Amplitude analysis
for exotic states

Alessandro Pilloni

APS-DNP, Pittsburgh, October 25th, 2017
Interpretations on the spectrum leads to understanding fundamental laws of nature.

Data

Lattice QCD

16 20 24

16 20 24

[111] $A_1$

[200] $A_1$

Fundamental properties, Model building

Hadron Spectroscopy

Meson

Baryon

Glueball

Hybrids

Tetraquark

Molecule

Hadroquarkonium

Experiment
Hadron Spectroscopy

Data

XYZ states

Esposito, AP, Polosa, Phys. Rept. 668

Fundamental properties, Model building
Hadron Spectroscopy

Data

Fundamental properties, Model building

XYZ states

Esposito, AP, Polosa, Phys. Rept. 668
Hadron Spectroscopy

Improvement needed! With great statistics comes great responsibility!

Data

Fundamental properties, Model building

Esposito, AP, Polosa, Phys. Rept. 668
Joint Physics Analysis Center

- Joint effort between *theorists* and *experimentalists* to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in *education* of further generations of hadron physics practitioners

- Effective Field Theories
- Analyticity+Unitarity
- Dispersion Relations
- Regge Theory

- Experiments
  - CLAS, GlueX, BESIII, COMPASS, LHCb, BaBar, Belle II, KLOE, MAMI Lattice

- Insight on QCD dynamics
- Fundamental parameters
  - Resonances, exotic states
**S-Matrix principles**

These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.

**Analyticity**

\[ A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon) \]

**Crossing**

\[ A(s, t) = \sum_l A_l(s) P_l(z_s) \]

**Unitarity**

These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.
Pole hunting

I sheet

Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet

II sheet

$^{3}\text{S}$ (bound state deuteron)

(resonances (poles))
Pole hunting

More complicated structure when more thresholds arise: two sheets for each new threshold

III sheet: usual resonances
IV sheet: cusps (virtual states)

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Triangle singularity

- Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438)
- However, this effects cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)
- But the cancellation can be spread in different channels, you might still see peaks in other channels!
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.

$$e^+ e^- \rightarrow Z_c(3900)^+ \pi^- \rightarrow J/\psi \pi^+ \pi^- \text{ and } (DD^*)^+ \pi^-$$

$M = 3888.7 \pm 3.4 \text{ MeV, } \Gamma = 35 \pm 7 \text{ MeV}$

Such a state would require a minimal 4q content and would be manifestly exotic.
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities $\rightarrow$ different natures

Triangle rescattering, logarithmic branching point

(anti)bound state, II/IV sheet pole («molecule»)

Resonance, III sheet pole («compact state»)

$u$: $D_0(2400)$

$D_1(2420)$

$Z_c(3900)$?

$u$: $Z_c(3900)$?

$\sigma, f_0(980)$

$\bar{D}$

$s$ $\pi$ $t$ $D^*$

$Y$ $D$

$AP \ et\ al. \ (JPAC), \ PLB772, \ 200$

$A.\ Pilloni$ – Amplitude analysis for exotic states
Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters.

\[ f_i(s, t, u) = 16\pi \left[ a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s' (s' - s)} \right) \right], \]

The scattering matrix is parametrized as \((t^{-1})_{ij} = K_{ij} - i \rho_i \delta_{ij}\)

Four different scenarios considered:

- **III**: the K matrix is \(\frac{g_i g_j}{M^2 - s'}\), this generates a pole in the closest unphysical sheet. The rescattering integral is set to zero.
- **III+tr.**: same, but with the correct value of the rescattering integral.
- **IV+tr.**: the K matrix is constant, this generates a pole in the IV sheet.
- **tr.**: same, but the pole is pushed far away by adding a penalty in the \(\chi^2\)
Singularities and lineshapes

Different lineshapes according to different singularities

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Fit: III

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Fit: III+tr.

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\( E_{\text{CM}} = 4.26 \text{ GeV} \)

\( E_{\text{CM}} = 4.23 \text{ GeV} \)
Fit: IV+tr.

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Fit: tr.
Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test.
Pole extraction

<table>
<thead>
<tr>
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<td>–</td>
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Not conclusive at this stage
Higher energies: Regge exchange

Resonances are poles in $s$ for fixed $l$

\[ A_l \sim \frac{g_1 g_2}{s_p - s} \]

Resonance exchange

dominate low energy region

Reggeons are poles in $l$ for fixed $s$

\[ A \sim \sum s^l \sim \beta(t) s^{\alpha(t)} \]

dominate high energy region

Resonance production

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Finite energy sum rules

See J. Nys and V. Mathieu talk on friday

\[ m(\eta\pi) < 3 \text{ (GeV/c}^2)^2 \]

\[ m(\eta\pi) \in [5-6] \text{ (GeV/c}^2)^2 \]

PWA in the low energy region

Resonance extraction

PWA

Regge exchanges at high energy

Analytically connected
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial waves amplitude according to the $N/D$ method
- We test against the $D$-wave data, where the $a_2$ and the $a'_2$ show up

A. Jackura, et al. (JPAC & COMPASS), 1707.02848
see talk on friday

Resonant content

$$D(s) = c_0 - c_1 s - \frac{c_2}{c_3 - s} - \frac{s}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s') N(s')}{s'(s' - s)}$$

The denominator $D(s)$ contains all the Final State Interactions constrained by unitarity \(\rightarrow\) universal
Searching for resonances in $\eta\pi$

Precise determination of pole position

Smooth «background»
Searching for resonances in $\eta\pi$

$m(a_2) = (1307 \pm 1 \pm 6) \text{ MeV} \quad m(a_2') = (1720 \pm 10 \pm 60) \text{ MeV}$

$\Gamma(a_2) = (112 \pm 1 \pm 8) \text{ MeV} \quad \Gamma(a_2') = (280 \pm 10 \pm 70) \text{ MeV}$

- The coupled channel analysis involving the exotic $P$-wave is ongoing, as well as the extension to the GlueX production mechanism and kinematics.

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Conclusions & prospects

• We aim at developing new theoretical tools, to get insight on QCD using first principles of QFT (unitarity, analyticity, crossing symmetry, low and high energy constraints,...) to extract the physics out of the data

• Many other ongoing projects (both for meson and baryon spectroscopy, and for high energy observables), with a particular attention to producing complete reaction models for the golden channels in exotic meson searches

Joint Physics Analysis Center

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J. Nys (Ghent U.)
L. Bibzrycki, R. Kaminski (Krakow)
M. Mikhasenko (Bonn U.)
L. Dai (FZ Julich)
I. Danilkin, A. Hiller Blin (Mainz U.)
A. Celentano (INFN-GE)
M. Albaladejo (Valencia U.)

Students, Postdocs, Faculties
BACKUP
Production

• ~120 Invited Talks and Seminars
• $O(10)$ ongoing analyses
• Summer Schools on Reaction Theory (IU, 2015 and 2017)
• Workshop “Future Directions in Hadron Spectroscopy” (JLab, 2014 and UNAM 2017)

<table>
<thead>
<tr>
<th>Process</th>
<th>Authors</th>
<th>ArXiv/Journal</th>
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<tbody>
<tr>
<td>FESR</td>
<td>V. Mathieu et al.</td>
<td>arXiv:1708.07779</td>
</tr>
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<td>$\pi N \rightarrow \eta \pi N$</td>
<td>A. Jackura et al.</td>
<td>arXiv:1707.02848</td>
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<td>$\gamma N \rightarrow \eta N$ vs. $\rightarrow \eta' N$</td>
<td>V. Mathieu et al.</td>
<td>arXiv:1704.07684</td>
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<td>$Z_c(3900)$</td>
<td>A. Pilloni et al.</td>
<td>PLB772, 200</td>
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<td>PRD95, 034014</td>
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<td>$\gamma p \rightarrow J/\psi p$</td>
<td>A. Blin et al.</td>
<td>PRD94, 034002</td>
</tr>
<tr>
<td>$K N \rightarrow K N$</td>
<td>C. Fernandez-Ramirez et al.,</td>
<td>PRD93, 034029; PRD93, 074015</td>
</tr>
<tr>
<td>$\gamma p \rightarrow \pi^0 p$</td>
<td>V. Mathieu et al.</td>
<td>PRD92, 074013</td>
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<tr>
<td>$\pi N \rightarrow \pi N$</td>
<td>V. Mathieu et al.</td>
<td>PRD92, 074004</td>
</tr>
<tr>
<td>$\eta \rightarrow \pi^+ \pi^- \pi^0$</td>
<td>P. Guo et al.</td>
<td>PRD92, 054016; PLB771, 497</td>
</tr>
<tr>
<td>$\omega, \phi \rightarrow \pi^+ \pi^- \pi^0$</td>
<td>I. Danilkin et al.</td>
<td>PRD91, 094029</td>
</tr>
<tr>
<td>$\gamma p \rightarrow K^+ K^- p$</td>
<td>M. Shi et al.</td>
<td>PRD91, 034007</td>
</tr>
</tbody>
</table>
Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

http://www.indiana.edu/~jpac/
Three-Body Unitarity

Original study by Amado/Aaron/Young
- 3-dimensional integral equation from unitarity constraint & BSE ansatz
- valid below break-up energies \( E < 3m \)
- analyticity constraints unclear

One has to begin with asymptotic states

\[
\hat{T} = \hat{T}_c + \hat{T}_d
\]

- \( \nu \) a general but cut-free (in the phys. region) function
- two-body interaction is parametrized by an “isobar”

\( = \text{has definite QN and correct r.h.-singularities w.r.t invariant mass} \)

- \( S \) and \( T \) are yet unknown functions

A. Pilloni – Amplitude analysis for exotic states

M. Mai

Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177
Three-Body Unitarity

3-body Unitarity (normalization condition ↔ phase space integral)

\[
\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle
\]

A general ansatz for the Isobar-spectator interaction

→ B & τ are unknown!!!
Three-Body Unitarity

$3 \rightarrow 3$ scattering amplitude is a 3-dimensional integral equation

- Imaginary parts ($B$, $\tau$, $S$) are fixed by unitarity/matching
  For simplicity $\nu=\lambda$ (full relations available)

$$\tau(\sigma(k)) = (2\pi)\delta^+(k^2 - m^2)S(\sigma(k))$$

$$-\frac{1}{S(\hat{p}^2)} = \sigma(k) - M_0^2 - \frac{1}{(2\pi)^3} \int d^3\ell \frac{\lambda^2}{2E_\ell(\sigma(k) - 4E_\ell^2 + i\epsilon)}$$

$$\langle q|B(s)|p \rangle = -\frac{\lambda^2}{2\sqrt{m^2 + Q^2}(E_Q - \sqrt{m^2 + Q^2 + i\epsilon})}$$

- un-subtracted dispersion relation
- one-$\pi$ exchange in TOPT
- real contributions can be added to $B$

A. M. Mai
Amplitude model

\[ f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\text{max}}} (2l + 1) \left( a_{l,i}^{(s)}(s)P_l(z_s) + a_{l,i}^{(t)}(t)P_l(z_t) + a_{l,i}^{(u)}(u)P_l(z_u) \right) \]

Khuri-Treiman

\[ f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} ds f_i(s, t(s, z_s), u(s, z_u)) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^{1} dz_s \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) = a_{0,i}^{(s)} + b_{0,i}(s) \]

\[ f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s P_l(z_s) \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) = b_{l,i}(s) \quad \text{for } l > 0. \]

\[ f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_{j}(s')b_{0,j}(s')}{s' - s}, \]

\[ f_i(s, t, u) = 16\pi \left[ a_{l,i}^{(t)}(t) + a_{l,i}^{(u)}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_{j}(s')b_{0,j}(s')}{s' (s' - s)} \right) \right], \]
Strategy

- We fit the following invariant mass distributions:
  - BESIII PRL110, 252001 $J/\psi \pi^+, J/\psi \pi^-, \pi^+\pi^-$ at $E_{CM} = 4.26$ GeV
  - BESIII PRL110, 252001 $J/\psi \pi^0$ at $E_{CM} = 4.23, 4.26, 4.36$ GeV
  - BESIII PRD92, 092006 $D^0D^{*+}, D^{*0}D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL115, 222002 $\bar{D}^0D^{*0}, \bar{D}^{*0}D^0$ at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL112, 022001 $\bar{D}^0D^{*+}, \bar{D}^{*0}D^+$ (single tag) at $E_{CM} = 4.26$ GeV
  - Belle PRL110, 252002 $J/\psi \pi^\pm$ at $E_{CM} = 4.26$ GeV
  - CLEO-c data PLB727, 366 $J/\psi \pi^\pm, J/\psi \pi^0$ at $E_{CM} = 4.17$ GeV

- Published data are not efficiency/acceptance corrected, → we are not able to give the absolute normalization of the amplitudes

- No given dependence on $E_{CM}$ is assumed – the couplings at different $E_{CM}$ are independent parameters
Strategy

- **Reducible** (incoherent) backgrounds are pretty flat and do not influence the analysis, except the peaking background in $\overline{D^0}D^*$, $\overline{D^*0}D^0$ (subtracted)

- Some information about angular distributions has been published, but it’s not constraining enough → we do not include in the fit

- Because of that, we approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters
To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction. We use the (few) existing data and VMD + pomeron inspired bkg to estimate the cross section.

GlueX data coming soon!

$J^P = (3/2)^-$

<table>
<thead>
<tr>
<th>$\sigma_s$ (MeV)</th>
<th>0</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>0.156$^{+0.029}_{-0.020}$</td>
<td>0.157$^{+0.039}_{-0.021}$</td>
<td>0.157$^{+0.037}_{-0.022}$</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>1.151$^{+0.018}_{-0.020}$</td>
<td>1.150$^{+0.018}_{-0.026}$</td>
<td>1.150$^{+0.015}_{-0.023}$</td>
</tr>
<tr>
<td>$\alpha'$ (GeV$^{-2}$)</td>
<td>0.112$^{+0.033}_{-0.054}$</td>
<td>0.111$^{+0.037}_{-0.064}$</td>
<td>0.111$^{+0.038}_{-0.054}$</td>
</tr>
<tr>
<td>$s_t$ (GeV$^2$)</td>
<td>16.8$^{+1.7}_{-0.9}$</td>
<td>16.9$^{+2.0}_{-1.6}$</td>
<td>16.9$^{+2.0}_{-1.1}$</td>
</tr>
<tr>
<td>$b_0$ (GeV$^{-2}$)</td>
<td>1.01$^{+0.47}_{-0.29}$</td>
<td>1.02$^{+0.61}_{-0.32}$</td>
<td>1.03$^{+0.49}_{-0.31}$</td>
</tr>
<tr>
<td>$B_{\psi p}$ (95% CL)</td>
<td>$\leq 29 %$</td>
<td>$\leq 30 %$</td>
<td>$\leq 23 %$</td>
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Hiller Blin, AP et al. (JPAC), PRD94, 034002
Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi \pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the $t_{12}$ scattering amplitude, the green curve is the $c_1 + H(s, D_1) + H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the $D_1$ (dashed) and the $D_0$ (dotted). Only for $D_1$ the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.
Lineshapes at 4230

Figure 8: Same as Figure 7, but for $E_{CM} = 4.23$ GeV.
Statistical analysis

Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

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PWA of $3\pi$ system

We start from $2^{-+}$, long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay

$$F_{LS}(s) = b_{LS}(s) + h_L \tilde{T}(s)c_{L'S'} + \frac{h_L \tilde{T}(s)}{\pi} \int_{s_{th}}^{\infty} \frac{\rho(s')b_{L'S'}(s')h_L(s')}{s' - s - i0} ds'$$

- The rescattering (Unitarisation) term has to be added to preserve unitarity.
- Shape of the background is fixed by projections of one-pion-exchange diagram
- Fit parameters are strengths of background for each channel, production constants $c_{LS}$ and $K$-matrix parameters.

### Details of one-pion-exchange amplitude calculations

- Pomeron trajectory $(s/s_0)^{\alpha(t)}$, $s_0 = 1$ GeV$^2$, $\alpha(t) = 1$.  
- Pion propagator is not "reggeized"
- Proton spin and structure is neglected
- Isobar decay amplitude is taken out, remaining isobar mass dependence is smeared out.

A. Jackura, M. Mikhasenko (JPAC), in progress

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PWA of $3\pi$ system

Model-II, 3 waves fit
$0.12 \text{ GeV}^2 < t' < 0.26 \text{ GeV}^2$, 3 poles, unitarized background

Spin-density matrix: Intensity, Real and Imaginary part of interferences.
PWA of $3\pi$ system

We start from $2^{-+}$, long standing puzzle about $\pi_2 (1670) - \pi_2 (1880)$ interplay

A. Jackura, M. Mikhasenko (JPAC), in progress
$KN$ scattering and the $\Lambda(1405)$

Coupled-channel $K$ matrix model (up to 13 channels per partial wave), analyticity in angular momentum enforced, fit to KSU partial waves

One of the $\Lambda(1405)$ poles is out of the trajectory → non 3-q state

Fernandez-Ramirez et al. (JPAC), PRD93, 034029
Fernandez-Ramirez et al. (JPAC), PRD93, 074015

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\[ \psi^{(i)} \to \pi^+ \pi^- \pi^0 \] within dual models

\[ A(s, t) = \frac{\Gamma(-J(s)) \Gamma(-J(t))}{\Gamma(-J(s) - J(t))} \]

Szczepaniak and Pennington, PLB737, 283