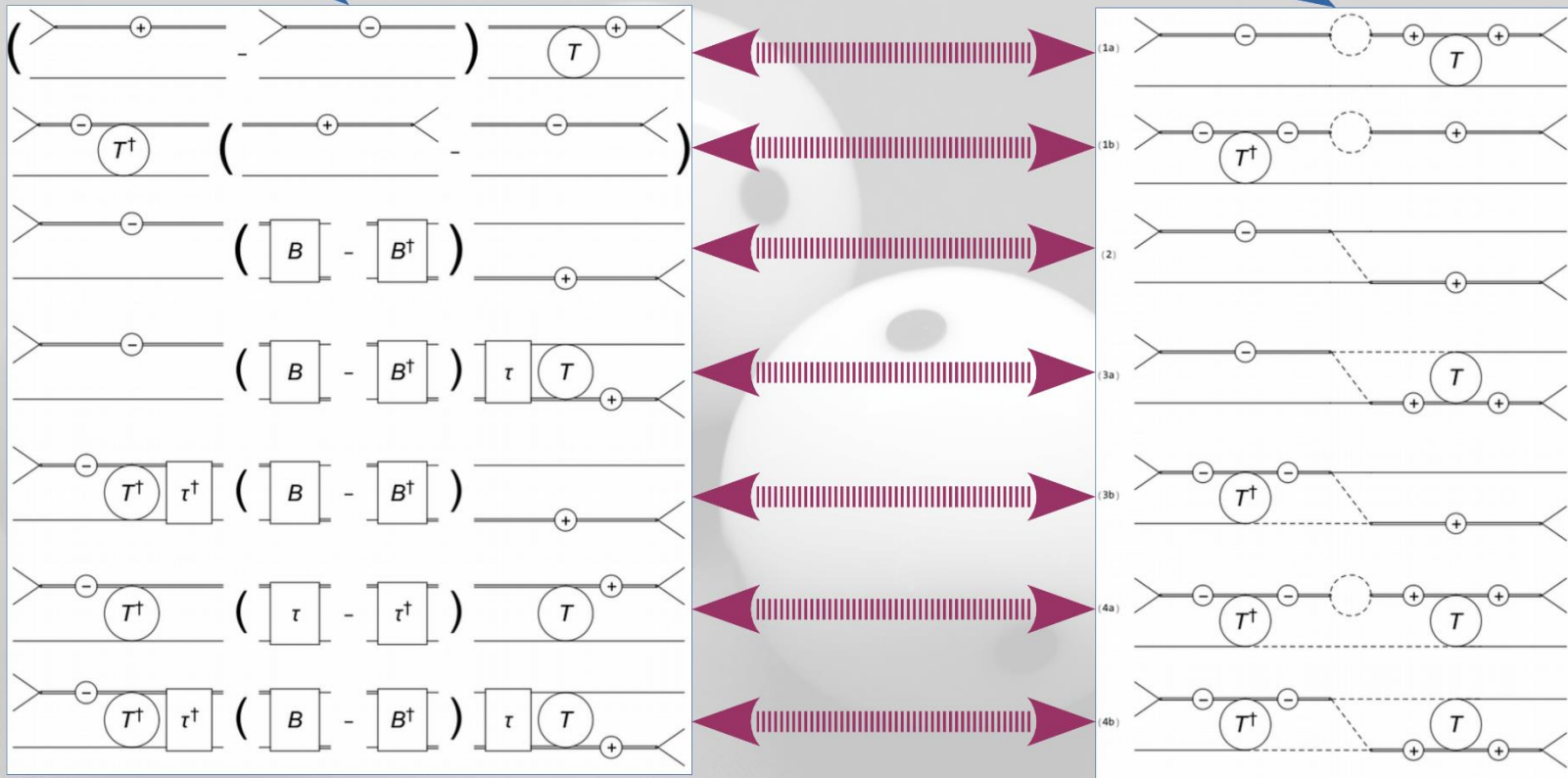
A general ansatz for the Isobar-spectator interaction

→ B & τ are unknown!!!



3-body Unitarity (normalization condition ↔ phase space integral)

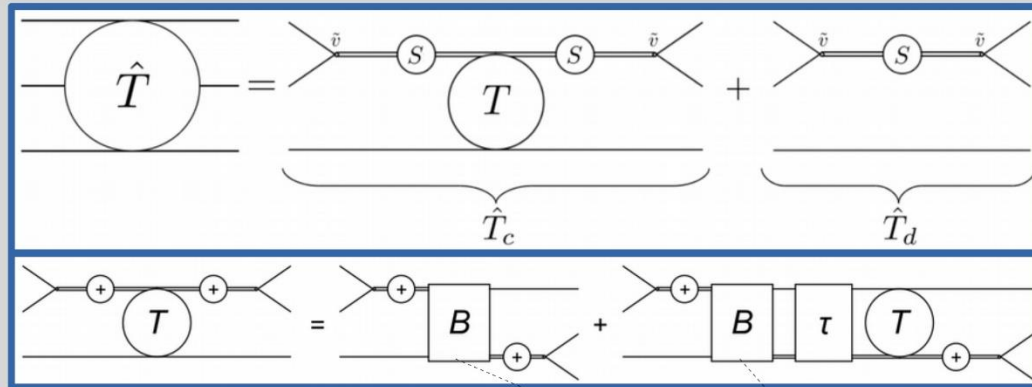
$$\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle$$



M. Mai

Three-Body Unitarity

3 → 3 scattering amplitude is a 3-dimensional integral equation



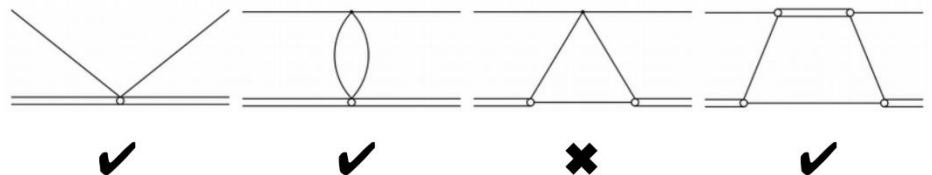
– Imaginary parts (**B**, **τ**, **S**) are fixed by **unitarity/matching**
 For simplicity $\nu=\lambda$ (full relations available)

$$\tau(\sigma(k)) = (2\pi)\delta^+(k^2 - m^2)S(\sigma(k))$$

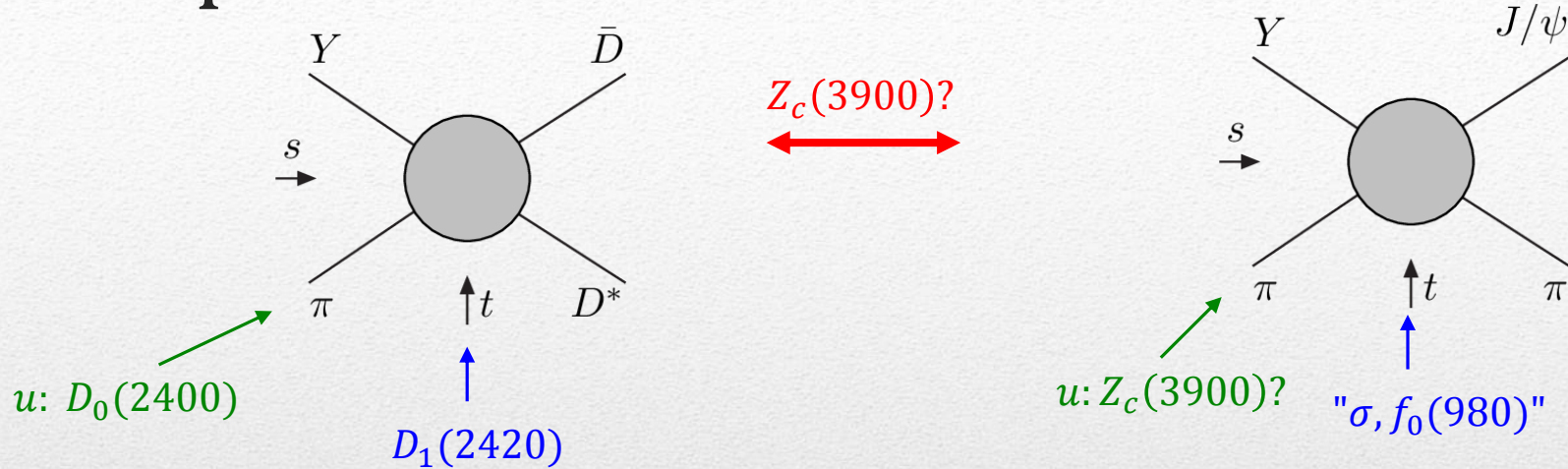
$$-\frac{1}{S(P^2)} = \sigma(k) - M_0^2 - \frac{1}{(2\pi)^3} \int d^3\ell \frac{\lambda^2}{2E_\ell(\sigma(k) - 4E_\ell^2 + i\epsilon)}$$

$$\langle q|B(s)|p \rangle = -\frac{\lambda^2}{2\sqrt{m^2+Q^2} \left(E_Q - \sqrt{m^2+Q^2} + i\epsilon \right)}$$

- un-subtracted dispersion relation
- one- π exchange in TOPT
- real contributions can be added to B



Amplitude model



$$f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1) \left(a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u) \right) \quad \text{Khuri-Treiman}$$

$$f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^1 dz_s f_i(s, t(s, z_s), u(s, z_s)) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^1 dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^1 dz_s P_l(z_s) \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s},$$

$$f_i(s, t, u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' (s' - s)} \right) \right],$$

Strategy

AP *et al.* (JPAC), arXiv:1612.06490

- We fit the following **invariant mass distributions**:
 - BESIII PRL110, 252001 $J/\psi \pi^+, J/\psi \pi^-, \pi^+ \pi^-$ at $E_{CM} = 4.26$ GeV
 - BESIII PRL110, 252001 $J/\psi \pi^0$ at $E_{CM} = 4.23, 4.26, 4.36$ GeV
 - BESIII PRD92, 092006 $\overline{D^0} D^{*+}, \overline{D^{*0}} D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
 - BESIII PRL115, 222002 $\overline{D^0} D^{*0}, \overline{D^{*0}} D^0$ at $E_{CM} = 4.23, 4.26$ GeV
 - ~~BESIII PRL112, 022001 $\overline{D^0} D^{*+}, \overline{D^{*0}} D^+$ (single tag) at $E_{CM} = 4.26$ GeV~~
 - ~~Belle PRL110, 252002 $J/\psi \pi^\pm$ at $E_{CM} = 4.26$ GeV~~
 - ~~CLEO-c data PLB727, 366 $J/\psi \pi^\pm, J/\psi \pi^0$ at $E_{CM} = 4.17$ GeV~~
- Published data are not efficiency/acceptance corrected,
→ we are **not able to give the absolute normalization** of the amplitudes
- No given dependence on E_{CM} is assumed – the couplings at different E_{CM} are independent parameters

Strategy

AP *et al.* (JPAC), PLB772, 200

- **Reducible** (incoherent) **backgrounds are pretty flat** and do not influence the analysis, except the peaking background in $\overline{D^0}D^{*0}, \overline{D^{*0}}D^0$ (subtracted)
- Some information about **angular distributions** has been published, but it's **not constraining** enough → we do not include in the fit
- Because of that, **we approximate all the particles to be scalar** – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

Pentaquark photoproduction

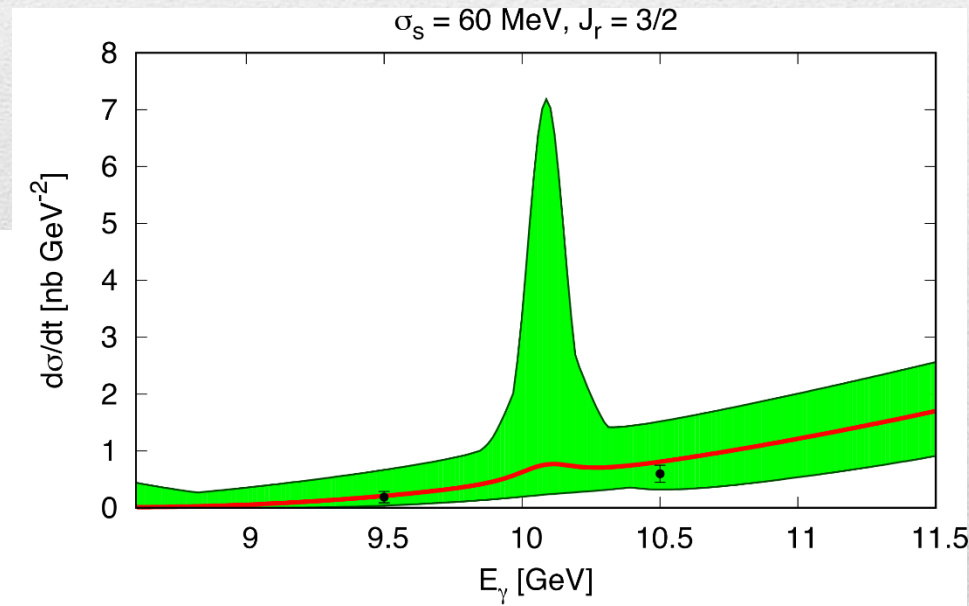
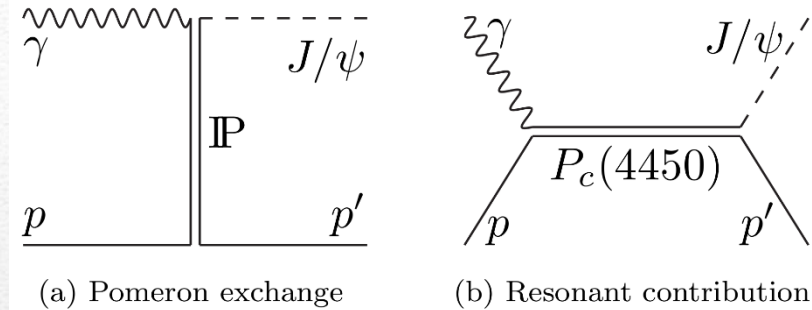
To exclude any rescattering mechanism,
we propose to search the $P_c(4450)$ state in
photoproduction

We use the (few) existing data and
VMD + pomeron inspired bkg
to estimate the cross section

GlueX data coming soon!

$$J^P = (3/2)^-$$

σ_s (MeV)	0	60	120
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$	$0.157^{+0.037}_{-0.022}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$	$1.150^{+0.015}_{-0.023}$
α' (GeV $^{-2}$)	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$	$0.111^{+0.038}_{-0.054}$
s_t (GeV 2)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$	$16.9^{+2.0}_{-1.1}$
b_0 (GeV $^{-2}$)	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$	$1.03^{+0.49}_{-0.31}$
$\mathcal{B}_{\psi p}$ (95% CL)	$\leq 29\%$	$\leq 30\%$	$\leq 23\%$



Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

Lineshapes at 4260

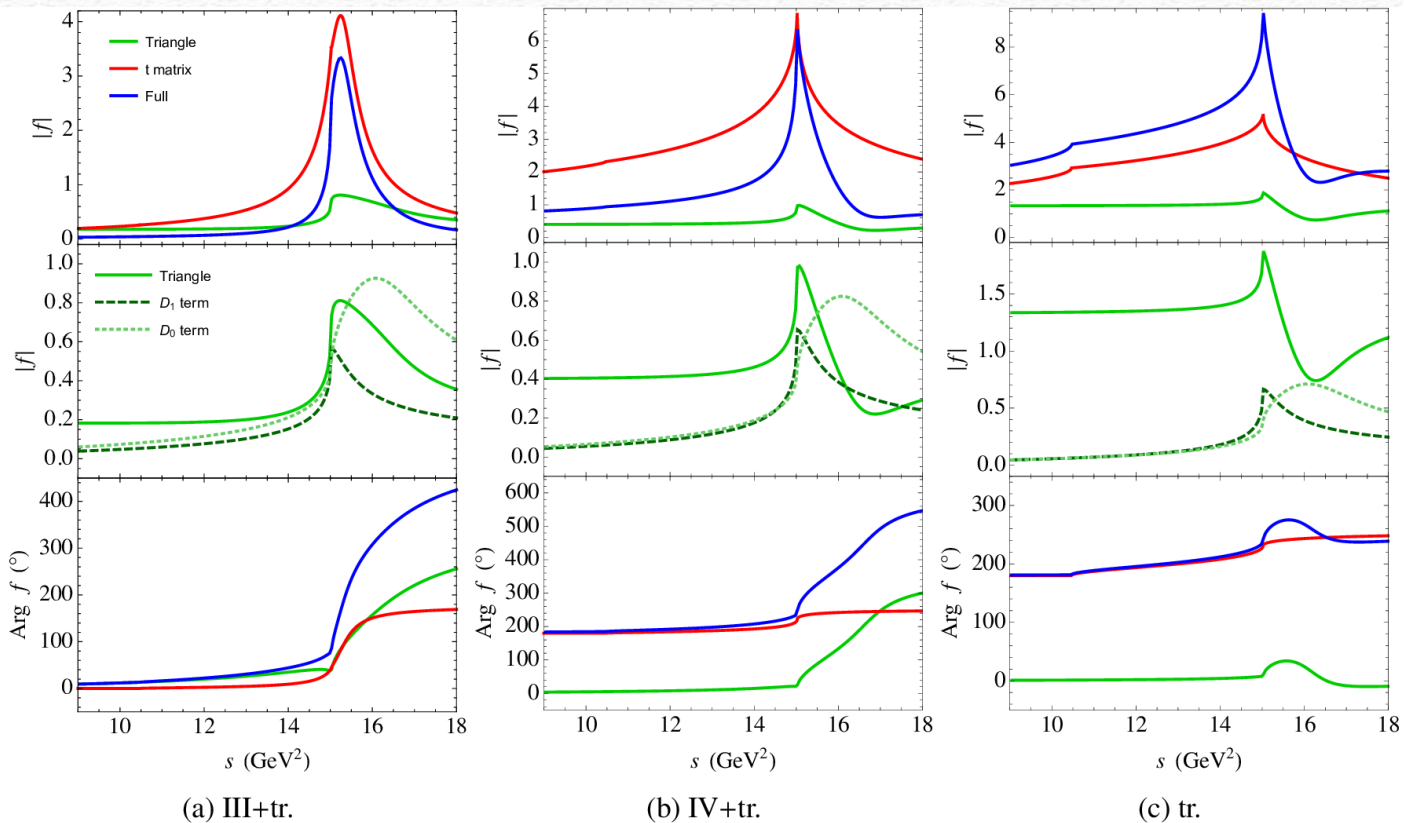


Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi\pi\pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the t_{12} scattering amplitude, the green curve is the $c_1 + H(s, D_1) + H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the D_1 (dashed) and the D_0 (dotted). Only for D_1 the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.

Lineshapes at 4230

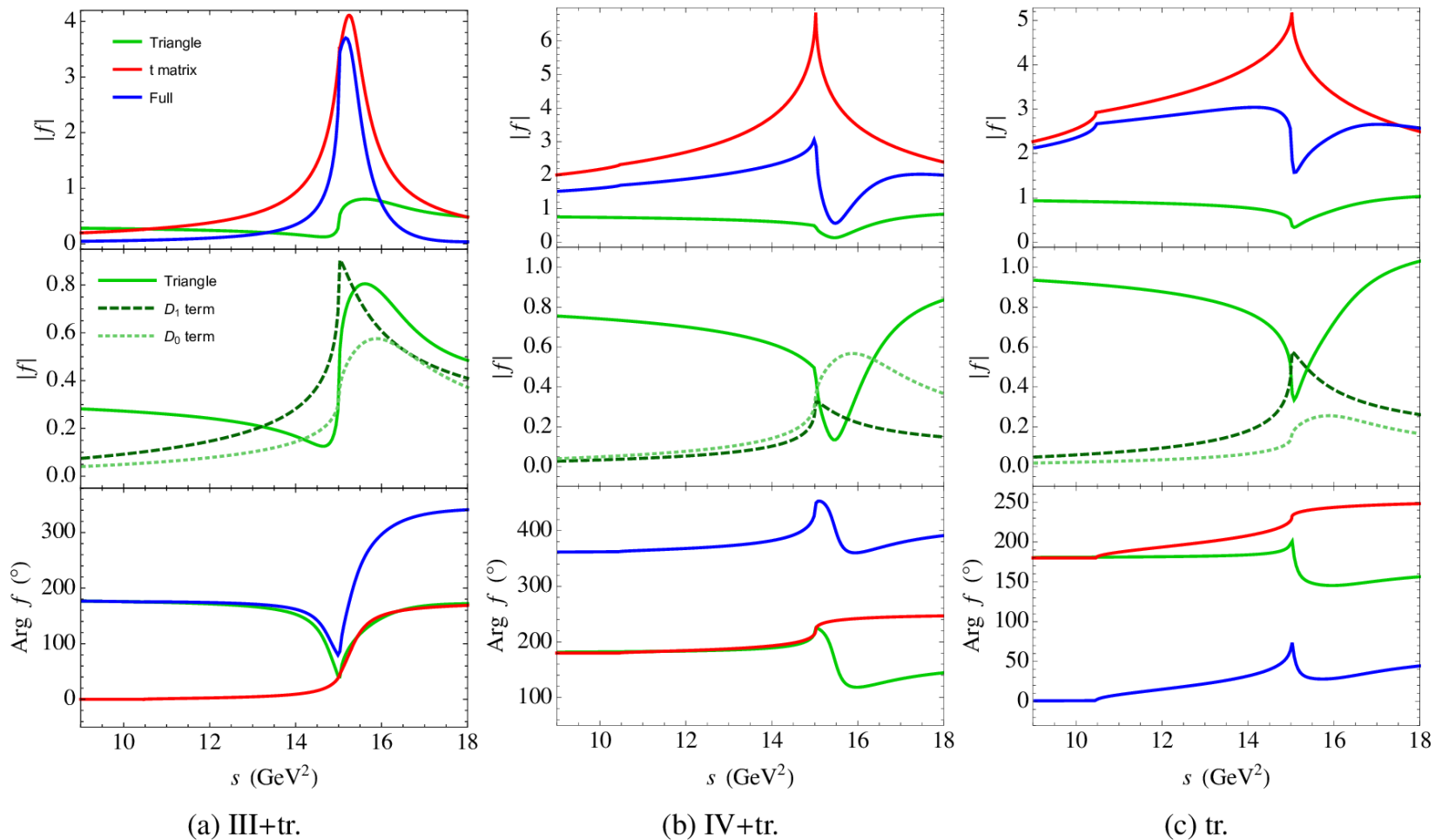
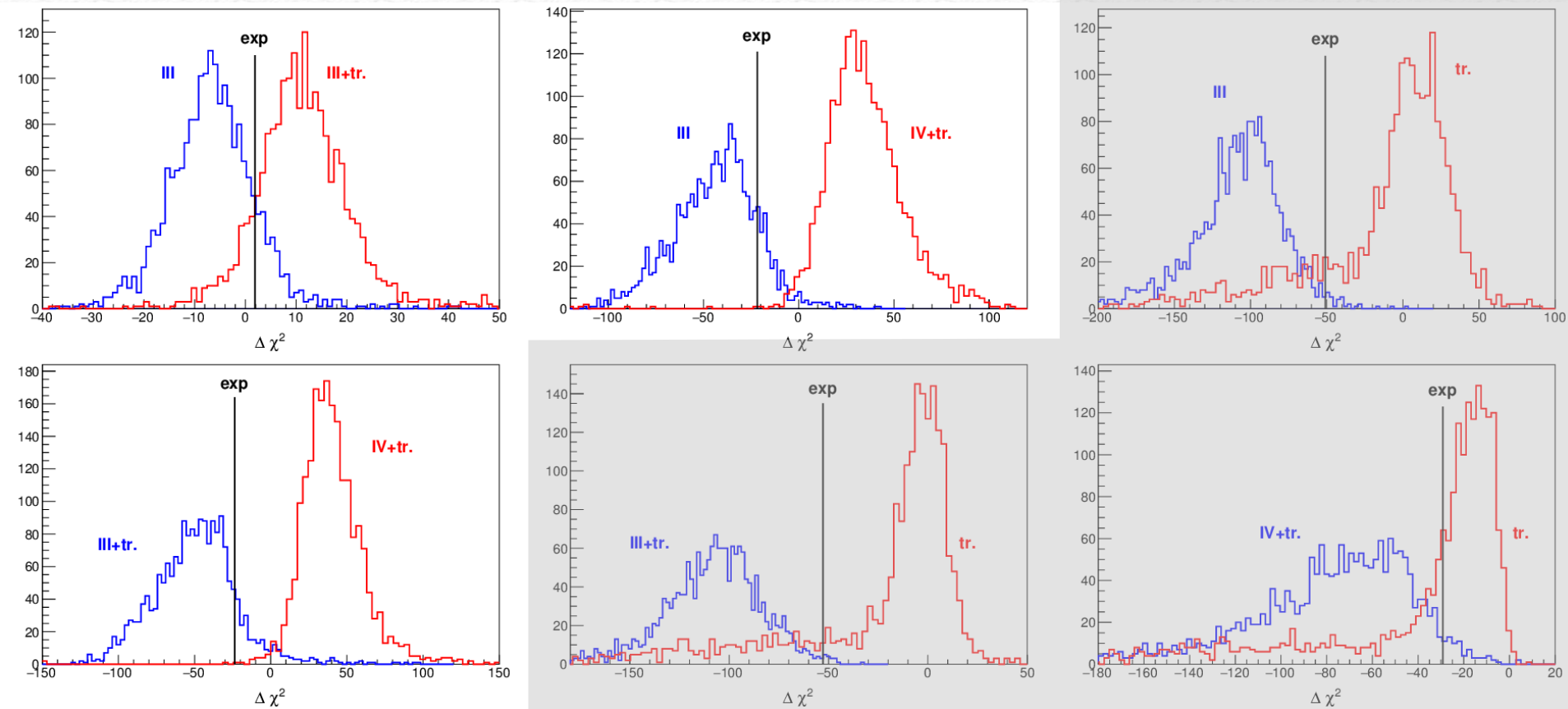


Figure 8: Same as Figure 7, but for $E_{CM} = 4.23$ GeV.

Statistical analysis



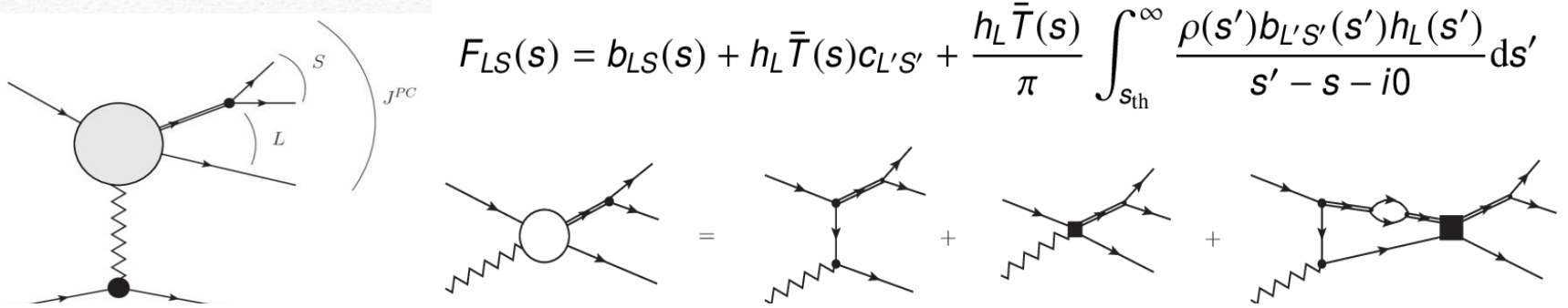
Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	" 2.4σ " (" 1.4σ ")
III+tr.	—	1.5σ (3.1σ)	" 2.6σ " (" 1.3σ ")
IV+tr.			" 2.1σ " (" 0.9σ ")

Not conclusive at this stage

PWA of 3π system

We start from 2^-+ , long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay



- The rescattering (Unitarisation) term has to be added to preserve unitarity.
- Shape of the background is fixed by projections of one-pion-exchange diagram
- Fit parameters are strengths of background for each channel, production constants c_{LS} and K-matrix parameters.

Details of one-pion-exchange amplitude calculations

- Pomeron trajectory $(s/s_0)^{\alpha(t)}$, $s_0 = 1 \text{ GeV}^2$, $\alpha(t) = 1$.
- Pion propagator is not "reggeized"
- Proton spin and structure is neglected
- Isobar decay amplitude is taken out, remaining isobar mass dependence is smeared out.

A. Jackura, M. Mikhasenko (JPAC), in progress

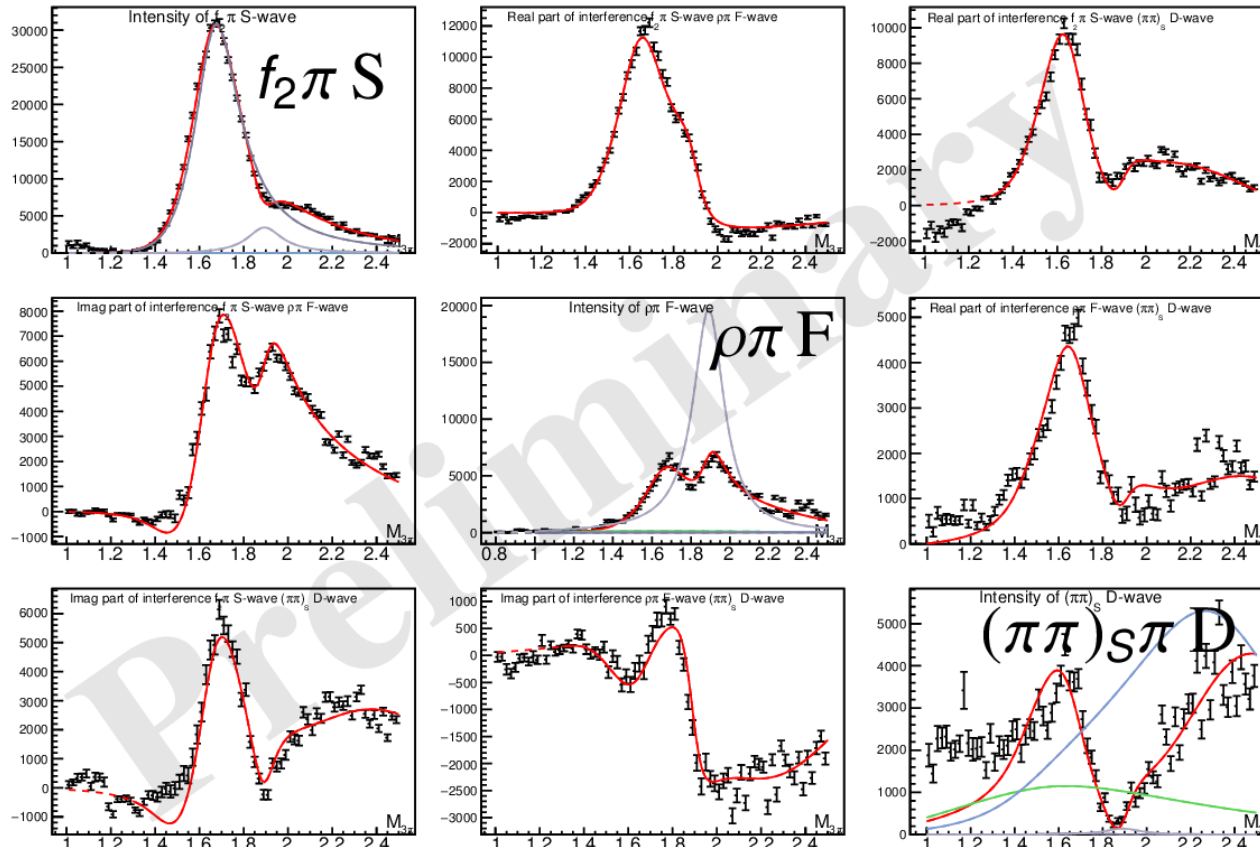
Prelimi

PWA of 3π system

Model-II, 3 waves fit

$0.12 \text{ GeV}^2 < t' < 0.26 \text{ GeV}^2$, 3 poles, unitarized background

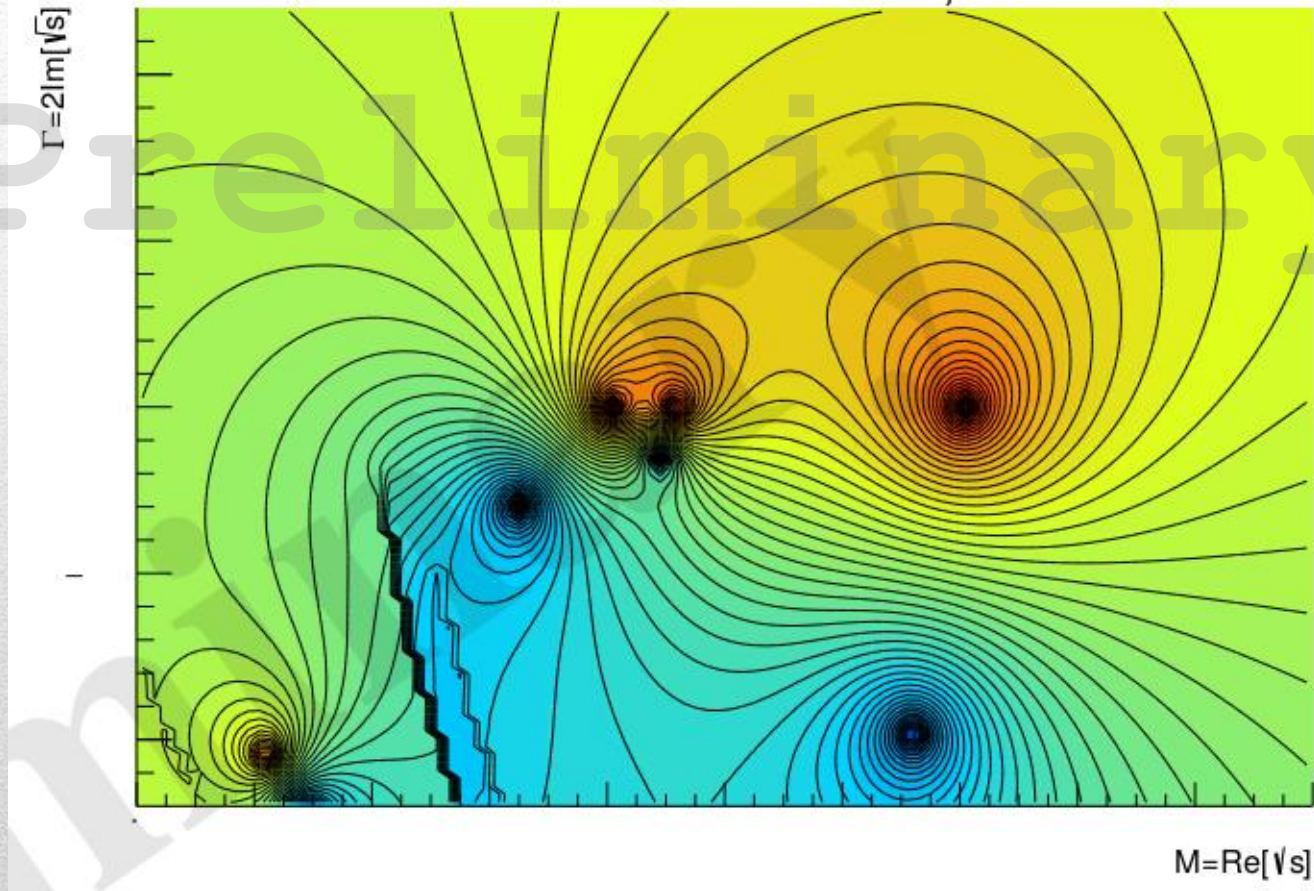
Spin-density matrix: Intensity, Real and Imaginary part of interferences.



A. Jackura

PWA of 3π system

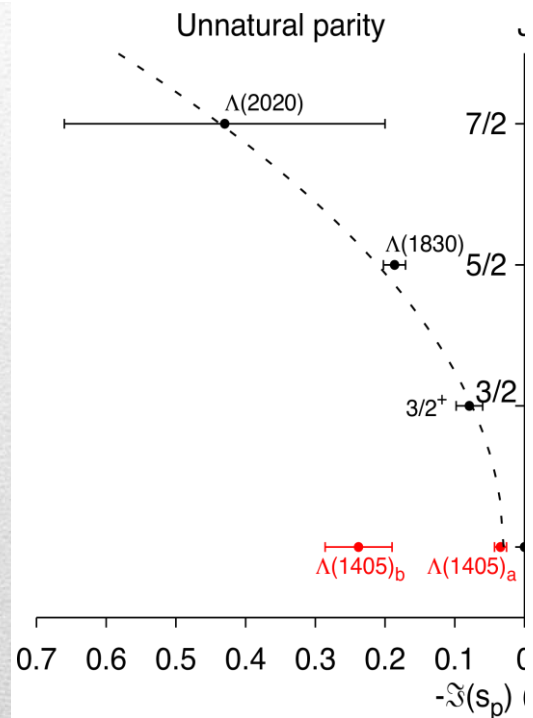
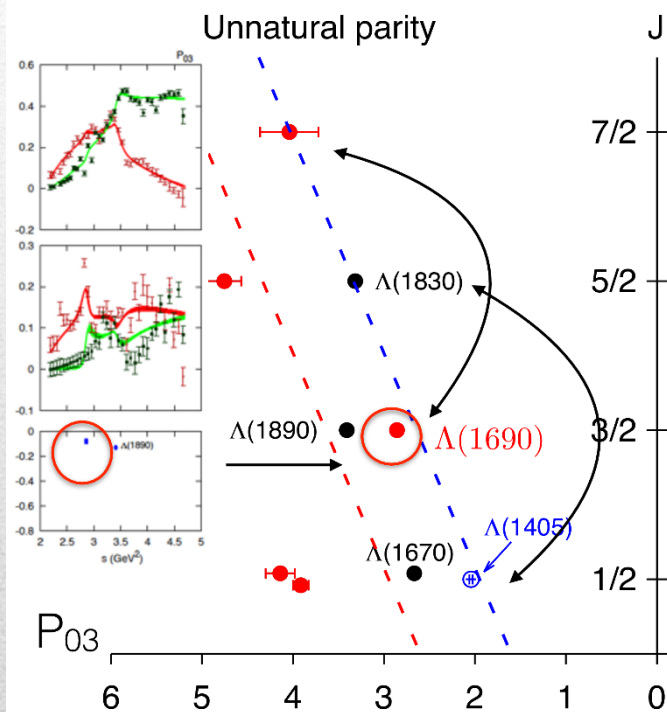
We start from 2^{-+} , long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay



A. Jackura, M. Mikhasenko (JPAC), in progress

KN scattering and the $\Lambda(1405)$

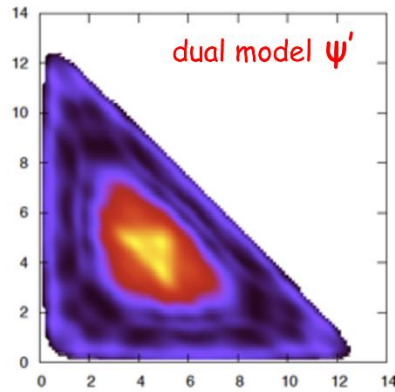
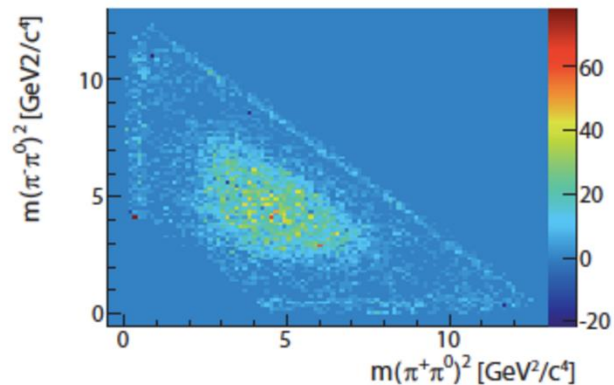
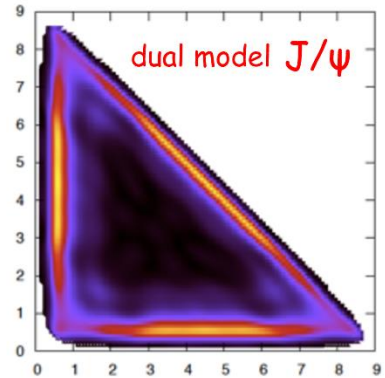
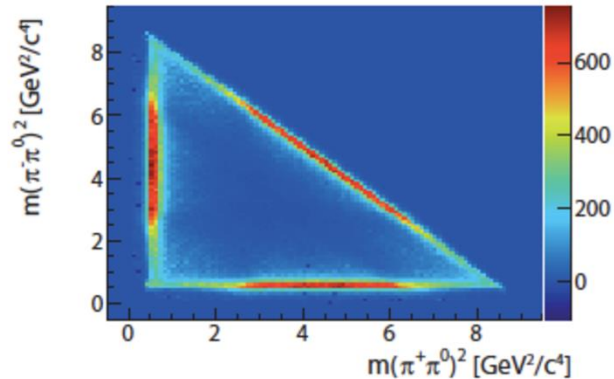
Coupled-channel K matrix model (up to 13 channels per partial wave), analyticity in angular momentum enforced, fit to KSU partial waves



One of the $\Lambda(1405)$ poles is out of the trajectory
 \rightarrow non 3-q state

Fernandez-Ramirez *et al.* (JPAC), PRD93, 034029
 Fernandez-Ramirez *et al.* (JPAC), PRD93, 074015

$\psi^{(\prime)} \rightarrow \pi^+ \pi^- \pi^0$ within dual models



$$A(s, t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s) - J(t))}$$

BESIII, Phys.Lett. B710 (2012) 594-599

Szczepaniak and Pennington, PLB737, 283