Why do we care about hadron spectroscopy?

• Because it allows us to understand how the QCD degrees of freedom manifest in nature. The role of models is crucial.

• Because we need a better understanding of hadron amplitudes if we want to reduce the «hadronic uncertainties» in precision physics (e.g. $\tau$ EDM, $g_\mu - 2$, CPV in hadronic $B$ decays...)

• (the honest answer would be «because we are nerds and we like it», but we cannot reply like this to funding agencies)
Outline

• The exotic landscape

• Amplitude analysis
  • The $S$-matrix principles
  • Case study for the $Z_c(3900)$
  • Three-body unitarity
  • The $Y$ states

• Modeling
  • Diquark-antidiquark & Molecules
  • Production at colliders
Hadron Spectroscopy

Meson  Baryon  Glueball

Hybrids  Tetraquark

Molecule

Data

Amplitude analysis

Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

Experiment

Lattice QCD
# Hadron Spectroscopy

\[ \rho(770) \quad I^G(J^{PC}) = 1^+(1^-^-) \]

**Review:** The \( \rho(770) \)

<table>
<thead>
<tr>
<th>( \rho(770) ) MASS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NEUTRAL ONLY, ( e^+ e^- )</td>
<td>775.26 ( \pm ) 0.25 MeV</td>
</tr>
<tr>
<td>CHARGED ONLY, ( \tau ) DECAYS and ( e^+ e^- )</td>
<td>775.11 ( \pm ) 0.34 MeV</td>
</tr>
<tr>
<td>MIXED CHARGES, OTHER REACTIONS</td>
<td>763.0 ( \pm ) 1.2 MeV</td>
</tr>
<tr>
<td>Mass ( m )</td>
<td></td>
</tr>
<tr>
<td>CHARGED ONLY, HADROPRODUCED</td>
<td>766.5 ( \pm ) 1.1 MeV</td>
</tr>
<tr>
<td>NEUTRAL ONLY, PHOTOPRODUCED</td>
<td>769.0 ( \pm ) 1.0 MeV</td>
</tr>
<tr>
<td>NEUTRAL ONLY, OTHER REACTIONS</td>
<td>769.0 ( \pm ) 0.9 MeV (( S = 1.4 ))</td>
</tr>
<tr>
<td>( m_{\rho(770)^0} - m_{\rho(770)^\pm} )</td>
<td>( -0.7 \pm 0.8 ) MeV (( S = 1.5 ))</td>
</tr>
<tr>
<td>( m_{\rho(770)^+} - m_{\rho(770)^-} )</td>
<td></td>
</tr>
</tbody>
</table>

| \( \rho(770) \) RANGE PARAMETER | 5.3\( ^{+0.9}_{-0.7} \) GeV\(^{-1} \) |

<table>
<thead>
<tr>
<th>( \rho(770) ) WIDTH</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NEUTRAL ONLY, ( e^+ e^- )</td>
<td>147.8 ( \pm ) 0.9 MeV (( S = 2.0 ))</td>
</tr>
<tr>
<td>CHARGED ONLY, ( \tau ) DECAYS and ( e^+ e^- )</td>
<td>149.1 ( \pm ) 0.8 MeV</td>
</tr>
<tr>
<td>MIXED CHARGES, OTHER REACTIONS</td>
<td>149.5 ( \pm ) 1.3 MeV</td>
</tr>
<tr>
<td>CHARGED ONLY, HADROPRODUCED</td>
<td>150.2 ( \pm ) 2.4 MeV</td>
</tr>
<tr>
<td>NEUTRAL ONLY, PHOTOPRODUCED</td>
<td>151.7 ( \pm ) 2.6 MeV</td>
</tr>
<tr>
<td>NEUTRAL ONLY, OTHER REACTIONS</td>
<td>150.9 ( \pm ) 1.7 MeV (( S = 1.1 ))</td>
</tr>
<tr>
<td>( \Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm} )</td>
<td>0.3 ( \pm ) 1.3 (( S = 1.4 ))</td>
</tr>
<tr>
<td>( \Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-} )</td>
<td>1.8 ( \pm ) 2.1</td>
</tr>
</tbody>
</table>
Hadron Spectroscopy

<table>
<thead>
<tr>
<th>$a_1(1260)$ Width</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VALUE (MeV)</strong></td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>250 to 600</td>
</tr>
<tr>
<td>367 ±9 +28 -25</td>
</tr>
<tr>
<td>410 ±31 ±30</td>
</tr>
<tr>
<td>520 - 680</td>
</tr>
<tr>
<td>480 ±20</td>
</tr>
<tr>
<td>580 ±41</td>
</tr>
<tr>
<td>460 ±85</td>
</tr>
<tr>
<td>814 ±36 ±13</td>
</tr>
</tbody>
</table>

A. Pilloni – The Quest for Exotic States
Hadron Spectroscopy

Data → Amplitude analysis → Fundamental properties, Model building

A. Pilloni – The Quest for Exotic States
Hadron Spectroscopy

Data

Fundamental properties, Model building

Improvement needed! With great statistics comes great responsibility!

Esposito, AP, Polosa, Phys. Rept. 668

A. Pilloni – The Quest for Exotic States
Hadron Spectroscopy

- Meson
- Baryon
- Glueball

Hybrids
Tetraquark

Molecule

Hadroquarkonium

Data
Amplitude analysis
Properties, Model building

Experiment
Lattice QCD
Quarkonium orthodoxy

\[ \alpha_s(M_Q) \sim 0.3 \]
(perturbative regime)

OZI-rule, QCD multipole

Heavy quark spin flip suppressed by quark mass,
approximate heavy quark spin symmetry (HQSS)

Potential models
(meaningful when \( M_Q \to \infty \))

\[
V(r) = -\frac{C_F \alpha_s}{r} + \sigma r
\]  
(Cornell potential)

Solve NR Schrödinger eq. \( \rightarrow \) spectrum

Effective theories
(HQET, NRQCD, pNRQCD...)

Integrate out heavy DOF
\( \downarrow \)
(spectrum), decay & production rates
Multiscale system

Systematically integrate out the heavy scale, $m_Q \gg \Lambda_{QCD}$

$m_Q \gg m_Q v \gg m_Q v^2$

Full QCD $\rightarrow$ NRQCD $\rightarrow$ pNRQCD

$m_b \sim 5$ GeV, $m_c \sim 1.5$ GeV

$v_b^2 \sim 0.1, v_c^2 \sim 0.3$

Factorization (to be proved) of universal LDMEs

Good description of many production channels, some known puzzles (polarizations)
A host of unexpected resonances have appeared decaying mostly into charmonium + light

Hardly reconciled with usual charmonium interpretation
**X(3872)**

- Discovered in $B \rightarrow K X \rightarrow K J/\psi \pi\pi$
- Quantum numbers $1^{++}$
- Very close to $DD^*$ threshold
- Too narrow for an above-threshold charmonium
- Isospin violation too big $\frac{\Gamma(X \rightarrow J/\psi \omega)}{\Gamma(X \rightarrow J/\psi \rho)} \sim 0.8 \pm 0.3$
- Mass prediction not compatible with $\chi_{c1}(2P)$

\[
M = 3871.68 \pm 0.17 \text{ MeV} \\
M_X - M_{DD^*} = -3 \pm 192 \text{ keV} \\
\Gamma < 1.2 \text{ MeV @90\%}
\]
$X(3872)$

Large prompt production at hadron colliders

$\sigma_B / \sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$

$\sigma_{PR} \times B(X \rightarrow J/\psi \pi \pi) = (1.06 \pm 0.11 \pm 0.15) \text{ nb}$

CMS, JHEP 1304, 154
Vector $Y$ states

Lots of unexpected $j^{PC} = 1^{--}$ states found in ISR/direct production (and nowhere else!) Seen in few final states, mostly $J/\psi \pi\pi$ and $\psi(2S) \pi\pi$

Not seen decaying into open charm pairs Large HQSS violation
New BESIII data show a peculiar lineshape for the $Y(4260)$

The state appear lighter and narrower, compatible with the ones in $h_c\pi\pi$ and $\chi_{c0}\omega$

A broader old-fashioned $Y(4260)$ is appearing in $D\bar{D}^*\pi$, maybe indicating a $\bar{D}D_1$ dominance

A. Pilloni – Amplitude analysis for exotic states
Charged Z states: $Z_c(3900), Z'_c(4020)$

Charged quarkonium-like resonances have been found, **4q needed**

Two states $J^{PC} = 1^{+-}$ appear slightly above $D(\ast)D^\ast$ thresholds

$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \pi^+\pi^-$ and $\rightarrow (DD^\ast)^+\pi^-$

$M = 3888.7 \pm 3.4$ MeV, $\Gamma = 35 \pm 7$ MeV

$e^+e^- \rightarrow Z'_c(4020)^+\pi^- \rightarrow h_c \pi^+\pi^-$ and $\rightarrow \bar{D}^*0D^*+\pi^-$

$M = 4023.9 \pm 2.4$ MeV, $\Gamma = 10 \pm 6$ MeV

---

A. Pilloni – The Quest for Exotic States
Interpretations on the spectrum leads to understanding fundamental laws of nature.
\[ f_l(E) = \lim_{\epsilon \to 0} f_l(E + i\epsilon) \]

These are constraints the amplitudes have to satisfy, but do not fix the dynamics.

Resonances (QCD states) are poles in the unphysical Riemann sheets.
Pole hunting

Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet
Pole hunting

More complicated structure when more thresholds arise: two sheets for each new threshold

III sheet: usual resonances
IV sheet: cusps (virtual states)

A. Pilloni – The Quest for Exotic States
The isobar model

Khuri-Treiman formalism was introduced to describe $K \rightarrow 3\pi$
Khuri and Treiman, PR119, 1115

Used recently for several reactions,
Niecknig and Kubis, JHEP 10, 142
Colangelo, et al., PRL118, 022001
AP et al. [JPAC], PLB772, 200
Albaladejo, AP et al. [JPAC], 1803.06027

The formalism implements the all-order rescattering in all the 3 channels at once
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.

$$e^+e^- \rightarrow Z_c(3900)^+\pi^- \rightarrow J/\psi \pi^+\pi^- \text{ and } \rightarrow (DD^*)^+\pi^-$$

$$M = 3888.7 \pm 3.4 \text{ MeV}, \Gamma = 35 \pm 7 \text{ MeV}$$

Such a state would require a minimal 4q content and would be manifestly exotic.
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to different singularities $\rightarrow$ different natures

\[ u: D_0(2400) \quad \text{and} \quad D_1(2420) \]

Triangle rescattering, logarithmic branching point

\[ Tornqvist, \ Z.\ Phys. \ C61, \ 525 \]
\[ Swanson, \ Phys.\ Rept. \ 429 \]
\[ Hanhart \ et\ al. \ PRL111, \ 132003 \]

\( (\text{anti})\text{bound state, II/IV sheet pole} \)
\( (\text{«molecule»}) \)

\[ D_1(2420) \quad Z_c(3900)? \]

\[ Y \quad \bar{D} \quad \pi \quad D^* \]

\[ u: Z_c(3900)? \quad "\sigma, f_0(980)" \]

Resonance, III sheet pole
\( (\text{«compact state»}) \)

\[ Maiani \ et\ al., \ PRD71, \ 014028 \]
\[ Faccini \ et\ al., \ PRD87, \ 111102 \]
\[ Esposito \ et\ al., \ Phys.\ Rept. \ 668 \]

A. Pilloni – The Quest for Exotic States
Amplitude model

\[ f_i(s, t, u) = 16\pi \sum_{l=0}^{L_{\text{max}}} (2l + 1) \left( a_{l,i}^{(s)}(s) P_l(z_s) + a_{l,i}^{(t)}(t) P_l(z_t) + a_{l,i}^{(u)}(u) P_l(z_u) \right) \]

\[ f_{0,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s f_i(s, t(s, z_s), u(s, z_s)) = a_{0,i}^{(s)} + \frac{1}{32\pi} \int_{-1}^{1} dz_s \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) = a_{0,i}^{(s)} + b_{0,i}(s) \]

\[ f_{l,i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s P_l(z_s) \left( a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) \right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s) \frac{1}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s' - s}, \]

\[ f_i(s, t, u) = 16\pi \left[ a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'(s' - s)} \right) \right], \]
Triangle singularity

Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438). However, this effect cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)

\[ f_{0,i}(s) = b_{0,i}(s) + \frac{t_{ij}}{\pi} \int_{s_i}^{s} ds' \frac{\rho_j(s')b_{0,j}(s')}{s' - s} \]

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!

Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo, Meissner, Wang, Yang PRD92, 071502
Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters.

\[
f_i(s, t, u) = 16\pi \left[ a^{(t)}_{0,i}(t) + a^{(u)}_{0,i}(u) + \sum_j t_{i,j}(s) \left( c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'(s'-s)} \right) \right],
\]

The scattering matrix is parametrized as \((t^{-1})_{ij} = K_{ij} - i \rho_i \delta_{ij}\)

Four different scenarios considered:

- «III»: the K matrix is \(\frac{g_i g_j}{M^2 - s}\), this generates a pole in the closest unphysical sheet. The rescattering integral is set to zero.
- «III+tr.»: same, but with the correct value of the rescattering integral.
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet.
- «tr.»: same, but the pole is pushed far away by adding a penalty in the \(\chi^2\)
Singularities and lineshapes

Different lineshapes according to different singularities

A. Pilloni – The Quest for Exotic States
Fit: III

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Fit: $III+tr.$

$E_{CM} = 4.26$ GeV

$E_{CM} = 4.23$ GeV

$E_{CM} = 4.26$ GeV

$E_{CM} = 4.23$ GeV

$E_{CM} = 4.26$ GeV

$E_{CM} = 4.23$ GeV
Fit: IV+tr.

$E_{CM} = 4.26$ GeV

$E_{CM} = 4.23$ GeV

A. Pilloni – The Quest for Exotic States
Fit: tr.
Pole extraction

<table>
<thead>
<tr>
<th>Scenario</th>
<th>III+tr.</th>
<th>IV+tr.</th>
<th>tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>$1.5\sigma$ ($1.5\sigma$)</td>
<td>$1.5\sigma$ ($2.7\sigma$)</td>
<td>“2.4\sigma” (“1.4\sigma”)</td>
</tr>
<tr>
<td>III+tr.</td>
<td>–</td>
<td>$1.5\sigma$ ($3.1\sigma$)</td>
<td>“2.6\sigma” (“1.3\sigma”)</td>
</tr>
<tr>
<td>IV+tr.</td>
<td>–</td>
<td>–</td>
<td>“2.1\sigma” (“0.9\sigma”)</td>
</tr>
</tbody>
</table>

Not conclusive at this stage

<table>
<thead>
<tr>
<th>$M$ (MeV)</th>
<th>III</th>
<th>III+tr.</th>
<th>IV+tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma$ (MeV)</td>
<td>48$^{+19}_{-14}$</td>
<td>85$^{+45}_{-26}$</td>
<td>240$^{+230}_{-130}$</td>
</tr>
</tbody>
</table>
Three-Body Unitarity

Amado, Aaron, Young (1968)
Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177
Three-Body Unitarity

Amado, Aaron, Young (1968)
Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177
Three-Body Unitarity

\[ \langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle \]

BS ansatz

Product of disconnected terms are source for the connected amplitude
Three-Body Unitarity

\[ \langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle = i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle \]

BS ansatz

Product of disconnected terms are source for the connected amplitude

\[ \langle q | B(s) | p \rangle = -\frac{\lambda^2}{2\sqrt{m^2+Q^2}} \frac{E_Q - \sqrt{m^2+Q^2+i\epsilon}}{E_Q} \]
The $a_1(1260)$

M. Mikhasenko, A. Jackura, AP, et al., in preparation

We can use these models to fit $\tau^- \rightarrow 2\pi^-\pi^+\nu$ and describe the $a_1(1260)$

The dispersed improved model describes better the data at threshold.
The $Y(4260)$

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in preparation

We start analyzing the single channel $e^+ e^- \rightarrow J/\psi \pi \pi$

We consider the amplitude in the elastic, quasi two-body approximation

LASSO method (linear penalization in the $\chi^2$) is helpful in constraining the number of resonances and parameters in the numerator
Hadron Spectroscopy

Meson  Baryon  Glueball

Tetraquark  Hybrids

Data -> Amplitude analysis -> Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

Experiment  Lattice QCD
Diquarks

Attraction and repulsion in 1-gluon exchange approximation is given by

\[ R = \frac{1}{2} \left( C_2(R_{12}) - C_2(R_1) - C_2(R_2) \right) \]

\[ R_1 = -\frac{4}{3}, \quad R_8 = +\frac{1}{6} \]

\[ R_3 = -\frac{2}{3}, \quad R_6 = +\frac{1}{3} \]

The singlet \( 1_c \) is attractive

A diquark in \( \bar{3}_c \) is attractive

Evidence (?) of diquarks in LQCD, Alexandrou, de Forcrand, Lucini, PRL 97, 222002

H-shape with a 4 quark system
Cardoso, Cardoso, Bicudo, PRD84, 054508
Tetraquark

In a constituent (di)quark model, we can think of a diquark-antidiquark compact state

\[[cq]_{S=0} + [\bar{c}\bar{q}]_{S=1} + h.c.\]

Maiani, Piccinini, Polosa, Riquer PRD71 014028
Faccini, Maiani, Piccinini, AP, Polosa, Riquer PRD87 111102
Maiani, Piccinini, Polosa, Riquer PRD89 114010

Spectrum according to color-spin hamiltonian
(all the terms of the Breit-Fermi hamiltonian are absorbed into a constant diquark mass):

\[H = \sum_{dq} m_{dq} + 2 \sum_{i<j} \kappa_{ij} \vec{S}_i \cdot \vec{S}_j \frac{\lambda_i^a \lambda_j^a}{2} \]

Decay pattern mostly driven by HQSS ✔
Fair understanding of existing spectrum ✔
A full nonet for each level is expected ✗

New ansatz: the diquarks are compact objects spacially separated from each other, only \(\kappa_{cq} \neq 0\)
Existing spectrum is fitted if \(\kappa_{cq} = 67\) MeV

A. Pilloni – The Quest for Exotic States
# Tetraquark

Maiani, Piccinini, Polosa, Riquer PRD89 114010

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>$cq\bar{c}ar{q}$</th>
<th>$c\bar{c}q\bar{q}$</th>
<th>Resonance Assig.</th>
<th>Decays</th>
</tr>
</thead>
<tbody>
<tr>
<td>0++</td>
<td>$</td>
<td>0, 0\rangle$</td>
<td>1/2$</td>
<td>0, 0\rangle + \sqrt{3}/2</td>
</tr>
<tr>
<td>0++</td>
<td>$</td>
<td>1, 1\rangle_0$</td>
<td>$\sqrt{3}/2</td>
<td>0, 0\rangle - 1/2</td>
</tr>
<tr>
<td>1++</td>
<td>$1/\sqrt{2}(</td>
<td>1, 0\rangle +</td>
<td>0, 1\rangle)$</td>
<td>$</td>
</tr>
<tr>
<td>1--</td>
<td>$1/\sqrt{2}(</td>
<td>1, 0\rangle -</td>
<td>0, 1\rangle)$</td>
<td>1/$\sqrt{2}(</td>
</tr>
<tr>
<td>1++</td>
<td>$</td>
<td>1, 1\rangle_1$</td>
<td>$1/\sqrt{2}(</td>
<td>1, 0\rangle +</td>
</tr>
<tr>
<td>2++</td>
<td>$</td>
<td>1, 1\rangle_2$</td>
<td>$</td>
<td>1, 1\rangle_2$</td>
</tr>
</tbody>
</table>

$H_{\text{eff}} = 2m_Q + \frac{B_Q}{2}L^2 - 3\kappa_{cq} + 2a_Y L \cdot S + b_Y \frac{\langle S_{12} \rangle}{4} + \kappa_{cq} [2(S_q \cdot S_c + S_{\bar{q}} \cdot S_{\bar{c}}) + 3]$

## Two different mass scenarios

| Label | $|S_Q, S_{\bar{Q}}; S, L\rangle_J$ |
|-------|----------------------------------|
| $Y_1$ | $|0, 0; 0, 1\rangle_1$ |
| $Y_2$ | $\sqrt{2}(|0, 1; 0, 1\rangle_1 + |0, 1; 1, 1\rangle_1)/\sqrt{2}$ |
| $Y_3$ | $|1, 1; 0, 0\rangle_1$ |
| $Y_4$ | $|1, 1; 2, 1\rangle_1$ |
| $Y_5$ | $|1, 1; 2, 3\rangle_1$ |

## Prediction for a high $Y_5$

\[ M_1 = 4008 \pm 40^{+114}_{-28}, \quad M_2 = 4230 \pm 8, \quad M_3 = 4341 \pm 8, \quad M_4 = 4643 \pm 9. \]

\[ M_1 = 4219.6 \pm 3.3 \pm 5.1, \quad M_2 = 4333.2 \pm 19.9, \quad M_3 = 4391.5 \pm 6.3, \quad M_4 = 4643 \pm 9. \]

\[ M_5 = \begin{cases} 
6539 \text{ MeV} & \text{SI(c1)} \\
6589 \text{ MeV} & \text{SI(c2)} \\
6862 \text{ MeV} & \text{SII(c1)} \\
6899 \text{ MeV} & \text{SII(c2)}
\end{cases} \]
Other models: Molecule

A deuteron-like meson pair, the interaction is mediated by the exchange of light mesons

• Some model-independent relations (Weinberg’s theorem) ✓
• Good description of decay patterns (mostly to constituents) and \( X(3872) \) isospin violation ✓
• States appear close to thresholds ✓ (but \( Z(4430) \) ×)
• Lifetime of constituents has to be \( \gg \frac{1}{m_\pi} \)
• Binding energy varies from \(-70\) to \(-0.1\) MeV, or even positive (repulsive interaction) ×
• Unclear spectrum (a state for each threshold?) – depends on potential models ×

\[
V_\pi(r) = \frac{g_{\pi N}^2}{3} (\hat{r}_1 \cdot \hat{r}_2) \left\{3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)\right\} \left(1 + \frac{3}{(m_\pi r)^2} + \frac{3}{m_\pi r}\right) + (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \frac{e^{-m_\pi r}}{r}
\]

Needs regularization, cutoff dependence
Prompt production of $X(3872)$

$X(3872)$ is the Queen of exotic resonances, the most popular interpretation is a $D^0\bar{D}^{0*}$ molecule (bound state, pole in the 1st Riemann sheet?) but it is copiously promptly produced at hadron colliders.

A solution can be FSI (rescattering of $DD^*$), which allow $k_{max}$ to be as large as $5m_\pi$, $\sigma(p\bar{p} \rightarrow DD^*|k < k_{max}) \approx 230$ nb

However, the rescattering is flawed by the presence of pions that interfere with $DD^*$ propagation. Estimating the effect of these pions increases $\sigma$, but not enough.

| $\sigma_{MC}(p\bar{p} \rightarrow DD^*|k < k_{max}) \approx 0.1$ nb |
| $\sigma_{exp}(p\bar{p} \rightarrow X(3872)) \approx 30 - 70$ nb!! |

Bignamini et al. PRL103 (2009) 162001

Esposito, Piccinini, AP, Polosa, JMP 4, 1569
Guerrieri, Piccinini, AP, Polosa, PRD90, 034003
Nuclear modification factors

What happens to molecules in heavy ion collisions?
We can use deuteron data to extract the values of the nuclear modification factors

\[ R_{CP} = \frac{N_{coll}^{P} \left( \frac{dN}{dp_T} \right)_{C}}{N_{coll}^{C} \left( \frac{dN}{dp_T} \right)_{P}} \]

\[ R_{AA} = \frac{\left( \frac{dN}{dp_T} \right)_{Pb-Pb}}{N_{coll} \left( \frac{dN}{dp_T} \right)_{pp}} \]

Larger than 1 at \( p_T > 2.5 \) GeV
We assume a pure Glauber model (RAA = 1) and a value RAA = 5 to rescale Pb-Pb data to pp

The $X(3872)$ is way larger than the extrapolated cross section

A. Pilloni – The Quest for Exotic States
Production of $Y(4260)$ and $P_c(4450)$

Given the new lineshape by BESIII, we need to rethink the binding energy of the $Y(4260)$

![Table]

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$X(3872)$</td>
<td>$\bar{D}^0D^*$</td>
<td>$\sim100$ keV</td>
<td>$\sim50$ MeV</td>
</tr>
<tr>
<td>$Y(4260)$</td>
<td>$\bar{D}D_1$</td>
<td>$\sim70$ MeV</td>
<td>$\sim400$ MeV</td>
</tr>
<tr>
<td>$P_c(4450)$</td>
<td>$\bar{D}^*\Sigma_c$</td>
<td>$\sim10$ MeV</td>
<td>$\sim150$ MeV</td>
</tr>
</tbody>
</table>

If the states are purely hadron molecule, all the properties depend on the position of the pole with respect to threshold – all the features are universal

What does the production of $X(3872)$ implies for the other states?

J. Nys and AP, to appear
Production of $Y(4260)$ and $P_c(4450)$

We can use Pythia to simulate the production of event, and calculate the relative production of $Y(4260)$ and $P_c(4450)$ with respect to the $X(3872)$.

We tune our MC on charm pair production for CDF data, $\sqrt{s} = 1.96$ TeV, $D^0, D^{*-}$: $|y| < 1, 5.5 < p_T < 20$ GeV.

For baryons we can double check with LHCb data.

CDF data, $\sqrt{s} = 1.96$ TeV, $D^0, D^{*-}$: $|y| < 1, 5.5 < p_T < 20$ GeV

LHCb, $\sqrt{s} = 7$ TeV, JHEP 1206, 141

all: $2 < y < 4, 3 < p_T < 12$ GeV
Production of $Y(4260)$ and $P_c(4450)$

Naively, the fragmentation function of the $D_1$ is 1/10 of the $D^*$, but the cross section scales as $k_{\text{max}}^3$.

Pythia $p\bar{p}$, $\sqrt{s} = 1.96$ TeV

$|y| < 0.6, 5 < p_T < 20$ GeV

The production of $Y(4260)$ is expected to be at worse comparable with the $X(3872)$.

<table>
<thead>
<tr>
<th></th>
<th>No FSI</th>
<th>With FSI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y(4260)/X$</td>
<td>23</td>
<td>0.75</td>
</tr>
<tr>
<td>$P_c(4450)/X$</td>
<td>1.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>

J. Nys and AP, to appear
Conclusions & prospects

• The discovery of exotic states has challenged the well established Charmonium framework
• Experiments are (too) prolific! Constant feedback on predictions
• Thorough amplitude analyses might shed some light on the microscopic nature of the new states
• The implementation of 3-body unitarity will be a major step to understand several of these phenomena
• Some fantasy needed, many phenomenological models introduced.
• Nuclei observation at hadron colliders can give an unexpected help in testing some phenomenological hypotheses for the XYZP states
• Search for exotic states in prompt production is a necessary step to improve our understanding of the sector

Thank you
BACKUP
Dictionary – Quark model

\[ L = \text{orbital angular momentum} \]
\[ S = \text{spin } q + \bar{q} \]
\[ J = \text{total angular momentum} \]
\[ = \text{exp. measured spin} \]
\[ I = \text{isospin } = 0 \text{ for quarkonia} \]

<table>
<thead>
<tr>
<th>( J^{PC} )</th>
<th>( L )</th>
<th>( S )</th>
<th>Charmonium ((c\bar{c}))</th>
<th>Bottomonium ((b\bar{b}))</th>
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</thead>
<tbody>
<tr>
<td>0^{--}</td>
<td>0 ((S\text{-wave}))</td>
<td>0</td>
<td>( \eta_c(nS) )</td>
<td>( \eta_b(nS) )</td>
</tr>
<tr>
<td>1{--}</td>
<td>0</td>
<td>1</td>
<td>( \psi(nS') )</td>
<td>( \Upsilon(nS) )</td>
</tr>
<tr>
<td>1^{+-}</td>
<td>0</td>
<td>0</td>
<td>( h_c(nP) )</td>
<td>( h_b(nP) )</td>
</tr>
<tr>
<td>0{++}</td>
<td>1 ((P\text{-wave}))</td>
<td>0</td>
<td>( \chi_{c0}(nP) )</td>
<td>( \chi_{b0}(nP) )</td>
</tr>
<tr>
<td>1{++}</td>
<td>1</td>
<td>1</td>
<td>( \chi_{c1}(nP) )</td>
<td>( \chi_{b1}(nP) )</td>
</tr>
<tr>
<td>2{++}</td>
<td>1</td>
<td>1</td>
<td>( \chi_{c2}(nP) )</td>
<td>( \chi_{b2}(nP) )</td>
</tr>
</tbody>
</table>

But \( J/\psi = \psi(1S), \psi' = \psi(2S) \)
Charged $Z$ states: $Z_b(10610), Z_b'(10650)$

Anomalous dipion width in $\Upsilon(5S)$, 2 orders of magnitude larger than $\Upsilon(nS)$

Moreover, observed $\Upsilon(5S) \rightarrow h_b(nP)\pi\pi$ which violates HQSS

$\Upsilon(5S) \rightarrow Z_b(10610)^+\pi^- \rightarrow \Upsilon(nS) \pi^+\pi^-, h_b(nP) \pi^+\pi^-$ and $\rightarrow (BB^*)^+\pi^-$

$M = 10607.2 \pm 2.0$ MeV, $\Gamma = 18.4 \pm 2.4$ MeV

$\Upsilon(5S) \rightarrow Z_b'(10650)^+\pi^- \rightarrow \Upsilon(nS) \pi^+\pi^-, h_b(nP) \pi^+\pi^-$ and $\rightarrow \bar{B}^*0 B^+\pi^-$

$M = 10652.2 \pm 1.5$ MeV, $\Gamma = 11.5 \pm 2.2$ MeV

2 twin resonances!
Pentaquarks!

LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

Two states seen in $\Lambda_b \to (J/\psi p) K^-$, evidence in $\Lambda_b \to (J/\psi p) \pi^-$

$M_1 = 4380 \pm 8 \pm 29$ MeV

$\Gamma_1 = 205 \pm 18 \pm 86$ MeV

$M_2 = 4449.8 \pm 1.7 \pm 2.5$ MeV

$\Gamma_2 = 39 \pm 5 \pm 19$ MeV

Quantum numbers

$J^P = \left( \frac{3^-}{2}, \frac{5^+}{2} \right)$ or $\left( \frac{3^+}{2}, \frac{5^-}{2} \right)$ or $\left( \frac{5^+}{2}, \frac{3^-}{2} \right)$

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by $\Lambda^*$ (model dependence?)

No obvious threshold nearby
Quantum numbers

\[ J^P = \left( \frac{3^-}{2}, \frac{5^+}{2} \right) \text{ or } \left( \frac{3^+}{2}, \frac{5^-}{2} \right) \text{ or } \left( \frac{5^+}{2}, \frac{3^-}{2} \right) \]

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by \( \Lambda^* \) (model dependence?)

No obvious threshold nearby
Charged $Z$ states: $Z(4430)$

$Z(4430)^+ \rightarrow \psi(2S) \pi^+$

$I^G J^{PC} = 1^+ 1^{-+}$

$M = 4475 \pm 7^{+15}_{-25}$ MeV

$\Gamma = 172 \pm 13^{+37}_{-34}$ MeV

Far from open charm thresholds

If the amplitude is a free complex number, in each bin of $m_{\psi',\pi^-}^2$, the resonant behaviour appears as well.
Other beasts

\( X(3915) \), seen in \( B \to X K \to J/\psi \omega \) and \( \gamma \gamma \to X \to J/\psi \omega \)

\( J^{PC} = 0^{++} \), candidate for \( \chi_{c0}(2P) \)

But \( X(3915) \not\to D\bar{D} \) as expected, and the hyperfine splitting

\( M(2^{++}) - M(0^{++}) \) too small

One/two peaks seen in \( B \to XK \to J/\psi \phi K \), close to threshold
$Y(4260) \rightarrow \bar{D}D_1$?

e$^+e^- \rightarrow Y(4260) \rightarrow \pi \bar{D}^0D^{**}$

$Z_c(3900) \rightarrow \bar{D}^0D^{**}$

$A = \frac{N_{|\cos \theta|>0.5} - N_{|\cos \theta|<0.5}}{N_{|\cos \theta|>0.5} + N_{|\cos \theta|<0.5}}$

<table>
<thead>
<tr>
<th>$D\bar{D}_1$ MC</th>
<th>$Z_c + ps$ MC</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.43±0.04</td>
<td>0.02±0.02</td>
<td>0.12±0.06</td>
</tr>
</tbody>
</table>

Not a lot of room for $\bar{D}D_1(2410)$

- $Y \rightarrow \pi \bar{D}D^*$ DATA
- $Y \rightarrow \bar{D}D_1(2420)$
- $Y \rightarrow \pi Z_c \rightarrow \pi \bar{D}D^*$ + $\pi \bar{D}D^*$ p.s.
Flavored $X(5568)$

- A flavored state seen in $B_s^0 \pi$ invariant mass by D0 (both $B_s^0 \rightarrow J/\psi \phi$ and $\rightarrow D_s \mu \nu$),
- not confirmed by LHCb or CMS
- (different kinematics? Compare differential distributions)

Controversy to be solved
Tetraquark: the $c\bar{c}s\bar{s}$ states

Good description of the spectrum but one has to assume the axial assignment for the $X(4274)$ to be incorrect (two unresolved states with $0^{++}$ and $2^{++}$)

Maiani, Polosa and Riquer, PRD 94, 054026

Much narrower than LHCb! Look for prompt!
$X(3872)$ on the lattice

There is only evidence (?) for the $X(3872)$ in the $I^G J^{PC} = 0^+ 1^{++}$ channel

Caveats:
- Small lattices, large artifacts
- Three body dynamics may play a role
- Interpretation of the overlap coefficients is questionable

Status of other XYZ on the lattice is even less clear

S. Prelovsek, L. Leskovec, PRL111, 192001
<table>
<thead>
<tr>
<th>State</th>
<th>M (MeV)</th>
<th>Γ (MeV)</th>
<th>J^{PC}</th>
<th>Process (mode)</th>
<th>Experiment (#σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(3823)</td>
<td>3823.1 ± 1.9</td>
<td>&lt; 24</td>
<td>?−</td>
<td>B → K(πχ_γ)</td>
<td>Bell(13)(4.0)</td>
</tr>
<tr>
<td>X(3872)</td>
<td>3871.68 ± 0.17</td>
<td>&lt; 1.2</td>
<td>1++</td>
<td>B → K(π−π−N J/ψ)</td>
<td>Balmer(5)(10), BaBar(5)(8.6)</td>
</tr>
<tr>
<td></td>
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<td>pp → (π+π−N J/ψ)</td>
<td>CDF(11.6), D0(5.2)</td>
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<tr>
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<td></td>
<td></td>
<td>pp → (π+π−N J/ψ)</td>
<td>LHCb(11)(11)(up)</td>
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<td></td>
<td>B → K(π+π−π0 N J/ψ)</td>
<td>Belle(4.3), BaBar(4.0)</td>
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<td></td>
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<td></td>
<td></td>
<td>B → K(π+π−N J/ψ)</td>
<td>Belle(5.5), BaBar(3.5)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>B → K(N J/ψ)</td>
<td>LHCb(2)(10)</td>
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<td></td>
<td>B → K(N 2S J/ψ)</td>
<td>BaBar(3.6), Belle(0.2)</td>
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<td></td>
<td>LHCb(2)(4.4)</td>
</tr>
<tr>
<td>Z_{c}(3900)</td>
<td>3888.7 ± 3.4</td>
<td>35 ± 7</td>
<td>1++</td>
<td>B → K(D¯D+)</td>
<td>Belle(6.4), BaBar(4.9)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Y(4260) → π− (D¯D+)</td>
<td>BES II(9)(8.9)</td>
</tr>
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<td></td>
<td>Y(4260) → π− (π+π−D$^{*}$)</td>
<td>BES III(10)(5)</td>
</tr>
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<td>CLEO data(7.5)</td>
</tr>
<tr>
<td>Z_{c}(4020)</td>
<td>4023.9 ± 2.4</td>
<td>10 ± 5</td>
<td>1++</td>
<td>Y(4260) → π− (π+π+h_{c})</td>
<td>BES II(10)(8.9)</td>
</tr>
<tr>
<td></td>
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<td>Y(4260) → π− (π+π−h_{c})</td>
<td>BES III(10)(5)</td>
</tr>
<tr>
<td>Y(3915)</td>
<td>3918.4 ± 1.9</td>
<td>25 ± 5</td>
<td>0++</td>
<td>B → K(ω J/ψ)</td>
<td>Belle(8), BaBar(19)</td>
</tr>
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<td>BES II(10)(10)</td>
</tr>
<tr>
<td>Z(3930)</td>
<td>3927.2 ± 2.6</td>
<td>24 ± 6</td>
<td>2++</td>
<td>e^{+}e^{-} → e^{+}e^{-} (ω J/ψ)</td>
<td>Belle(5.3), BaBar(5.8)</td>
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<td>X(3940)</td>
<td>3942^{+8}_{−8}</td>
<td>37^{+17}_{−17}</td>
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<td>e^{+}e^{-} → J/ψ (D¯D+)</td>
<td>Belle(5)(6)</td>
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<tr>
<td>Y(4008)</td>
<td>3891 ± 42</td>
<td>255 ± 42</td>
<td>1−</td>
<td>e^{+}e^{-} → (π+π−N J/ψ)</td>
<td>Belle(7.4)</td>
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<td>Z(4050)</td>
<td>4051^{+24}_{−24}</td>
<td>82^{+31}_{−35}</td>
<td>?−</td>
<td>B^0 → K^+(π+π_{c1})</td>
<td>Belle(5.0), BaBar(1.1)</td>
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<tr>
<td>Y(4140)</td>
<td>4145.6 ± 3.6</td>
<td>14.3 ± 5.9</td>
<td>?−</td>
<td>B^{+} → K^{+}(N J/ψ)</td>
<td>CDF(5.0), Belle(1.9), LHCb(3.1), CMS(5.5)</td>
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<td>D0(3.1)</td>
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<td>X(4160)</td>
<td>4156^{+25}_{−25}</td>
<td>130^{+13}_{−13}</td>
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<td>Z(4200)</td>
<td>4196^{+35}_{−35}</td>
<td>370^{+39}_{−110}</td>
<td>1−</td>
<td>B^0 → K^−(π+π−N J/ψ)</td>
<td>Belle(7)(2)</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>State</th>
<th>M (MeV)</th>
<th>Γ (MeV)</th>
<th>J^{PC}</th>
<th>Process (mode)</th>
<th>Experiment (#σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(4220)</td>
<td>4196^{+35}_{−35}</td>
<td>39 ± 32</td>
<td>1−</td>
<td>e^{+}e^{-} → (π+π−h_{c})</td>
<td>BES III data(4.5)</td>
</tr>
<tr>
<td>Y(4230)</td>
<td>4230 ± 38</td>
<td>12 ± 12</td>
<td>1−</td>
<td>e^{+}e^{-} → (π+π−h_{c})</td>
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<tr>
<td>Z(4240)</td>
<td>4248_{-45}^{+185}</td>
<td>171_{-72}^{+321}</td>
<td>?−</td>
<td>B^0 → K^−(π+π−h_{c1})</td>
<td>Belle(5.0), BaBar(2.0)</td>
</tr>
<tr>
<td>Y(4260)</td>
<td>4250 ± 9</td>
<td>108 ± 12</td>
<td>1−</td>
<td>e^{+}e^{-} → (π+π−h_{c})</td>
<td>BES II(8)(11), BES III(15), BES II(10)(up)</td>
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<td>e^{+}e^{-} → (ω J/ψ)</td>
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<td>Belle(8.2)</td>
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<td>BES III(5.8), BaBar(2.5)</td>
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<tr>
<td>Z_{b}(10610)</td>
<td>10607 ± 2.0</td>
<td>18.4 ± 2.4</td>
<td>1++</td>
<td>Y(5S) → π(π Y(n S))</td>
<td>Belle(10)(10)</td>
</tr>
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<td>BES III(16)</td>
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<td>BES III(8)</td>
</tr>
<tr>
<td>Z_{b}(10650)</td>
<td>10652 ± 1.5</td>
<td>11.5 ± 2.2</td>
<td>1++</td>
<td>Y(5S) → π(π Y(n S))</td>
<td>Belle(10)(10)</td>
</tr>
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<td>BES III(16)</td>
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<td>BES III(8.8)</td>
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</tbody>
</table>

A. Pilloni – The Quest for Exotic States

Esposito, AP, Polosa, Phys.Rept. 668
Guerrieri, AP, Piccinini, Polosa, IJMPA 30, 1530002

63
Joint Physics Analysis Center

- Joint effort between theorists and experimentalists to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in education of further generations of hadron physics practitioners

Effective Field Theories
Analyticity+Unitarity
Dispersion Relations
Regge Theory

Insight on QCD dynamics
Fundamental parameters
Resonances, exotic states

Experiments
CLAS, GlueX, BESIII, COMPASS,
LHCb, BaBar, Belle II, KLOE, MAMI
Lattice
Joint Physics Analysis Center

A. Jackura, N. Sherrill, G. Fox, T. Londergan (IU), E. Passemar, A. Szczepaniak (IU/JLab)
R. Workman (GWU), M. Döring (GWU/JLab)
V. Mathieu, V. Pauk, A. Pilloni, V. Mokeev (JLab)
P. Guo (Cal. State U.)

L. Bibzrycki, R. Kaminski (Krakow)
J. Nys (Ghent U.)
M. Mikhasenko (Bonn U.)
L. Dai (FZ Julich)
I. Danilkin, A. Hiller Blin (Mainz U.)
A. Celentano (INFN-GE)
M. Albaladejo (Valencia U.)

Students, Postdocs, Faculties

A. Pilloni – JPAC program for Hadron Spectroscopy
Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

http://www.indiana.edu/~jpac/
Strategy

- We fit the following invariant mass distributions:
  - BESIII PRL110, 252001 $J/\psi \pi^+, J/\psi \pi^-, \pi^+\pi^-$ at $E_{CM} = 4.26$ GeV
  - BESIII PRL110, 252001 $J/\psi \pi^0$ at $E_{CM} = 4.23, 4.26, 4.36$ GeV
  - BESIII PRD92, 092006 $D^0 D^{*+}, D^{*0} D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL115, 222002 $D^0 D^{*0}, D^{*0} D^0$ at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL112, 022001 $D^0 D^{*+}, D^{*0} D^+$ (single tag) at $E_{CM} = 4.26$ GeV
  - Belle PRL110, 252002 $J/\psi \pi^{\pm}$ at $E_{CM} = 4.26$ GeV
  - CLEO-c data PLB727, 366 $J/\psi \pi^{\pm}, J/\psi \pi^0$ at $E_{CM} = 4.17$ GeV

- Published data are not efficiency/acceptance corrected, → we are not able to give the absolute normalization of the amplitudes

- No given dependence on $E_{CM}$ is assumed – the couplings at different $E_{CM}$ are independent parameters
Strategy

- **Reducible (incoherent) backgrounds are pretty flat** and do not influence the analysis, except the peaking background in $\bar{D}^0 D^0$, $D^*\bar{D}^0$ (subtracted)

- Some information about **angular distributions** has been published, but it’s **not constraining** enough $\rightarrow$ we do not include in the fit

- Because of that, **we approximate all the particles to be scalar** – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters
Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test
To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction.

\[ \langle \lambda_{\psi}\lambda_{p'} | T_r | \lambda_{\gamma}\lambda_p \rangle = \frac{\langle \lambda_{\psi}\lambda_{p'} | T_{\text{dec}} | \lambda_R \rangle}{M_r^2 - W^2 - i\Gamma_r M_r} \]

Hadronic part

- 3 independent helicity couplings, \( g_{\lambda_{\psi}\lambda_{p'}} \approx g \)
- \( g \) extracted from total width and (unknown) branching ratio

\[ \Gamma_\gamma = 4\pi\alpha \Gamma_{\psi p} \left( \frac{f_\psi}{M_\psi} \right)^2 \left( \frac{\tilde{p}_i}{\tilde{p}_f} \right)^{2\ell+1} \times \frac{4}{6} \]

Hiller Blin, AP et al. (JPAC), PRD94, 034002
The background is described via an Effective Pomeron, whose parameters are fitted to high energy data from Hera.

\[ \langle \lambda_\psi \lambda_{p'} | T_P | \lambda_\gamma \lambda_p \rangle = \]

\[ iA \left( \frac{s - s_t}{s_0} \right)^\alpha(t) e^{b_0(t-t_{\text{min}})} \delta\lambda_p \lambda_{p'} \delta\lambda_\psi \lambda_\gamma \]

Asymptotic + Effective threshold
Helicity conservation

Hiller Blin, AP et al. (JPAC), PRD94, 034002

A. Pilloni – The Quest for Exotic States
Pentaquark photoproduction

\[ J^P = (3/2)^- \]

\[ J^P = (3/2)^- \]

Hiller Blin, AP et al. (JPAC), PRD94, 034002

A. Pilloni – The Quest for Exotic States
Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi \pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the $t_{12}$ scattering amplitude, the green curve is the $c_1 + H(s, D_1) + H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the $D_1$ (dashed) and the $D_0$ (dotted). Only for $D_1$ the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.
Lineshapes at 4230

Figure 8: Same as Figure 7, but for $E_{CM} = 4.23$ GeV.
Statistical analysis

Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>III+tr.</th>
<th>IV+tr.</th>
<th>tr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>III</td>
<td>1.5σ (1.5σ)</td>
<td>1.5σ (2.7σ)</td>
<td>“2.4σ” (&quot;1.4σ&quot;)</td>
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<tr>
<td>III+tr.</td>
<td>–</td>
<td>1.5σ (3.1σ)</td>
<td>“2.6σ” (&quot;1.3σ&quot;)</td>
</tr>
<tr>
<td>IV+tr.</td>
<td>Not conclusive at this stage</td>
<td>“2.1σ” (&quot;0.9σ&quot;)</td>
<td></td>
</tr>
</tbody>
</table>
The $a_1(1260)$

M. Mikhasenko, A. Jackura, AP, et al., in preparation
The $a_1(1260)$

M. Mikhasenko, A. Jackura, AP, et al., in preparation
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the $N/D$ method
- We test against the $D$-wave data, where the $a_2$ and the $a'_2$ show up

A. Jackura, M. Mikhasenko, AP et al. (JPAC & COMPASS), PLB779, 464-472

Production amplitude

Scattering amplitude
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the $N/D$ method
- We test against the $D$-wave data, where the $a_2$ and the $a'_2$ show up

A. Jackura, M. Mikhasenko, AP et al. (JPAC & COMPASS), PLB779, 464-472

$$t(s) = \frac{N(s)}{D(s)}$$

The $D(s)$ has only right hand cuts; it contains all the Final State Interactions constrained by unitarity $\rightarrow$ universal

$$\text{Im } D(s) = -\rho N(s)$$
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the $N/D$ method
- We test against the $D$-wave data, where the $a_2$ and the $a'_2$ show up

A. Jackura, M. Mikhasenko, AP et al. (JPAC & COMPASS), PLB779, 464-472

$$t(s) = \frac{N(s)}{D(s)}$$

The $n(s), N(s)$ have left hand cuts only, they depend on the exchanges \(\rightarrow\) process-dependent, smooth
Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi} \int_{s_i}^{\infty} \frac{\rho(s') N(s')}{s' (s' - s)} ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s} \quad \text{OR} \quad K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

$$\rho(s) N(s) = g \frac{\lambda^{(2l+1)/2} (s, m_\pi^2, m_\eta^2)}{(s + s_R)^7}$$
Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity $\Rightarrow$ universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi} \int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'(s' - s)} ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s} \quad \text{OR} \quad K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

The $n(s)$ is process-dependent, smooth

$$n(s) = \sum_j a_j T_j(\omega(s)) \quad \omega(s) = \frac{s}{s + s_0}$$
Searching for resonances in $\eta\pi$

Precise determination of pole position

Smooth «background»
Searching for resonances in $\eta \pi$
Searching for resonances in $\eta\pi$

$$m(a_2) = (1307 \pm 1 \pm 6) \text{ MeV} \quad m(a_2') = (1720 \pm 10 \pm 60) \text{ MeV}$$

$$\Gamma(a_2) = (112 \pm 1 \pm 8) \text{ MeV} \quad \Gamma(a_2') = (280 \pm 10 \pm 70) \text{ MeV}$$

- The coupled channel analysis involving the exotic $P$-wave is ongoing, as well as the extension to the GlueX production mechanism and kinematics.
The $Y(4260)$

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in preparation

\[ f(s) = \frac{N(s)}{K^{-1}(s) - \frac{i}{2} \rho_3(s)}, \]

Same game, we start analyzing the single channel $e^+ e^- \rightarrow J/\psi \pi\pi$ data

We consider the amplitude in the elastic, quasi two-body approximation

\[ \rho_2(s') = \int_{4m_{\pi}^2}^{(\sqrt{s' - M_{J/\psi}})^2} ds' \frac{(s', s_{\pi\pi}, m_{J/\psi}^2)}{2\pi} \frac{\lambda^{1/2}(s', s_{\pi\pi}, m_{J/\psi}^2)}{8\pi s'} \frac{\lambda^{1/2}(s_{\pi\pi}, m_{\pi}^2, m_{\pi}^2)}{8\pi s_{\pi\pi}} |t_{2 \rightarrow 2}(s_{\pi\pi})|^2 \]

Need model for the Dalitz distribution
Models

Meson/Baryon+continuum
Ferretti et al., PRC88, 015207
Ferretti et al., PRD90, 094022

Hybrids/BO tetraquarks
Kou and Pene, PLB631, 164
Braaten, PRL111, 162003
Berwein et al., PRD92, 114019

Molecule
Tornqvist, Z.Phys. C61, 525
Braaten and Kusunoki, PRD69 074005
Swanson, Phys.Rept. 429 243-305

Hadroquarkonium
Dubynskiy et al., PLB 666, 344
Dubynskiy and Voloshin, PLB 671, 82
Li and Voloshin, MPLA29, 1450060

Diquark-Antidiquark
Maiani, et al. PRD71, 014028
Faccini, AP, et al. PRD87, 111102
Maiani, et al. PRD89, 114010
Maiani, et al., PLB778, 247

Hybridized Tetraquaks
Esposito, AP, Polosa
PLB758, 292

Kinematical effects
Szczepaniak, PLB747, 410
Szczepaniak, PLB757, 61
Guo et al., PRD92, 071502
Swanson, IJMPE25, 1642010
Weinberg theorem

Resonant scattering amplitude

\[ f(ab \rightarrow c \rightarrow ab) = -\frac{1}{8\pi E_{CM}} g^2 \frac{1}{(p_a + p_b)^2 - m_c^2} \]

with \( m_c = m_a + m_b - B \), and \( B, T \ll m_{a,b} \)

\[ f(ab \rightarrow c \rightarrow ab) = -\frac{1}{16\pi (m_a + m_b)^2} g^2 \frac{1}{B + T} \]

This has to be compared with the potential scattering for slow particles \((kR \ll 1, \text{ being } R \sim 1/m_\pi \text{ the range of interaction})\) in an attractive potential \( U \) with a superficial level at \(-B\)

\[ f(ab \rightarrow ab) = -\frac{1}{\sqrt{2\mu}} \frac{\sqrt{B - i\sqrt{T}}}{B + T}, \quad B = \frac{g^4}{512\pi^2} \frac{\mu^5}{(m_a m_b)^2} \]

This corresponds to the pure molecular interpretation of the \( X(3872) \)

Weinberg, PR 130, 776
Weinberg, PR 137, B672
Polosa, PLB 746, 248
Three-Body Unitarity

Imaginary parts of $B$, $\tau$, $S$ are fixed by unitarity and matching (for simplicity $\nu = \lambda$)

$$\tau(\sigma(k)) = (2\pi)^3 \delta^+(k^2 - m^2) S(\sigma(k))$$

$$-\frac{1}{S(P^2)} = \sigma(k) - M_0^2 - \frac{1}{(2\pi)^3} \int d^3 \ell \frac{\lambda^2}{2E_\ell (\sigma(k) - 4E_\ell^2 + i\epsilon) \sqrt{m^2 + Q^2}}$$

- in the rest-frame of isobar (Lorentz invariance!)
- twice subtracted dispersion relation in $\sigma(k) = (P - k)^2$

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2 + Q^2}(E_Q - \sqrt{m^2 + Q^2} + i\epsilon)}$$

- un-subtracted dispersion relation
- one-$\pi$ exchange in TOPT
- real contributions can be added to $B$

The freedom of adding real terms to $B$ allows us to use this solution as a flexible parametrization

Numerics in progress:
- D. Sadasivan, M. Mai, AP, M. Doring, A. Szczepaniak for the $a_1(1260)$ and $a_1(1420)$
- Alternative approach based on $N/D$:
  - A. Jackura, AP et al. (JPAC) for the $X(3872)$
  - J.M. Alarcon, E. Passemar, AP, C. Weiss for the nucleon isoscalar vector form factor
This means that IF you can consider the pion exchange as a contact interaction, the amplitude is determined by the pole close to threshold.

This loop is now divergent, I need to renormalize the integral.
I can put the pole where I want.

Complex $s$
Weinberg and amplitudes

A. Jackura, AP et al., in progress

BUT the $D^*$ actually decays into $D\pi$ and the system is constrained by 3-body unitarity

The position of the pole can be calculated given a model for the simple pion exchange

The simplest model leads to a convergent dispersion relation, the pole position is determined
One can check whether this purely molecular amplitude is consistent or not with data

Complex $s$  

Short cut of real pion exchange

polar?
Hadro-charmonium

Born in the context of QCD multipole expansion

\[ H_{\text{eff}} = -\frac{1}{2} a_\psi E_i^a E_i^a \]
\[ a_\psi = \langle \psi | (t_c^a - t_c^{a'}) r_i G r_i (t_c^a - t_c^{a'}) | \psi \rangle \]

the chromoelectric field interacts with soft light matter (highly excited light hadrons)

A bound state can occur via Van der Waals-like interactions

Expected to decay into core charmonium + light hadrons,
Decay into open charm exponentially suppressed
Counting rules

- Exotic states can be produced in threshold regions in $e^+e^-$, electroproduction, hadronic beam facilities and are best characterized by cross section ratios
- Two examples:
  1) \[
  \frac{\sigma(e^+e^-\rightarrow Z_c^+ \pi^-)}{\sigma(e^+e^-\rightarrow \mu^+ \mu^-)} \propto \frac{1}{s^6} \text{ as } s \rightarrow \infty
  \]
  2) \[
  \frac{\sigma(e^+e^-\rightarrow Z_c^+(\bar{c}c\bar{d}u)+\pi^-(\bar{u}d))}{\sigma(e^+e^-\rightarrow \Lambda_c(cud)+\Lambda_c(\bar{c} \bar{u}d))} \rightarrow \text{const} \text{ as } s \rightarrow \infty
  \]
- Ratio numerically smaller if $Z_c$ behaves like weakly-bound dimeson molecule instead of diquark-antidiquark bound state due to weaker meson color van der Waals forces

Different estimates close to thesholds, and in presence of annihilating $q \bar{q}$

Guo, Meissner, Wang, Yang, 1607.04020
Voloshin PRD94, 074042
Tetraquark: the $Y(4220)$

\[ \langle \chi_c(p) \omega(\eta, q) | Y(\lambda, P) \rangle = g_\chi \eta \cdot \lambda, \]
\[ \langle Z_c'(\eta, q) \pi(p) | Y(\lambda, P) \rangle = g_Z \eta \cdot \lambda \frac{P \cdot p}{f_\pi M_Y}, \]
\[ \langle h_c(\eta, q) \sigma(p) | Y(\lambda, P) \rangle = g_h \epsilon_{\mu \nu \rho \sigma} \eta^\mu \lambda^\nu \frac{P^\rho q^\sigma}{P \cdot q}, \]
\[ \langle \pi(q) \pi(p) | \sigma(P) \rangle = \frac{P^2}{2f_\pi}, \]

A state apparently breaking HQSS has been observed

\[ \frac{\Gamma (Y(4220) \rightarrow \chi_c \omega)}{\Gamma (Y(4220) \rightarrow h_c \pi^+ \pi^-)} = (13.4 \pm 3.6) \times R_{YZ} = 2.3 \pm 1.2. \]
\[ \frac{\Gamma (Y(4220) \rightarrow Z_c' \pi^\mp \rightarrow h_c \pi^+ \pi^-)}{\Gamma (Y(4220) \rightarrow h_c \sigma \rightarrow h_c \pi^+ \pi^-)} = 4.8 \pm 3.5, \]

Compatible to be the $Y_3$ state

Faccini, Filaci, Guerrieri, AP, Polosa, PRD 91, 117501

A. Pilloni – The Quest for Exotic States
Tetraquark: the $b$ sector

Ali, Maiani, Piccinini, Polosa, Riquer PRD91 017502

\[ M(Z_b') - M(Z_b) = 2\kappa_b \]
\[ M(Z_c') - M(Z_c) = 2\kappa_c \sim 120 \text{ MeV} \]
\[ \kappa_b : \kappa_c = M_c : M_b \sim 0.30 \]

\[ 2\kappa_b \sim 36 \text{ MeV}, \text{ vs. } 45 \text{ MeV (exp.)} \]

\[ Z_b = \frac{\alpha |1_{q\bar{q}}0_{b\bar{b}}\rangle - \beta |0_{q\bar{q}}1_{b\bar{b}}\rangle}{\sqrt{2}} \]
\[ Z_b' = \frac{\alpha |1_{q\bar{q}}0_{b\bar{b}}\rangle + \beta |0_{q\bar{q}}1_{b\bar{b}}\rangle}{\sqrt{2}} \]

Data on $\Upsilon(5S) \to \Upsilon(nS)\pi\pi$ and $\Upsilon(5S) \to h_b(nP)\pi\pi$ strongly favor $\alpha = \beta$
$Z_c(3900) \to \eta_c \rho$

Esposito, Guerrieri, AP, PLB 746, 194-201

If tetraquark

Kinematics with HQSS, dynamics estimated according to Brodsky et al., PRL113, 112001

$$A = \langle \chi_{cc}|\chi_c \otimes \chi_c \rangle \langle \phi_{cc}|\hat{T}_{\perp HQS} \phi[cq][\bar{c}q]\rangle + O \left( \frac{\Lambda_{QCD}}{m_c} \right)$$

Uncertainty $\sim 25\%$

Clebsch-Gordan

Reduced matrix element
- approximated as a constant
- or $\propto \psi_{cc}(r_Z)$

<table>
<thead>
<tr>
<th></th>
<th>Kinematics only type I</th>
<th>Dynamics included type I</th>
<th>Kinematics only type II</th>
<th>Dynamics included type II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$BR(Z_c \to \eta_c \rho)$</td>
<td>$(3.3^{+2.9}_{-1.4}) \times 10^2$</td>
<td>$(2.3^{+3.3}_{-1.4}) \times 10^2$</td>
<td>$0.27^{+0.40}_{-0.17}$</td>
<td></td>
</tr>
<tr>
<td>$BR(Z_c \to J/\psi \pi)$</td>
<td>$0.41^{+0.96}_{-0.17}$</td>
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<td></td>
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<tr>
<td>$BR(Z_c' \to \eta_c \rho)$</td>
<td>$(1.2^{+2.8}_{-0.5}) \times 10^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$BR(Z_c' \to h_c \pi)$</td>
<td></td>
<td>$6.6^{+56.8}_{-5.8}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If molecule

Non-Relativistic Effective Theory, HQET+NRQCD and Hidden gauge Lagrangian

Uncertainty estimated with power counting at NLO

$$\mathcal{L}_{Z_c^{(r)}} = \frac{z^{(r)}}{2} \left< Z_{\mu,ab}^{(r)} \tilde{H}_{2b} \gamma^\mu \tilde{H}_{1a} \right> + h.c.,$$

$$\mathcal{L}_{c\bar{c}} = \frac{g_2}{2} \left< \bar{\psi} H_{1a} \gamma^\mu H_{2a} \right> + \frac{g_1}{2} \left< \bar{\chi}_\mu H_{1a} \gamma^\mu H_{2a} \right> + h.c.,$$

$$\mathcal{L}_{\psi DD^*} = i\beta \left< H_{1b} \gamma^\mu \left( \mathcal{O}_\mu - \rho_\mu \right)_{ba} \tilde{H}_{1a} \right> + i\lambda \left< H_{1b} \sigma^{\mu\nu} F_{\mu\nu} (\rho)_{ba} \tilde{H}_{1a} \right> + h.c.,$$

$$\frac{\text{BR}(Z_c \rightarrow \eta_c \rho)}{\text{BR}(Z_c \rightarrow J/\psi \pi)} = (4.6^{+2.5}_{-1.7}) \times 10^{-2} ; \quad \frac{\text{BR}(Z_c' \rightarrow \eta_c \rho)}{\text{BR}(Z_c' \rightarrow h_c \pi)} = (1.0^{+0.5}_{-0.4}) \times 10^{-2} .$$

$$\frac{\text{BR}(Z_c \rightarrow h_c \pi)}{\text{BR}(Z_c' \rightarrow h_c \pi)} = 0.34^{+0.21}_{-0.13} ; \quad \frac{\text{BR}(Z_c \rightarrow J/\psi \pi)}{\text{BR}(Z_c' \rightarrow J/\psi \pi)} = 0.35^{+0.49}_{-0.21} .$$
Dynamical movie \( Z^+(4430) \)

- Since this is still a \( 3 \leftrightarrow \bar{3} \) color interaction, just use the Cornell potential:

\[
V(r) = -\frac{4}{3} \frac{\alpha_s}{r} + br + \frac{32\pi \alpha_s}{9m_{cq}^2} \left( \frac{\sigma}{\sqrt{\pi}} \right)^3 e^{-\sigma^2 r^2} S_{cq} \cdot S_{c\bar{q}},
\]

- Use that the kinetic energy released in \( \bar{B}^0 \rightarrow K^- Z^+(4430) \) converts into potential energy until the diquarks come to rest
- Hadronization most effective at this point (WKB turning point)

\[
B(Z^+(4430) \rightarrow \psi(2S) \pi^+) \Bigg/ \frac{B(Z^+(4430) \rightarrow J/\psi \pi^+)}{B(Z^+(4430) \rightarrow J/\psi \pi^+)} \sim 72 > 10 \text{ exp.}
\]

\( r_Z = 1.16 \text{ fm}, \langle r_{\psi(2S)} \rangle = 0.80 \text{ fm}, \langle r_{J/\psi} \rangle = 0.39 \text{ fm} \)

Brodsky, Hwang, Lebed PRL 113 112001

-e.g. Barnes et al., PRD 72, 054026
Towards hybridized tetraquarks

The absence of many of the predicted states might point to the need for selection rules. It is unlikely that the many close-by thresholds play no role whatsoever. All the well assessed 4-quark resonances lie close and above some meson-meson thresholds: We introduce a mechanism that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data.

<table>
<thead>
<tr>
<th>Resonance</th>
<th>Thr.</th>
<th>$\delta$ (MeV)</th>
<th>$\Lambda \sqrt{\delta}$ (MeV)</th>
<th>$\Gamma$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3872)$</td>
<td>$\bar{D}^0 D^{*0}$</td>
<td>0$^\dagger$</td>
<td>0$^\dagger$</td>
<td>0$^\dagger$</td>
</tr>
<tr>
<td>$Z_c(3900)$</td>
<td>$\bar{D}^0 D^{*+}$</td>
<td>7.8</td>
<td>27.9</td>
<td>27.9</td>
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<tr>
<td>$Z'_c(4020)$</td>
<td>$\bar{D}^{*0} D^{++}$</td>
<td>6.7</td>
<td>25.9</td>
<td>24.8$^|$</td>
</tr>
<tr>
<td>$X(4140)$</td>
<td>$J/\psi \phi$</td>
<td>$a)$ 31.6</td>
<td>52.7</td>
<td>28.0</td>
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<td></td>
<td></td>
<td>$b)$ 30.1</td>
<td>54.7</td>
<td>83.0</td>
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<tr>
<td>$Z_b(10610)$</td>
<td>$\bar{B}^0 B^{*+}$</td>
<td>2.7</td>
<td>16.6</td>
<td>18.4</td>
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<tr>
<td>$Z'_b(10650)$</td>
<td>$\bar{B}^{*0} B^{**}$</td>
<td>1.8</td>
<td>13.4</td>
<td>11.5</td>
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<tr>
<td>$X(5568)$</td>
<td>$B_s^0 \pi^+$</td>
<td>61.4</td>
<td>78.4</td>
<td>21.9</td>
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<tr>
<td>$X_{bs}$</td>
<td>$B^+ \bar{K}^0$</td>
<td>5.8</td>
<td>24.1</td>
<td>—</td>
</tr>
</tbody>
</table>

We introduce a mechanism that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data.
Hybridized tetraquarks

The absence of many of the predicted states might point to the need for **selection rules**
It is unlikely that the **many close-by thresholds** play no role whatsoever
All the well assessed 4-quark resonances lie close and **above** some meson-meson thresholds:
We introduce a **mechanism** that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data

Let $P$ and $Q$ be orthogonal subspaces of the Hilbert space

\[ H = H_{PP} + H_{QQ} \]

We have the (weak) scattering length $a_P$ in the open channel.

We add an off-diagonal $H_{QP}$ which connects the two subspaces
Hybridized tetraquarks

\[ \Gamma = -16\pi^3 \rho \zeta(T) \sim 16\pi^4 \rho |H_{PQ}|^2 \delta \left( \frac{p_1^2}{2M} + \frac{p_2^2}{2M} - \delta \right) \]

The expected width is the average over momenta that allow for the existence of a tetraquark \( p < \bar{p} = 50 \div 100 \) MeV

\[ \Gamma \sim A\sqrt{\delta} \]

We therefore expect to see a level if:

- \( \delta > 0 \) the state lies above threshold
- \( \delta < \frac{\bar{p}^2}{2M} \), only the closest threshold contributes
- The states \( \psi_Q \) and \( \psi_P \) are orthogonal

\( X(3872)^+ \) falls below threshold, \( M(1^{++}) < M(D^{*+}\bar{D}^0) \)

\( \delta < 0 \), so \( a > 0 \) → Repulsive interaction

No charged partners of the \( X(3872) \)!
Hybridized tetraquarks

The model works only if no direct transition between closed channel levels can occur. This prevents the straightforward generalization to $L = 1$ and radially excited states (like the $Y_s$ or the $Z(4430)$).

In this picture, a $[bu][s\bar{d}]$ state with resonance parameters of the $X(5568)$ observed by D0 is not likely.

Also, one has to ensure the orthogonality between the two Hilbert subspaces $P$ and $Q$. This might affect the estimate for the $X(4140)$.

All the resonances can be fitted with

$$A = (10.3 \pm 1.3) \text{ MeV}^{1/2}$$

$$\chi^2/\text{DOF} = 1.2/5$$
Baryonium

A structure $[cq][\bar{c}\bar{q}]$ can explain the dominance of baryon channel

Isospin violation expected,
\[ \alpha_s(m_c) \ll 1 \]

\[ B(Y(4660) \to \Lambda_c^+\Lambda_c^-) \]
\[ \frac{B(Y(4660) \to \psi(2S)\pi\pi)}{B(Y(4660) \to \psi(2S)\pi\pi)} = 25 \pm 7 \]

Cotugno, Faccini, Polosa, Sabelli,
PRL 104, 132005
\[ Y(4260) \rightarrow \gamma X(3872) \]


F. Piccinini

**BESIII:** \[ e^+e^- \rightarrow Y(4260) \rightarrow X(3872)\gamma \]

With \[ \mathcal{B}[X(3872) \rightarrow \pi^+\pi^- J/\psi] = 5\% \]

\[
\frac{\mathcal{B}[Y(4260) \rightarrow \gamma X(3872)]}{\mathcal{B}(Y(4260) \rightarrow \pi^+\pi^- J/\psi)} = 0.1
\]

Strong indication that \( Y(4260) \) and \( X(3872) \) share a similar structure
Tuning of MC

Monte Carlo simulations

- We compare the $D^0 D^{*-}$ pairs produced as a function of relative azimuthal angle with the results from CDF:

Such distributions of charm mesons are available at Tevatron
No distribution has been published (yet) at LHC

The c-cbar run underestimate the low angles (low-$k_0$) region!
Prompt production of $X(3872)$

$$\sigma(\bar{p}p \to X) \sim \left| \int d^3k \langle X | D^0 \bar{D}^*0(k) \rangle \langle D^0 \bar{D}^*0(k) | \bar{p}p \rangle \right|^2$$

$$\leq \int_{\mathcal{R}} d^3k \left| \Psi(k) \right|^2 \int_{\mathcal{R}} d^3k \left| \langle D^0 \bar{D}^*0(k) | \bar{p}p \rangle \right|^2$$

The essence of the argument is that one has to look at the integral of the wave function

$$\int_{\mathcal{R}} d^3k \psi(k)$$

The estimate of the $k_{max}$ has been brought back

Albaladejo et al. arXiv:1709.09101

Esposito et al. arXiv:1709.09631

W. Wang arXiv:1709.10382
Prompt production of $X(3872)$

However, the integral of the wave function may not be well defined. For example, if one considers the wave function in the scattering length approximation,

$$\psi(k) = \frac{1}{\pi} \frac{a^{3/2}}{a^2 k^2 + 1}$$

it’s not integrable

A physical value should rather be based on expectation values which involve $|\psi(k)|^2$

For example, an estimate using the virial theorem gives $k \sim 100$ MeV for the deuteron.

Moreover, the wave function may change sign, which makes the integral nonmonotone. What’s the right $R$ then?
The argument is about the value of a nonnormalizable wave function. Any argument about where the wave function is localized must be calculated for the modulus square.

$$\sigma(pp \to X) \sim \left| \int \frac{d^3k}{(2\pi)^3} \langle X|D^0\bar{D}^{*0}(k)\rangle \langle D^0\bar{D}^{*0}(k)|pp \rangle \right|^2$$
Tuning pions

This picture could spoil existing meson distributions used to tune MC.
We verify this is not the case up to an overall $K$ factor.

Guerrieri, Piccinini, AP, Polosa, PRD90, 034003

Neither at CDF...

...nor at ATLAS.
Notes from the Editors: Highlights of the Year

Published December 30, 2013 | Physics 6, 130 (2013) | DOI: 10.1103/Physics.6.130

Physics looks back at the standout stories of 2013.

As 2013 draws to a close, we look back on the research covered in Physics that really made waves in and beyond the physics community. In thinking about which stories to highlight, we considered a combination of factors: popularity on the website, a clear element of surprise or discovery, or signs that the work could lead to better technology. On behalf of the Physics staff, we wish everyone an excellent New Year.

– Matteo Rini and Jessica Thomas

Four-Quark Matter

Quarks come in twos and threes—or so nearly every experiment has told us. This summer, the BESIII Collaboration in China and the Belle Collaboration in Japan reported they had sorted through the debris of high-energy electron-positron collisions and seen a mysterious particle that appeared to contain four quarks. Though other explanations for the nature of the particle, dubbed $Z_c(3900)$, are possible, the “tetraquark” interpretation may be gaining traction: BESIII has since seen a series of other particles that appear to contain four quarks.
Doubly charmed states

For example, we proposed to look for doubly charmed states, which in tetraquark model are $[cc]_{s=1}[\bar{q}q]_{s=0,1}$.

These states could be observed in $B_c$ decays @LHC and sought on the lattice.

Esposito, Papinutto, AP, Polosa, Tantalo, PRD88 (2013) 054029

Preliminary results on spectrum for $m_\pi = 490$ MeV, $32^3 \times 64$ lattice, $a = 0.075$ fm

Guerrieri, Papinutto, AP, Polosa, Tantalo, PoS LATTICE2014 106
$T$ states production

\[ \bar{b} \rightarrow \bar{c}, c, \bar{s}, u, d, s \]

\[ \bar{b} \rightarrow \bar{c}, \bar{u}, \bar{d}, \bar{s}, \bar{c} \]

\[ b \rightarrow c, \bar{u}, \bar{c}, d, \bar{c}, \bar{u}, \bar{d}, \bar{s}, u, d, s \]

\[ b \rightarrow c, \bar{u}, \bar{c}, d, \bar{c}, \bar{u}, \bar{d}, \bar{s}, u, d, s \]

$D^0, D^-, D_s^-$

$T_s^+, T_s^{++}, T_s^{++}$

$T^0, T^+, T_s^+$

$p, n, \Lambda, \Sigma, \Xi ...$
Prompt production of $X(3872)$

$X(3872)$ is the Queen of exotic resonances, the most popular interpretation is a $D^0\bar{D}^{0*}$ molecule (bound state, pole in the 1$^{\text{st}}$ Riemann sheet?)

We aim to evaluate prompt production cross section at hadron colliders via Monte-Carlo simulations

Q. What is a molecule in MC? A. «Coalescence» model

\[
\sigma(p\bar{p} \rightarrow X(3872)) \sim \int d^3k \ |\langle X|D\bar{D}^*\rangle\langle D\bar{D}^*|p\bar{p}\rangle|^2 < \int_{k<k_{\text{max}}} d^3k \ |\langle D\bar{D}^*|p\bar{p}\rangle|^2
\]

This should provide an upper bound for the cross section

Bignamini, Piccinini, Polosa, Sabelli PRL103 (2009) 162001
Kadastic, Raidan, Strumia PLB683 (2010) 248
Estimating $k_{max}$

The binding energy is $E_B \approx -0.16 \pm 0.31$ MeV: very small! In a simple square well model this corresponds to:

$$\sqrt{\langle k^2 \rangle} \approx 50 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 10 \text{ fm}$$

(binding energy reported in Kamal Seth's talk is $E_B \approx -0.013 \pm 0.192$ MeV:)

$$\sqrt{\langle k^2 \rangle} \approx 30 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 30 \text{ fm}$$

to compare with deuteron: $E_B = -2.2$ MeV

$$\sqrt{\langle k^2 \rangle} \approx 80 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 4 \text{ fm}$$

We assume $k_{max} \sim \sqrt{\langle k^2 \rangle} \approx 50$ MeV, some other choices are commented later
We tune our MC to reproduce CDF distribution of $\frac{d\sigma}{d\Delta\phi} (p\bar{p} \rightarrow D^0 D^{*-})$

We get $\sigma(p\bar{p} \rightarrow DD^*|k < k_{max}) \approx 0.1 \text{ nb} @ \sqrt{s} = 1.96 \text{ TeV}$

Experimentally $\sigma(p\bar{p} \rightarrow X(3872)) \approx 30 - 70 \text{ nb}!!!
Estimating $k_{max}$

A solution can be FSI (rescattering of $DD^*$), which allow $k_{max}$ to be as large as $5m_\pi \sim 700$ MeV

$$\sigma(p\bar{p} \rightarrow DD^* | k < k_{max}) \approx 230 \text{ nb}$$

Artoisenet and Braaten, PRD81, 114018

However, the applicability of Watson theorem is challenged by the presence of pions that interfere with $DD^*$ propagation

Bignamini, Grinstein, Piccinini, Polosa, Riquer, Sabelli, PLB684, 228-230

FSI saturate unitarity bound? Influence of pions small?

Artoisenet and Braaten, PRD83, 014019

Guo, Meissner, Wang, Yang, JHEP 1405, 138; EPJC74 9, 3063; CTP 61 354

use $E_{max} = M_X + \Gamma_X$ for above-threshold unstable states

With different choices, 2 orders of magnitude uncertainty, limits on predictive power
A new mechanism?

In a more billiard-like point of view, the comoving pions can elastically interact with $D(D^*)$, and slow down the pairs $DD^*$

The mechanism also implies: $D$ mesons actually “pushed” inside the potential well (the classical 3-body problem!)

$X(3872)$ is a real, negative energy bound state (stable).

It also explains a small width $\Gamma_X \sim \Gamma_{D^*} \sim 100$ keV

By comparing hadronization times of heavy and light mesons, we estimate up to $\sim 3$ collisions can occur before the heavy pair to fly apart

We get $\sigma(p\bar{p} \rightarrow X(3872)) \sim 5$ nb, still not sufficient to explain all the experimental cross section
Hybridized tetraquarks – Selection rules

• Consider the down quark part of the $X(3872)$ in the diquarkonium picture:
  \[ \Psi_d = X_d = [cd]_0[\bar{c}\bar{d}]_1 + [cd]_1[\bar{c}\bar{d}]_0 \sim (D^*-D^+ - D^{*-}D^-) + i(\psi \times \rho^0 - \psi \times \omega^0) \]

  Fierz rearrangement

• The closest threshold from below is \( \Psi_m \sim \bar{D}^0 D^{*0} \rightarrow \Psi_d \perp \Psi_m \checkmark \)

• But if we consider the up quark part of the $X(3872)$:
  \[ \Psi_u = X_u = [cu]_0[\bar{c}\bar{u}]_1 + [cu]_1[\bar{c}\bar{u}]_0 \sim (\bar{D}^{*0}D^0 - D^{*0}\bar{D}^0) - i(\psi \times \rho^0 + \psi \times \omega^0) \]

• But then \( \Psi_d \not\perp \Psi_m \times \)

• Only $X_d$ is produced via this mechanism \( \rightarrow \) isospin violation
  \( \rightarrow \) no hyperfine neutral doublet

• $X_b$
  (A) Diquark model predicts $M(X_b) \simeq M(Z_b) \simeq (10607 \pm 2)$ MeV
  (B) The closest orthogonal threshold is $M(B^0 B^{*0}) = (10604.4 \pm 0.3)$ MeV
  (C) This could either be above threshold (very narrow state) or below (no state at all)
  (D) Experimentally the diquark model overpredicts the mass of the $X$:
  \[ M(Z_c) - M(X) \simeq 32 \text{ MeV} \]
  (E) We favor the below threshold scenario \( \rightarrow \) no $X_b$ should be seen
Production of hybridized tetraquarks

Going back to $pp(\bar{p})$ collisions, we can imagine hadronization to produce a state

$$|\psi\rangle = \alpha|[qQ][\bar{q}\bar{Q}]_c\rangle + \beta|\bar{q}q)(\bar{Q}Q)\rangle_o + \gamma|\bar{q}Q)(\bar{Q}q)\rangle_o$$

If $\beta, \gamma \gg \alpha$, an initial tetraquark state is not likely to be produced.
The open channel mesons fly apart (see MC simulations).

$\alpha$ expected to be small in Large N limit, Maiani, Polosa, Riquer JHEP 1606, 160

No prompt production without hybridization mechanism!

Note that only the $X(3872)$ has been observed promptly so far...

...and a narrow $X(4140)$ not compatible with the LHCb one $\Rightarrow$ needs confirmation