The Quest for Exotic States

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Prologue

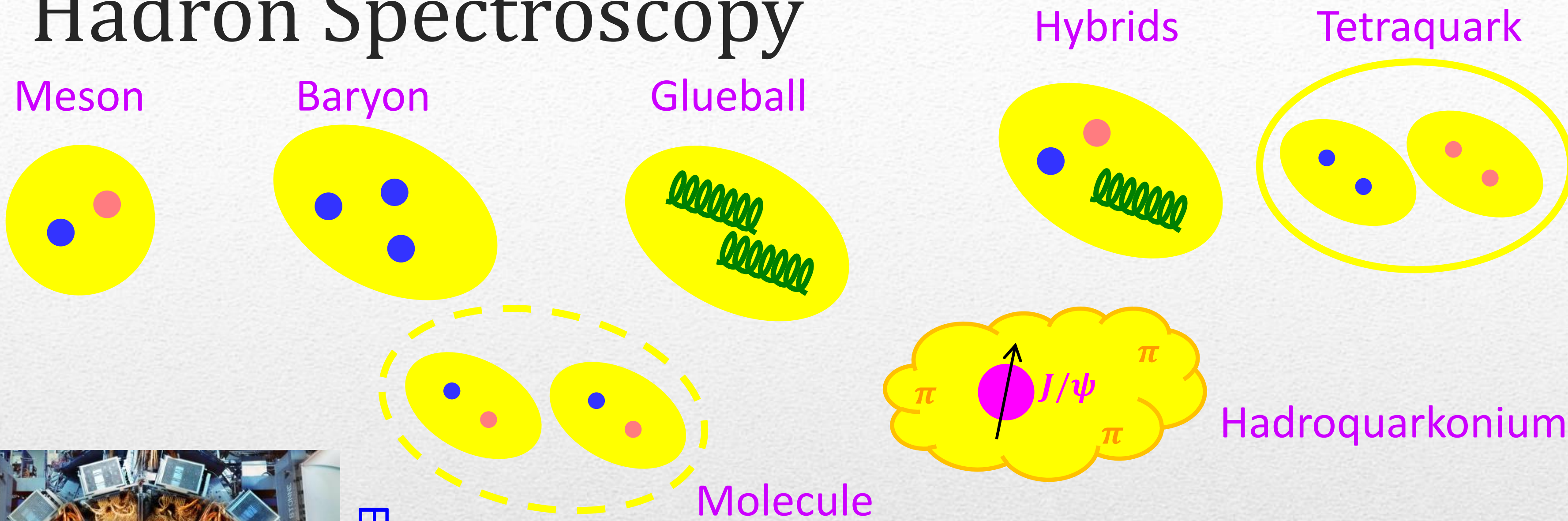
Why do we care about hadron spectroscopy?

- Because it allows us to understand how the QCD degrees of freedom manifest in nature. The role of models is crucial
- Because we need a better understanding of hadron amplitudes if we want to reduce the «hadronic uncertainties» in precision physics (e.g. τ EDM, $g_\mu - 2$, CPV in hadronic B decays...)
- (the honest answer would be «because we are nerds and we like it», but we cannot reply like this to funding agencies)

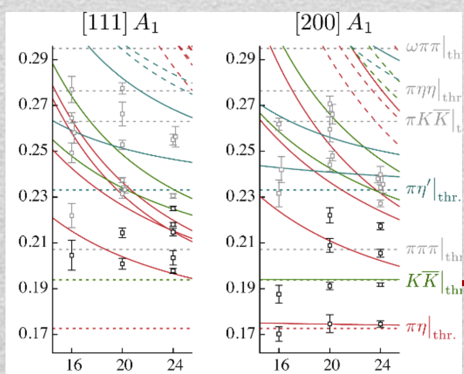
Outline

- The exotic landscape
- Amplitude analysis
  - The S-matrix principles
  - Case study for the $Z_c(3900)$
  - The $\eta\pi$ system
  - Three-body unitarity
  - The Y states
- Modeling
  - Diquark-antidiquark & Molecules
  - Production at colliders

Hadron Spectroscopy



Experiment



Lattice QCD

Data → Amplitude analysis → Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

Hadron Spectroscopy

$\rho(770)$ $\quad I^G(J^{PC}) = 1^+(1^{--})$

Review: The $\rho(770)$

$\rho(770)$ MASS

NEUTRAL ONLY, e^+e^-	775.26 ± 0.25 MeV
CHARGED ONLY, τ DECAYS and e^+e^-	775.11 ± 0.34 MeV
MIXED CHARGES, OTHER REACTIONS	763.0 ± 1.2 MeV
Mass m	
CHARGED ONLY, HADROPRODUCED	766.5 ± 1.1 MeV
NEUTRAL ONLY, PHOTOPRODUCED	769.0 ± 1.0 MeV
NEUTRAL ONLY, OTHER REACTIONS	769.0 ± 0.9 MeV (S = 1.4)
$m_{\rho(770)^0} - m_{\rho(770)^\pm}$	-0.7 ± 0.8 MeV (S = 1.5)
$m_{\rho(770)^+} - m_{\rho(770)^-}$	
$\rho(770)$ RANGE PARAMETER	$5.3^{+0.9}_{-0.7}$ GeV^{-1}

$\rho(770)$ WIDTH

NEUTRAL ONLY, e^+e^-	147.8 ± 0.9 MeV (S = 2.0)
CHARGED ONLY, τ DECAYS and e^+e^-	149.1 ± 0.8 MeV
MIXED CHARGES, OTHER REACTIONS	149.5 ± 1.3 MeV
CHARGED ONLY, HADROPRODUCED	150.2 ± 2.4 MeV
NEUTRAL ONLY, PHOTOPRODUCED	151.7 ± 2.6 MeV
NEUTRAL ONLY, OTHER REACTIONS	150.9 ± 1.7 MeV (S = 1.1)
$\Gamma_{\rho(770)^0} - \Gamma_{\rho(770)^\pm}$	0.3 ± 1.3 (S = 1.4)
$\Gamma_{\rho(770)^+} - \Gamma_{\rho(770)^-}$	1.8 ± 2.1

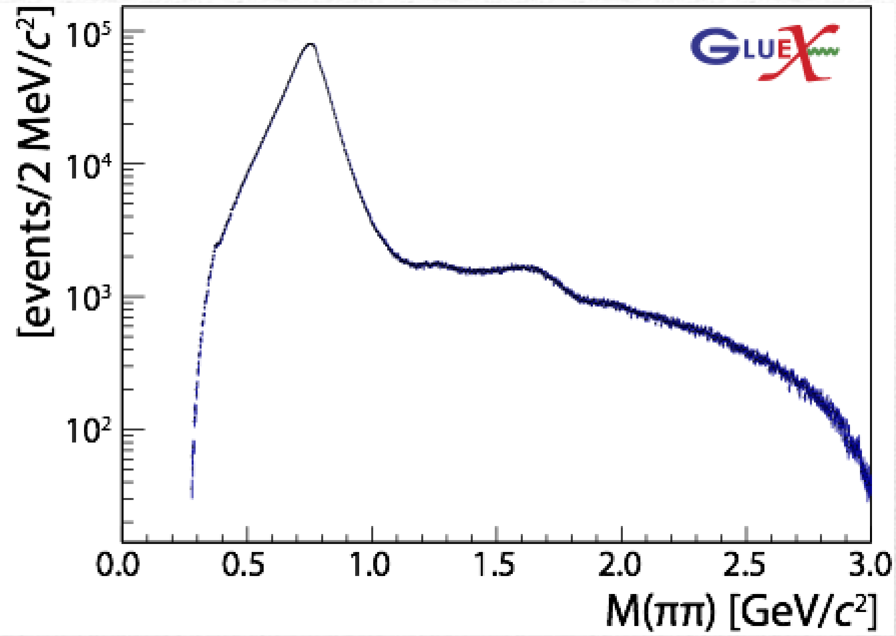
Hadron Spectroscopy

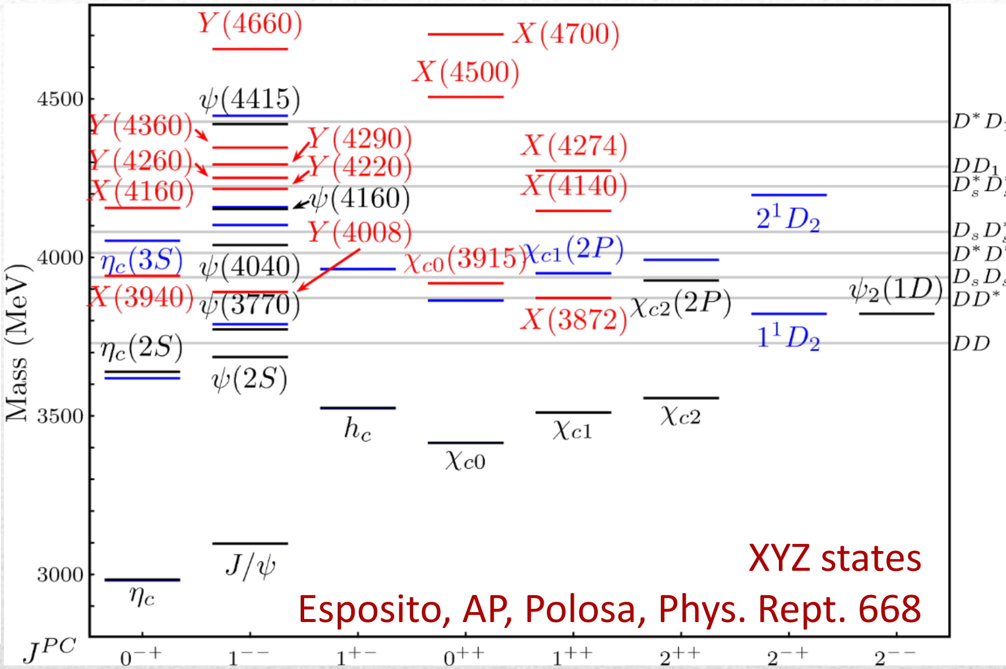
$a_1(1260)$ WIDTH

INSPIRE search

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
250 to 600	**OUR ESTIMATE**				
$367 \pm 9 ^{+28}_{-25}$	420k	ALEKSEEV	2010	COMP	$190\ \pi^- \to \pi^-\pi^-\pi^+ Pb'$
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$410 \pm 31 \pm 30$		1 AUBERT	2007AU	BABR	$10.6\ e^+ e^- \to \rho^0 \rho^\pm \pi^\mp \gamma$
520 - 680	6360	2 LINK	2007A	FOCS	$D^0 \to \pi^-\pi^+\pi^-\pi^+$
480 ± 20		3 GOMEZ-DUMM	2004	RVUE	$\tau^+ \to \pi^+\pi^+\pi^-\nu_\tau$
580 ± 41	90k	SALVINI	2004	OBLX	$\overline{p}\, p \to 2\pi^+ 2\pi^-$
460 ± 85	205	4 DRUTSKOY	2002	BELL	$B^{(*)} K^- K^{*0}$
$814 \pm 36 \pm 13$	37k	5 ASNER	2000	CLE2	$10.6\ e^+ e^- \to \tau^+\tau^-,\ \tau^- \to \pi^-\pi^0\pi^0\nu_\tau$

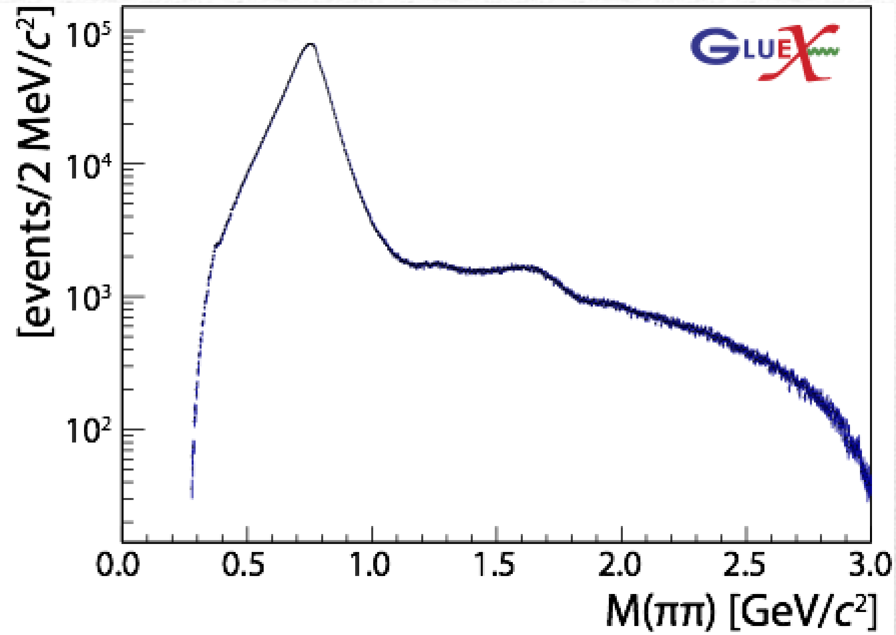
Hadron Spectroscopy

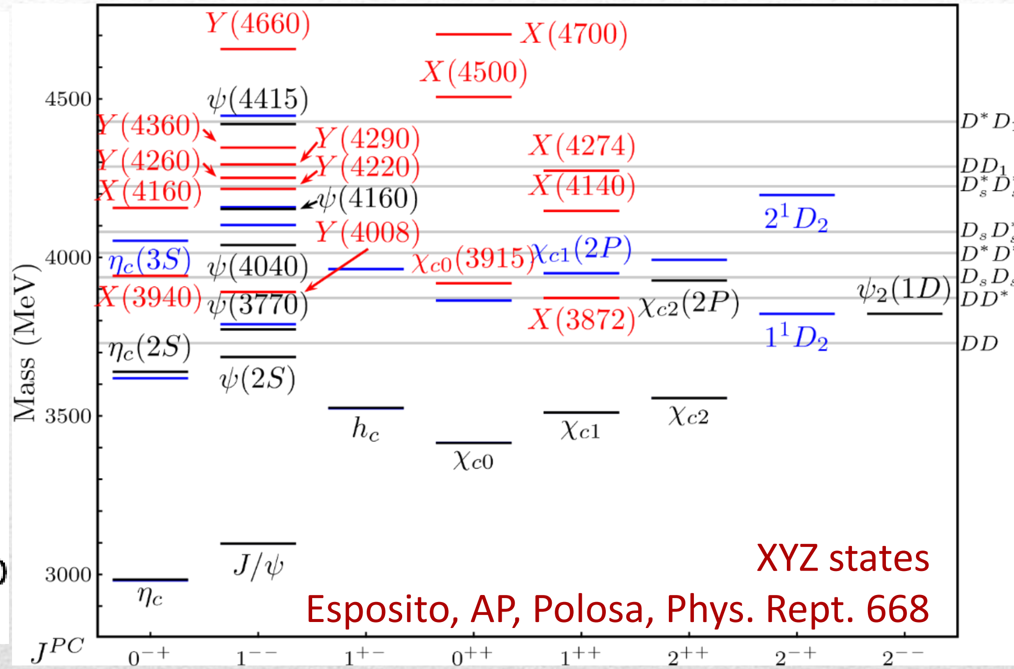


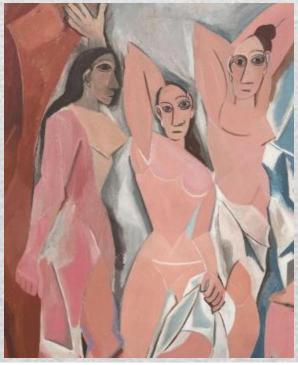


Data → Amplitude analysis → Fundamental properties, Model building

Hadron Spectroscopy

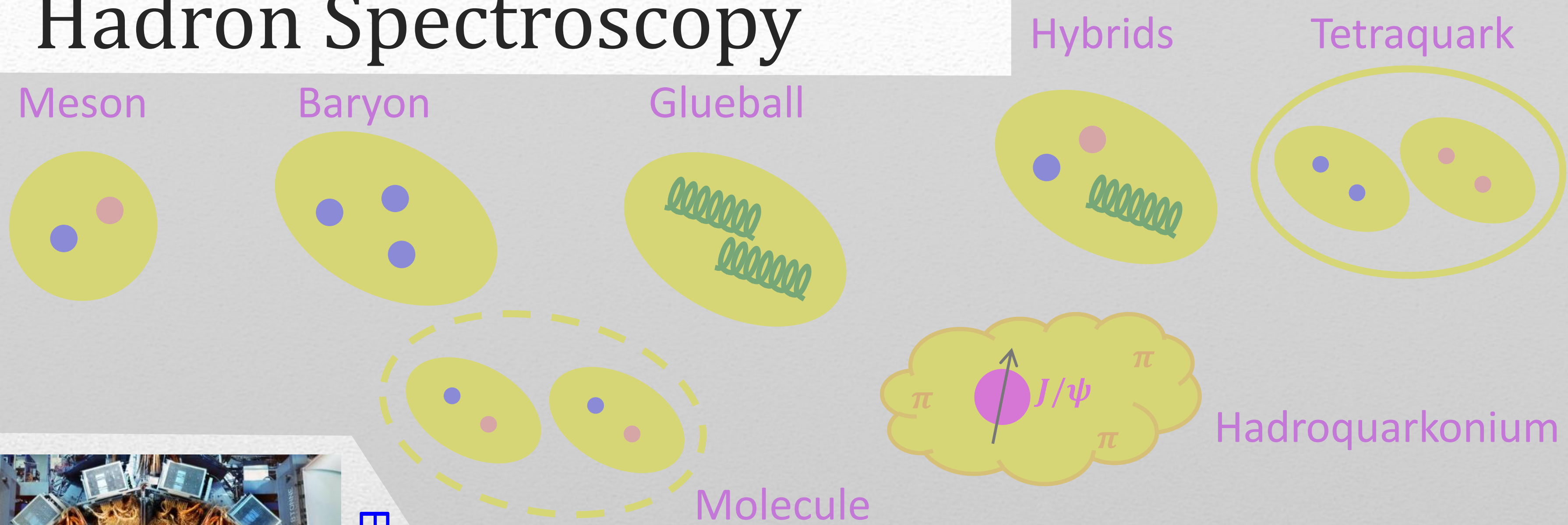




Data →  → Fundamental properties, Model building

Improvement needed! With great statistics comes great responsibility!

Hadron Spectroscopy



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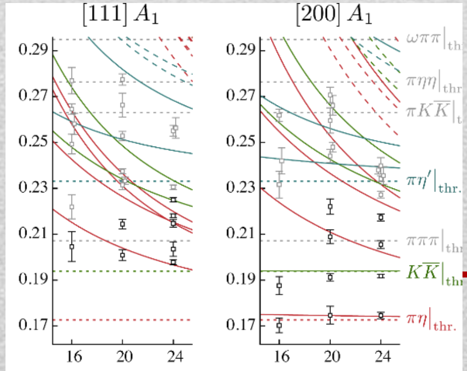
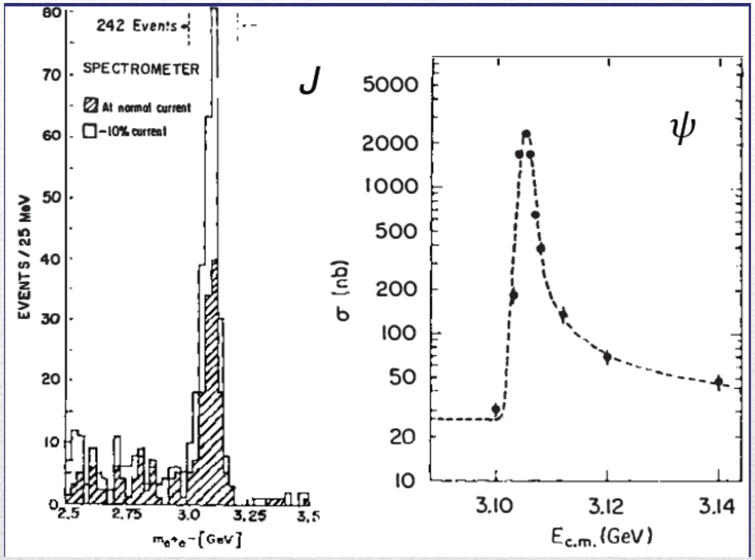
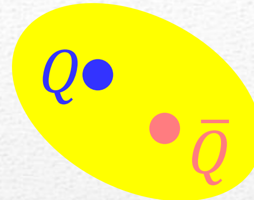


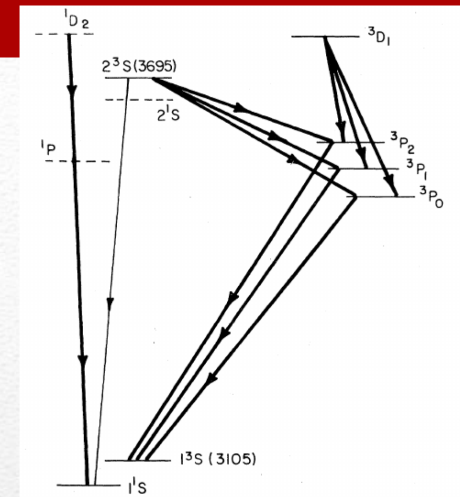
Lattice QCD

Data → Amplitude analysis → Properties, Model building

Quarkonium orthodoxy







Potential models

(meaningful when $M_Q \to \infty$)

$$V(r) = -\frac{C_F \alpha_s}{r} + \sigma r$$

(Cornell potential)

Solve NR Schrödinger eq. → spectrum

$$\alpha_s(M_Q) \sim 0.3$$

(perturbative regime)

OZI-rule, QCD multipole

Effective theories

(HQET, NRQCD, pNRQCD...)

Integrate out heavy DOF

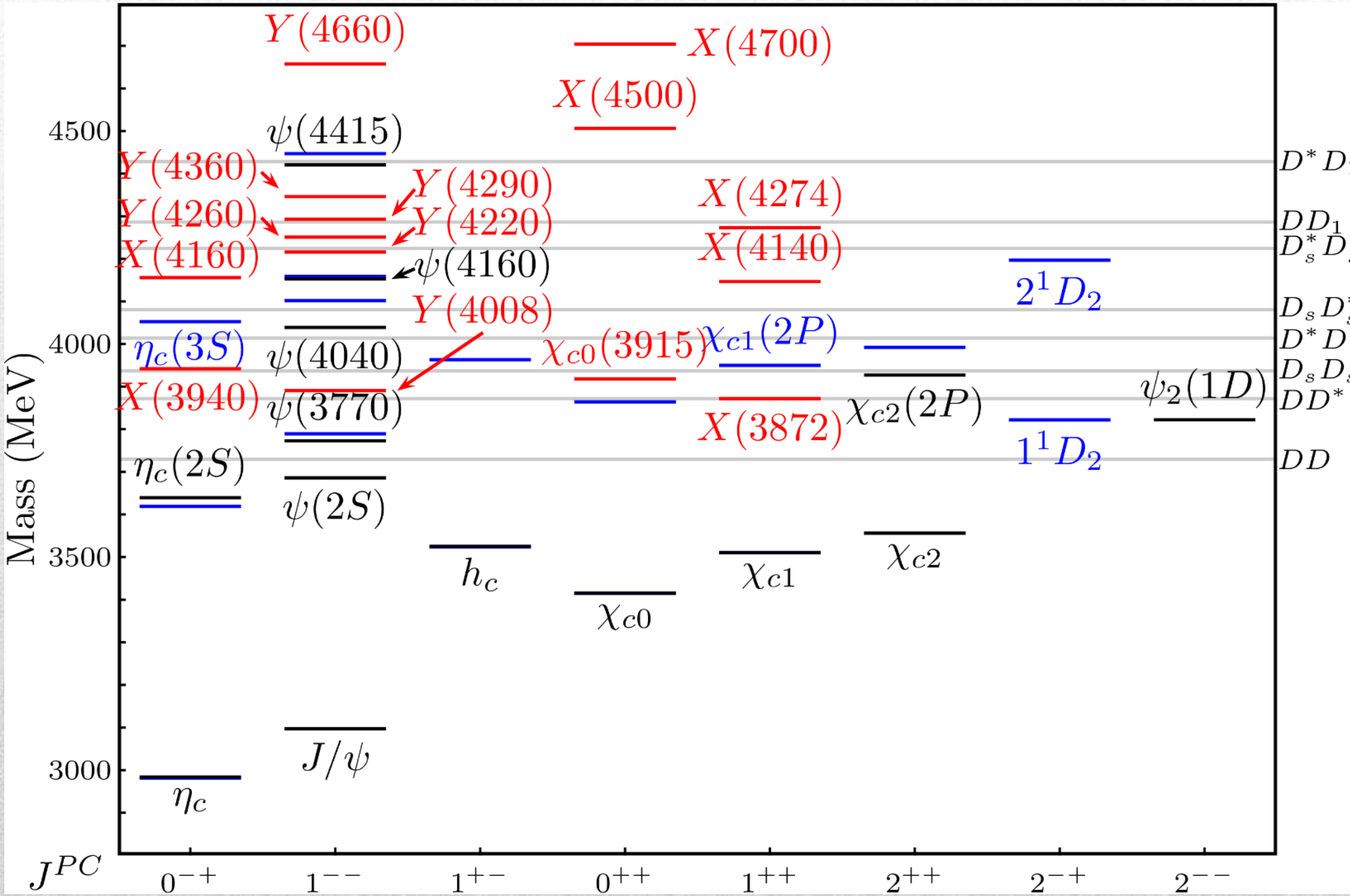
↓

(spectrum), decay & production rates

Heavy quark spin flip suppressed by quark mass, approximate heavy quark spin symmetry (HQSS)

Exotic landscape

Esposito, AP, Polosa, Phys.Rept. 668

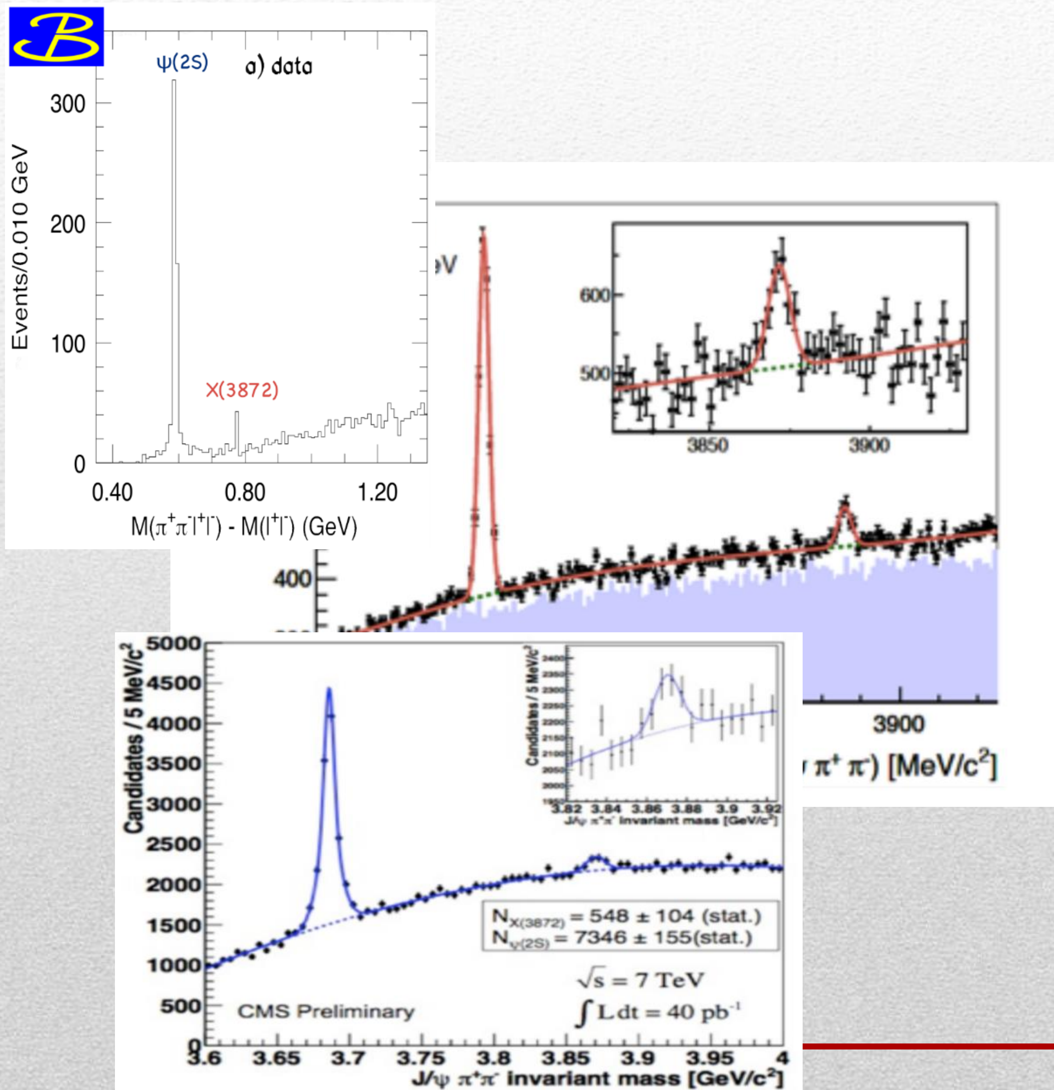


A host of unexpected resonances have appeared

decaying mostly into charmonium + light

Hardly reconciled with usual charmonium interpretation

$X(3872)$



- Discovered in
  $B \to K\,X \to K\,J/\psi\,\pi\pi$
- Quantum numbers 1^{++}
- Very close to DD^* threshold
- Too narrow for an above-treshold charmonium
- Isospin violation too big
  $\frac{\Gamma(X\to J/\psi\ \omega)}{\Gamma(X\to J/\psi\ \rho)} \sim 0.8 \pm 0.3$
- Mass prediction not compatible with $\chi_{c1}(2P)$

$M = 3871.68 \pm 0.17\ \text{MeV}$
$M_X - M_{DD^*} = -3 \pm 192\ \text{keV}$
$\Gamma < 1.2\ \text{MeV}\ @90\%$

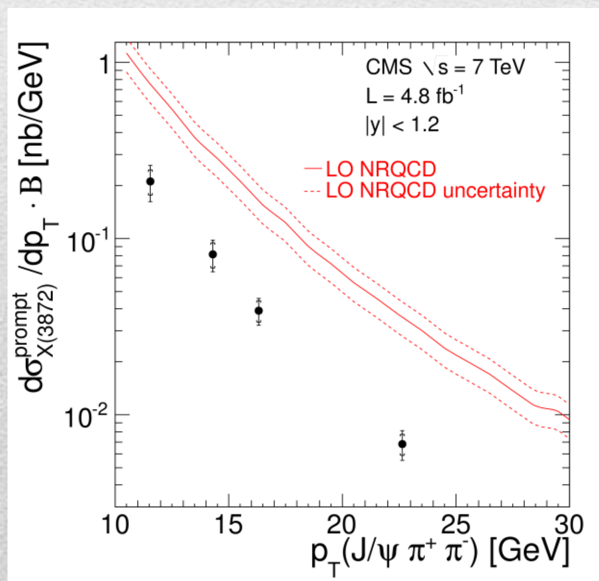
$X(3872)$

Large prompt production
at hadron colliders

$\sigma_B/\sigma_{TOT} = (26.3 \pm 2.3 \pm 1.6)\%$

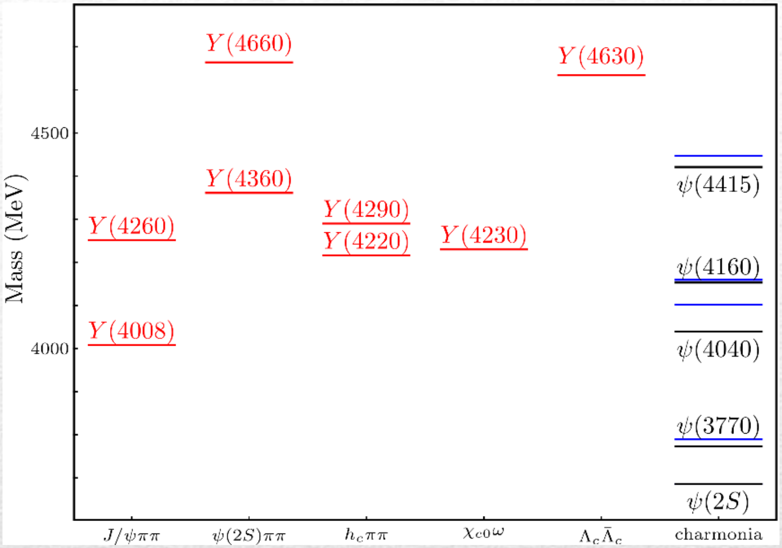
$\sigma_{PR} \times B(X \to J/\psi\pi\pi)$
$= (1.06 \pm 0.11 \pm 0.15)$ nb

CMS, JHEP 1304, 154



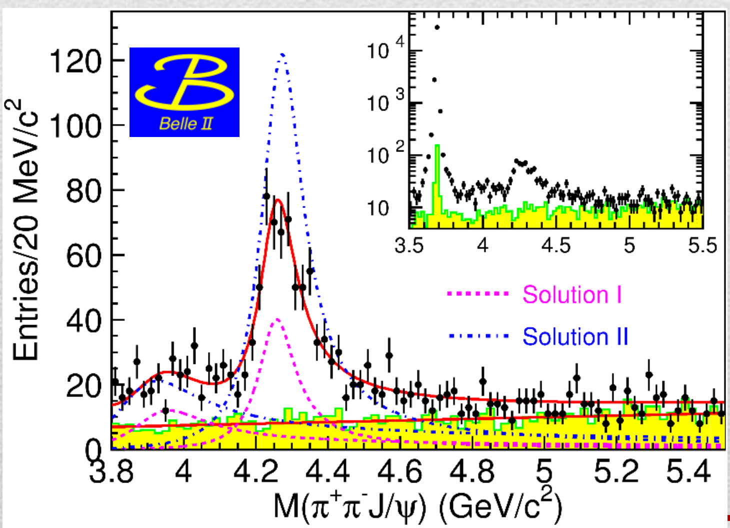
B decay mode	X decay mode	product branching fraction ($\times 10^5$)		B_{fit}	R_{fit}
K^+X	$X \to \pi\pi J/\psi$	$\mathbf{0.86 \pm 0.08}$	(*BABAR*[26], Belle[25])	$0.081^{+0.019}_{-0.031}$	1
		$0.84 \pm 0.15 \pm 0.07$	*BABAR*[26]		
		$0.86 \pm 0.08 \pm 0.05$	Belle[25]		
K^0X	$X \to \pi\pi J/\psi$	$\mathbf{0.41 \pm 0.11}$	(*BABAR*[26], Belle[25])		
		$0.35 \pm 0.19 \pm 0.04$	*BABAR*[26]		
		$0.43 \pm 0.12 \pm 0.04$	Belle[25]		
$(K^+\pi^-)_{NR}X$	$X \to \pi\pi J/\psi$	$0.81 \pm 0.20^{+0.11}_{-0.14}$	Belle[106]		
$K^{*0}X$	$X \to \pi\pi J/\psi$	< 0.34, 90% C.L.	Belle[106]		
KX	$X \to \omega J/\psi$	$R = 0.8 \pm 0.3$	*BABAR*[33]	$0.061^{+0.024}_{-0.036}$	$0.77^{+0.28}_{-0.32}$
K^+X		$0.6 \pm 0.2 \pm 0.1$	*BABAR*[33]		
K^0X		$0.6 \pm 0.3 \pm 0.1$	*BABAR*[33]		
KX	$X \to \pi\pi\pi^0 J/\psi$	$R = 1.0 \pm 0.4 \pm 0.3$	Belle[32]		
K^+X	$X \to D^{*0}\bar{D}^0$	$\mathbf{8.5 \pm 2.6}$	(*BABAR*[38], Belle[37])	$0.614^{+0.166}_{-0.074}$	$8.2^{+2.3}_{-2.8}$
		$16.7 \pm 3.6 \pm 4.7$	*BABAR*[38]		
		$7.7 \pm 1.6 \pm 1.0$	Belle[37]		
K^0X	$X \to D^{*0}\bar{D}^0$	$\mathbf{12 \pm 4}$	(*BABAR*[38], Belle[37])		
		$22 \pm 10 \pm 4$	*BABAR*[38]		
		$9.7 \pm 4.6 \pm 1.3$	Belle[37]		
K^+X	$X \to \gamma J/\psi$	$\mathbf{0.202 \pm 0.038}$	(*BABAR*[35], Belle[34])	$0.019^{+0.005}_{-0.009}$	$0.24^{+0.05}_{-0.06}$
K^+X		$0.28 \pm 0.08 \pm 0.01$	*BABAR*[35]		
		$0.178^{+0.048}_{-0.044} \pm 0.012$	Belle[34]		
K^0X		$0.26 \pm 0.18 \pm 0.02$	*BABAR*[35]		
		$0.124^{+0.076}_{-0.061} \pm 0.011$	Belle[34]		
K^+X	$X \to \gamma\psi(2S)$	$\mathbf{0.44 \pm 0.12}$	*BABAR*[35]	$0.04^{+0.015}_{-0.020}$	$0.51^{+0.13}_{-0.17}$
K^+X		$0.95 \pm 0.27 \pm 0.06$	*BABAR*[35]		
		$0.083^{+0.198}_{-0.183} \pm 0.044$	Belle[34]		
		$R' = 2.46 \pm 0.64 \pm 0.29$	LHCb[36]		
K^0X		$1.14 \pm 0.55 \pm 0.10$	*BABAR*[35]		
		$0.112^{+0.357}_{-0.290} \pm 0.057$	Belle[34]		
K^+X	$X \to \gamma\chi_{c1}$	$< 9.6 \times 10^{-3}$	Belle[23]	$< 1.0 \times 10^{-3}$	< 0.014
K^+X	$X \to \gamma\chi_{c2}$	< 0.016	Belle[23]	$< 1.7 \times 10^{-3}$	< 0.024
KX	$X \to \gamma\gamma$	$< 4.5 \times 10^{-3}$	Belle[111]	$< 4.7 \times 10^{-4}$	$< 6.6 \times 10^{-3}$
KX	$X \to \eta J/\psi$	< 1.05	*BABAR*[112]	< 0.11	< 1.55
K^+X	$X \to p\bar{p}$	$< 9.6 \times 10^{-4}$	LHCb[110]	$< 1.6 \times 10^{-4}$	$< 2.2 \times 10^{-3}$

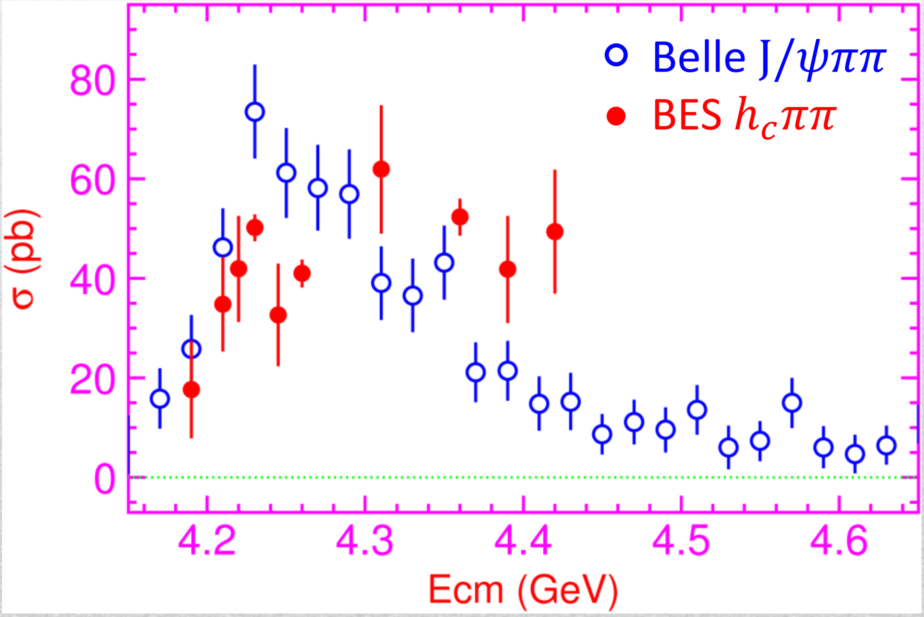
Vector Y states



Lots of unexpected $J^{PC} = 1^{--}$ states
found in ISR/direct production (and nowhere else!)
Seen in few final states,
mostly $J/\psi\ \pi\pi$ and $\psi(2S)\ \pi\pi$

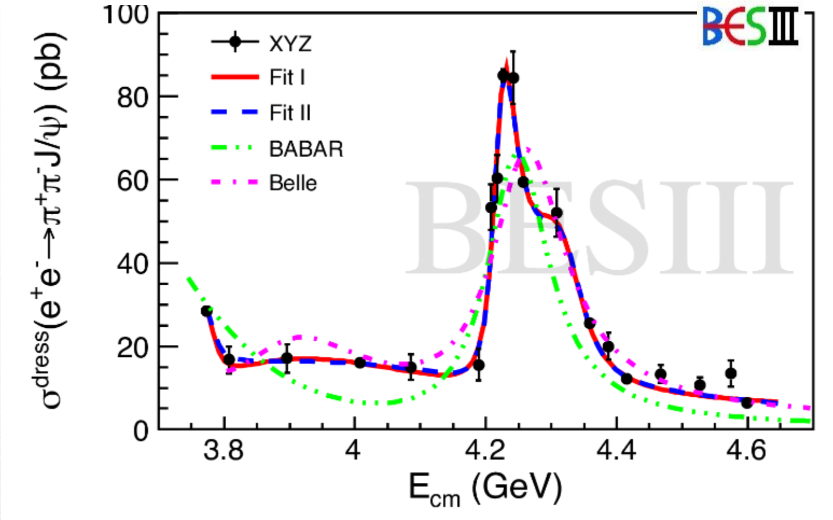
Not seen decaying into open charm pairs
Large HQSS violation





Vector Y states in BESIII

BESIII, PRL118, 092001 (2017) $e^+e^- \to J/\psi\, \pi\pi$



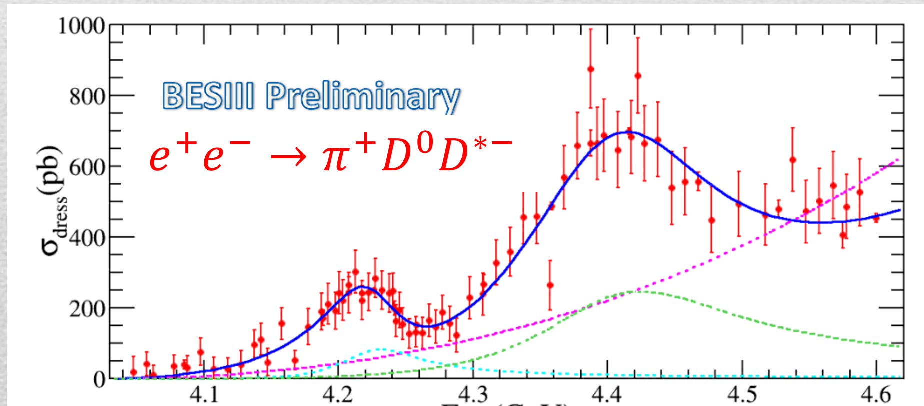
BESIII, PRL118, 092002 (2017)

Parameters	Solution I	Solution II
$\Gamma_{e^+e^-}\mathcal{B}[\psi(3770)\to\pi^+\pi^- J/\psi]$		0.5 ± 0.1 (0
$\Gamma_{e^+e^-}\mathcal{B}(R_1\to\pi^+\pi^- J/\psi)$	$8.8^{+1.5}_{-2.2}$ $(\cdots)$	$6.8^{+1.1}_{-1.5}$ $(\cdots)$
$\Gamma_{e^+e^-}\mathcal{B}(R_2\to\pi^+\pi^- J/\psi)$	13.3 ± 1.4 (12.0 ± 1.0)	9.2 ± 0.7 (8.9 ± 0.6)
$\Gamma_{e^+e^-}\mathcal{B}(R_3\to\pi^+\pi^- J/\psi)$	21.1 ± 3.9 (17.9 ± 3.3)	$1.7^{+0.8}_{-0.6}$ $(1.1^{+0.5}_{-0.4})$
ϕ_1	-58 ± 11 (-33 ± 8)	-116^{+9}_{-10} (-81^{+7}_{-8})
ϕ_2	-156 ± 5 (-132 ± 3)	68 ± 24 (107 ± 20)

New BESIII data show a peculiar lineshape for the $Y(4260)$

The state appear lighter and narrower, compatible with the ones in $h_c\pi\pi$ and $\chi_{c0}\omega$

A broader old-fashioned $Y(4260)$ is appearing in $\bar{D}D^*\pi$, maybe indicating a $\bar{D}D_1$ dominance

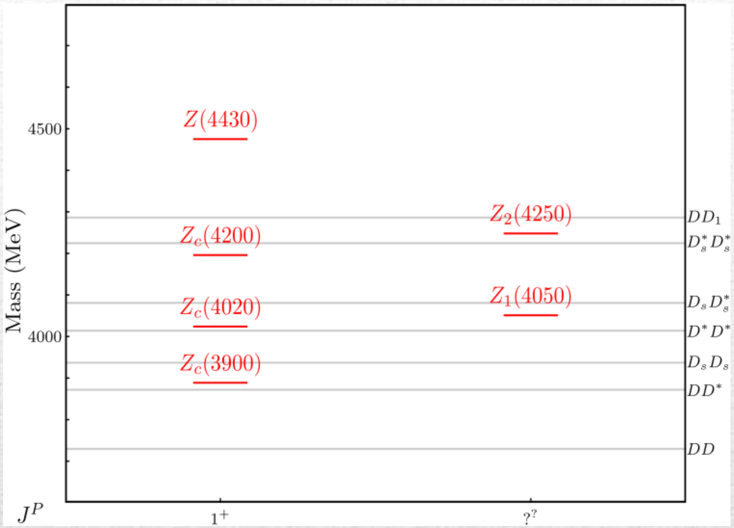


Fit with a constant (pink dashed triple-dot line) and two constant width relativistic BW functions (green dashed double-dot line and aqua dashed line).

$M(Y(4220))=(4224.8\pm5.6\pm4.0)$ MeV/c^2, $\Gamma(Y(4220))=(72.3\pm9.1\pm0.9)$ MeV.
$M(Y(4390))=(4400.1\pm9.3\pm2.1)$ MeV/c^2, $\Gamma(Y(4220))=(181.7\pm16.9\pm7.4)$ MeV.

Charged Z states: $Z_c(3900), Z_c'(4020)$

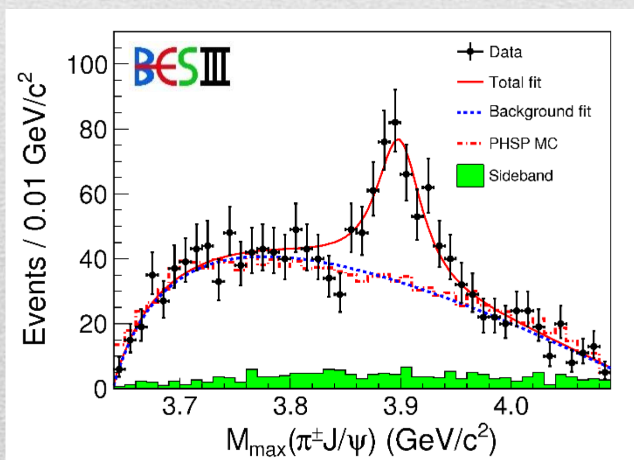
Charged quarkonium-like resonances have been found, **4q needed**

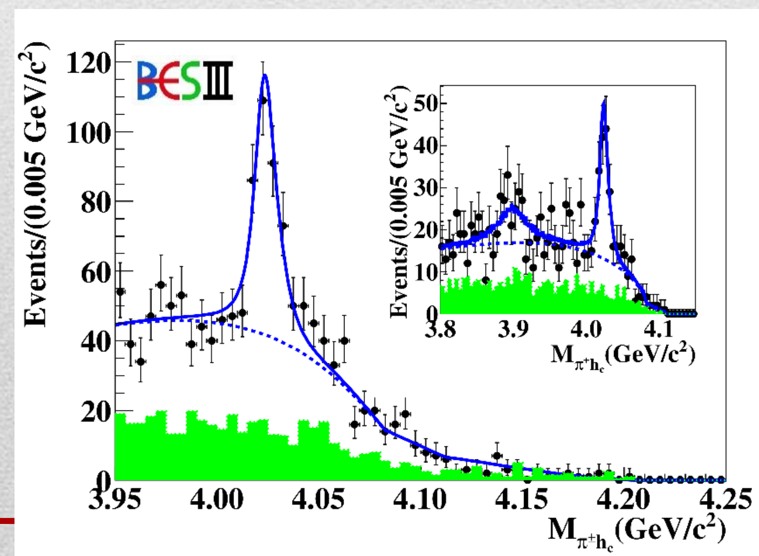


Two states $J^{PC} = 1^{+-}$ appear
slightly above $D^{(*)}D^*$ thresholds

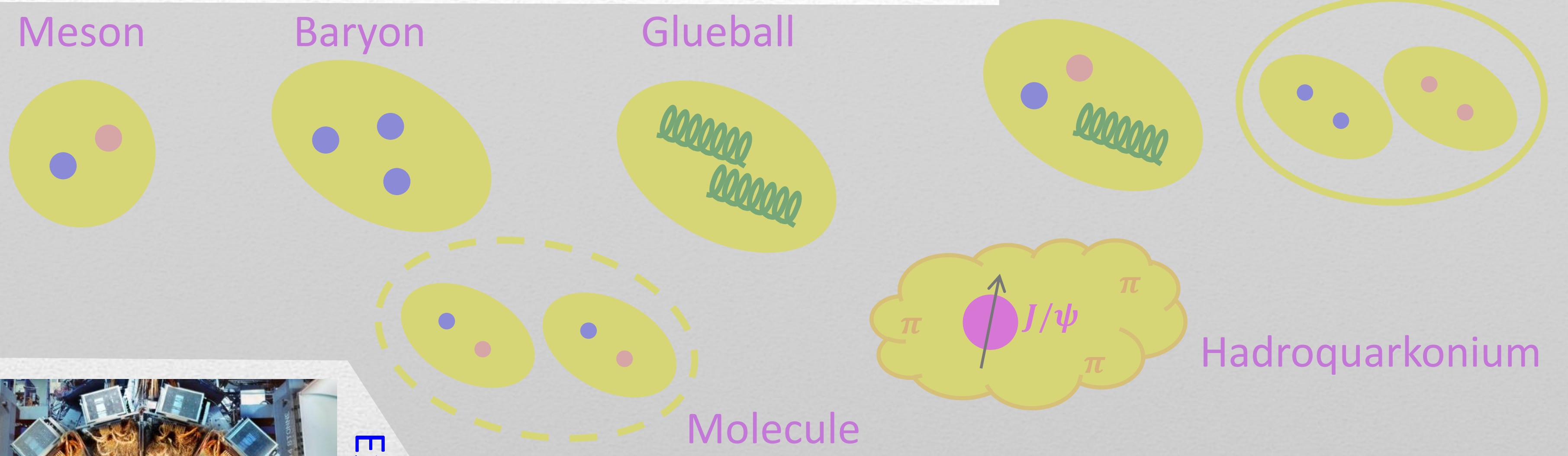
$e^+e^- \to Z_c(3900)^+\pi^- \to J/\psi\, \pi^+\pi^-$ and $\to (DD^*)^+\pi^-$
$M = 3888.7 \pm 3.4$ MeV, $\Gamma = 35 \pm 7$ MeV

$e^+e^- \to Z_c'(4020)^+\pi^- \to h_c\, \pi^+\pi^-$ and $\to \bar{D}^{*0}D^{*+}\pi^-$
$M = 4023.9 \pm 2.4$ MeV, $\Gamma = 10 \pm 6$ MeV





Hadron Spectroscopy



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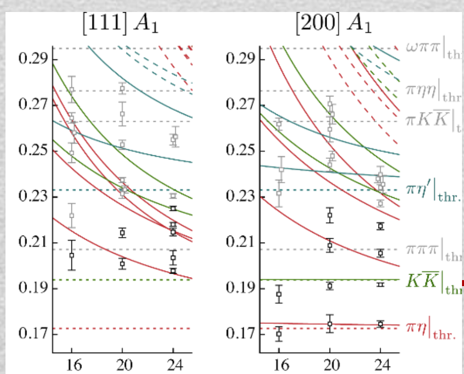
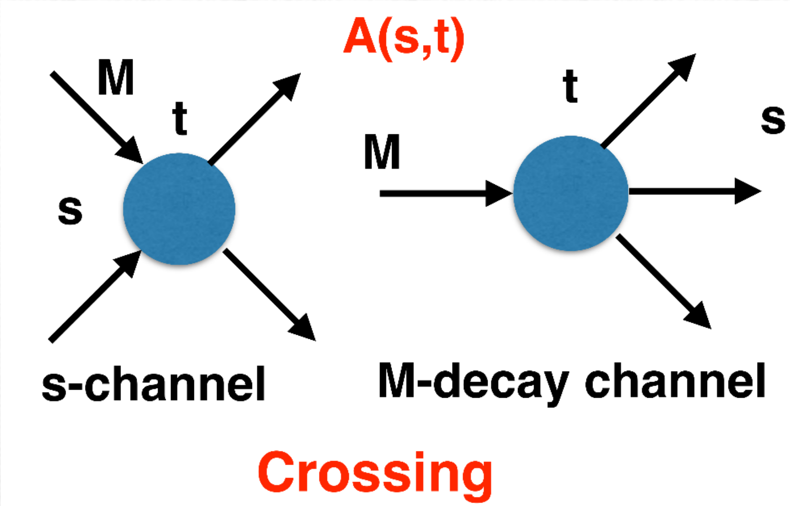


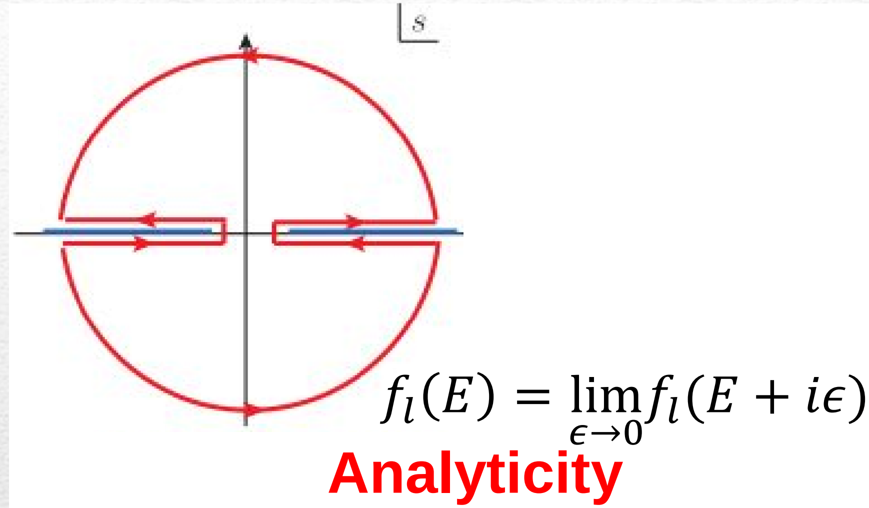
Lattice QCD

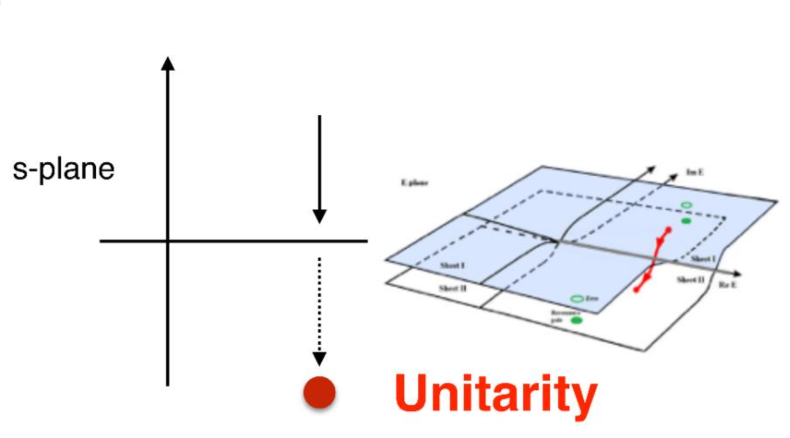
Data → Amplitude analysis → Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

S-Matrix principles





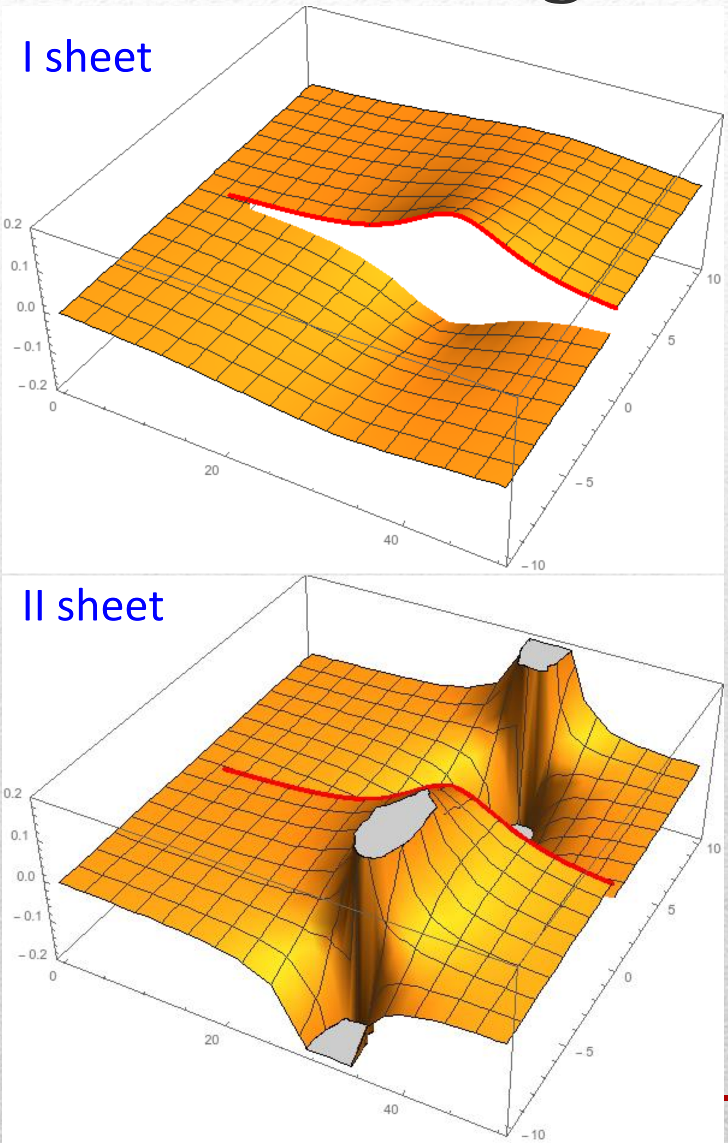


+ Lorentz, discrete & global symmetries

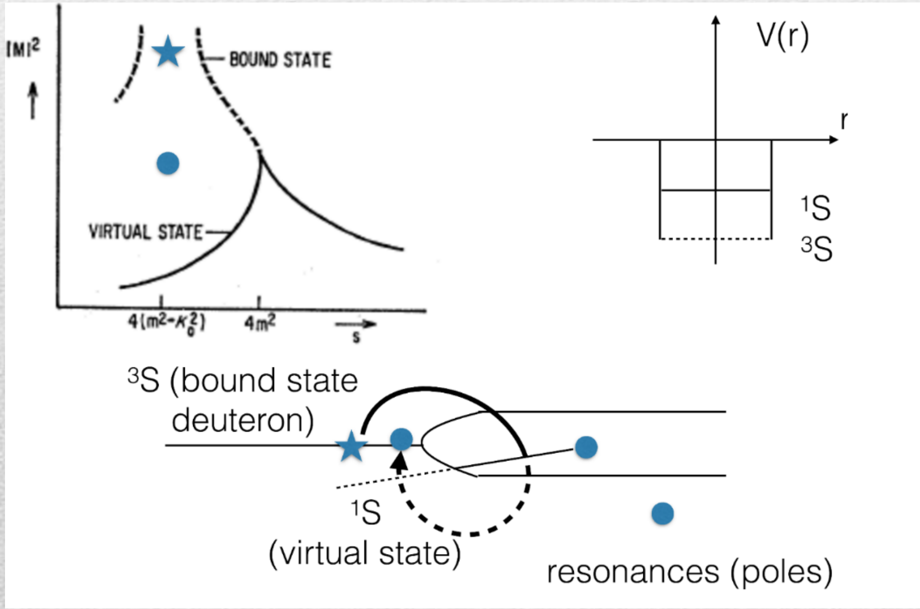
These are constraints the amplitudes have to satisfy, but do not fix the dynamics

Resonances (QCD states) are poles in the unphysical Riemann sheets

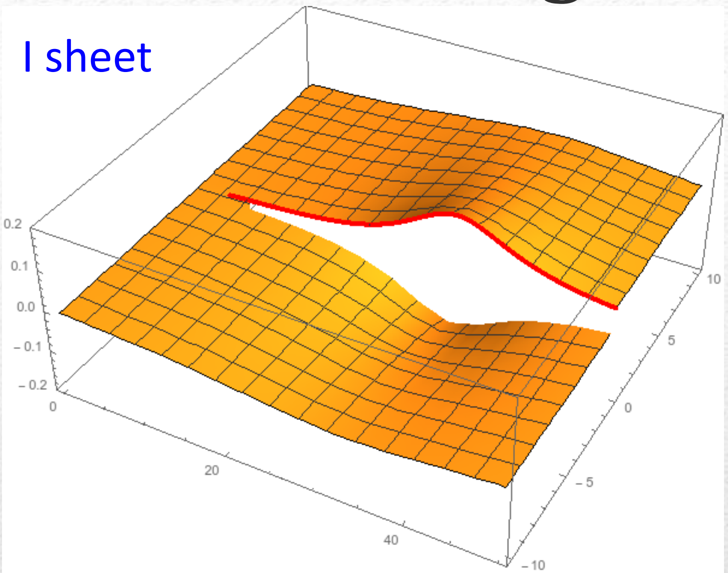
Pole hunting

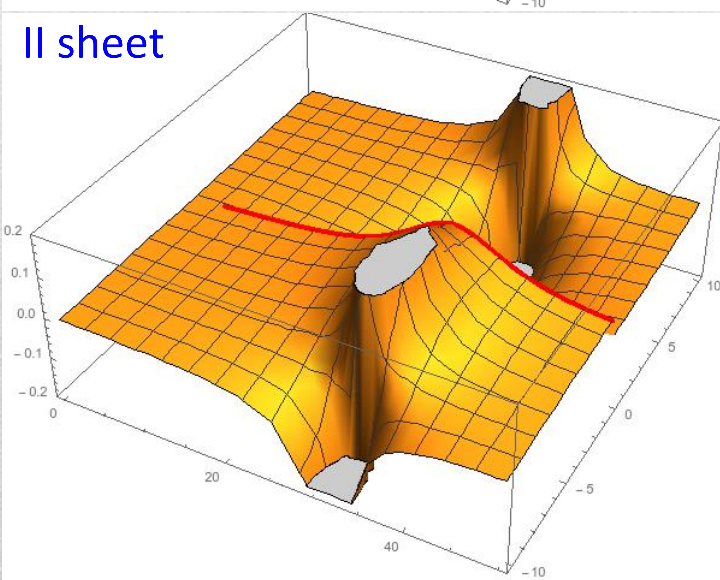


Bound states on the real axis 1st sheet
Not-so-bound (virtual) states on the real axis 2nd sheet



Pole hunting

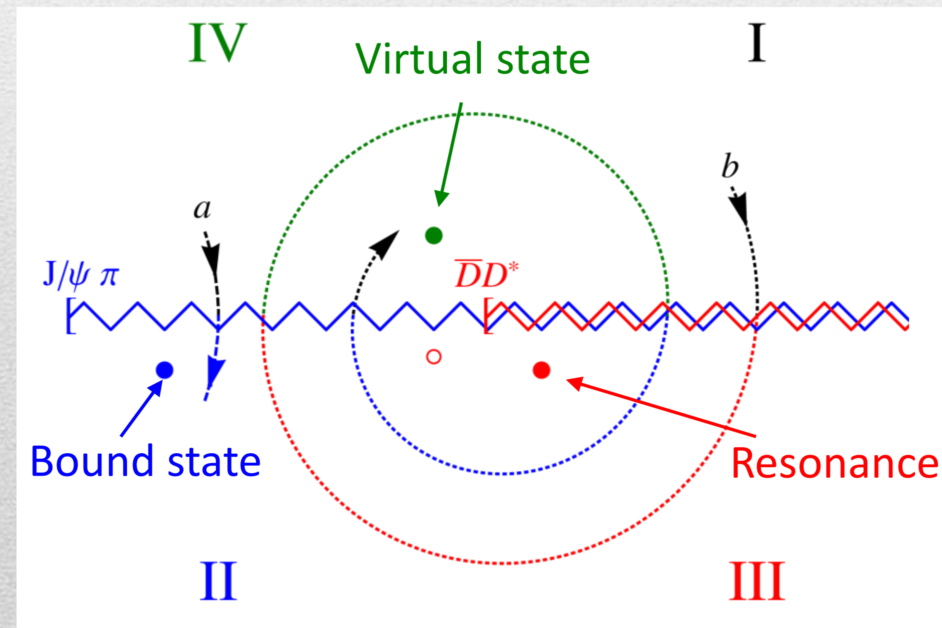




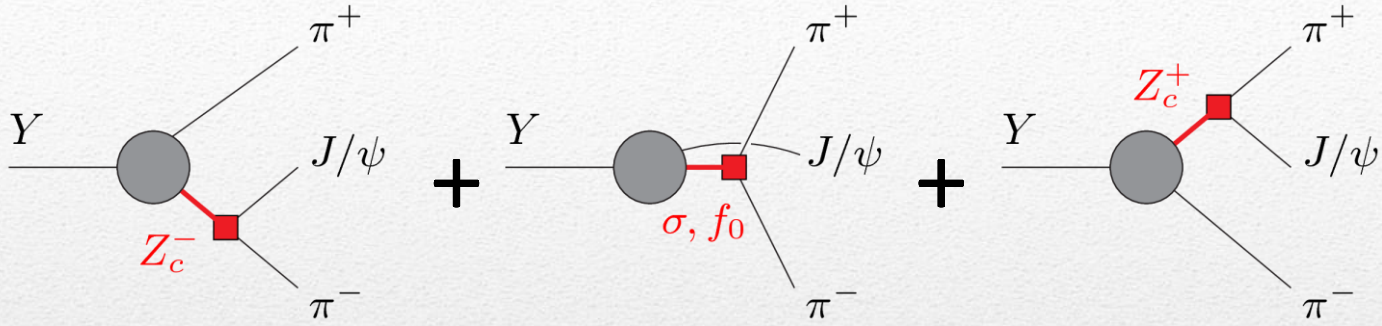
More complicated structure when more thresholds arise: two sheets for each new threshold

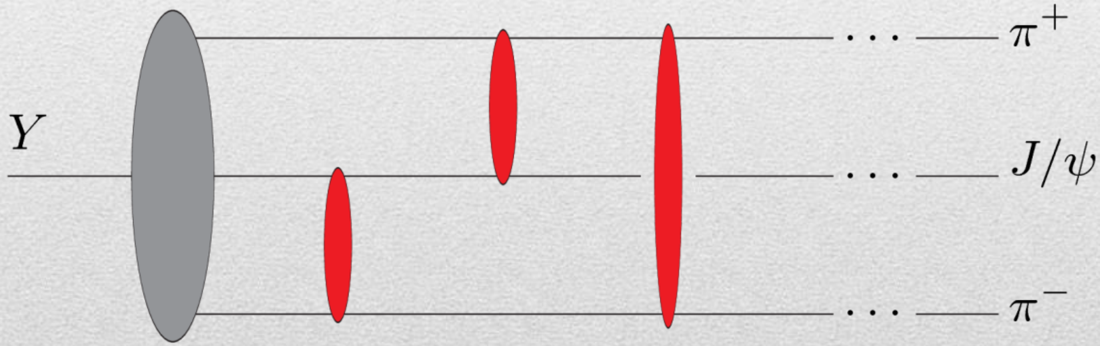
III sheet: usual resonances

IV sheet: cusps (virtual states)



The isobar model





Khuri-Treiman formalism was introduced to describe $K \to 3\pi$

Khuri and Treiman, PR119, 1115

Used recently for several reactions,

Niecknig and Kubis, JHEP 10, 142

Colangelo, *et al.*, PRL118, 022001

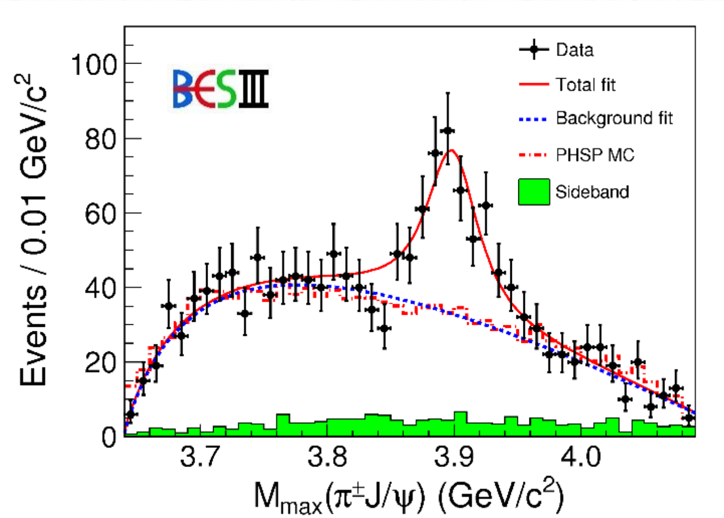
AP *et al.* [JPAC], PLB772, 200

Albaladejo, AP *et al.* [JPAC], 1803.06027

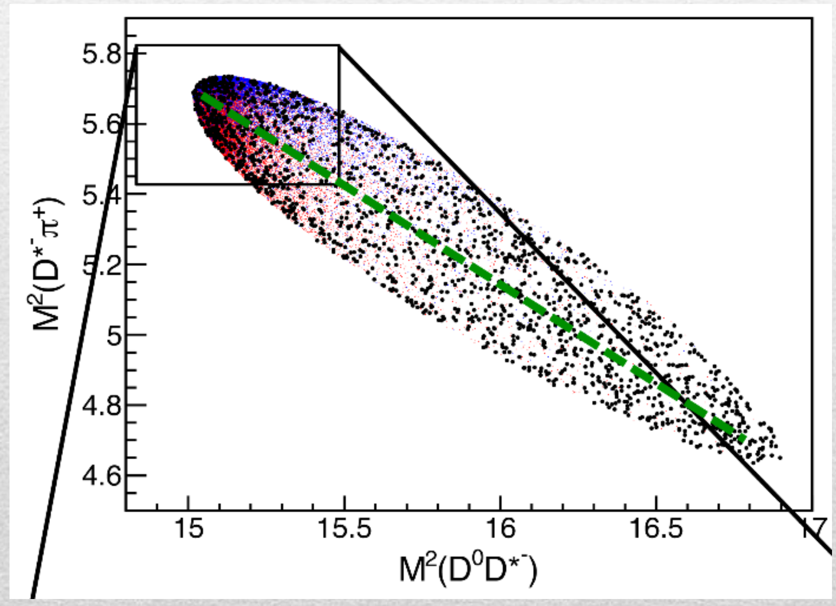
The formalism implements the all-order rescattering in all the 3 channels at once

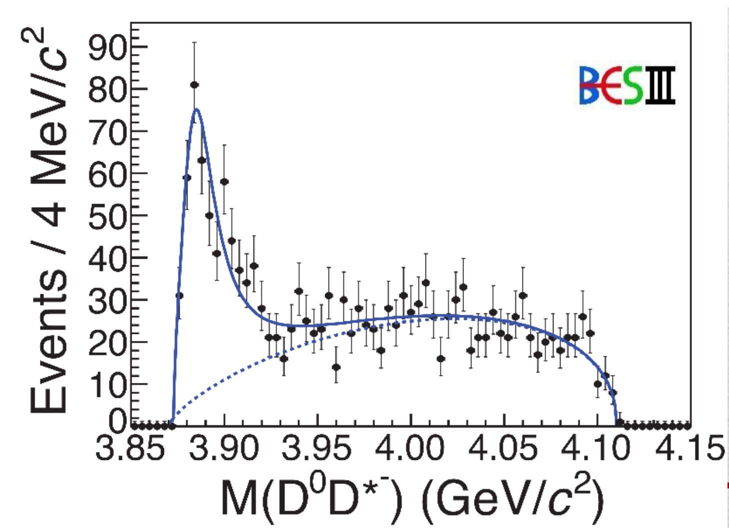
Example: The charged $Z_c(3900)$

A charged charmonium-like resonance has been claimed by BESIII in 2013.



$e^+e^- \to Z_c(3900)^+\pi^- \to J/\psi\,\pi^+\pi^-$ and $\to (DD^*)^+\pi^-$
$M = 3888.7 \pm 3.4$ MeV, $\Gamma = 35 \pm 7$ MeV



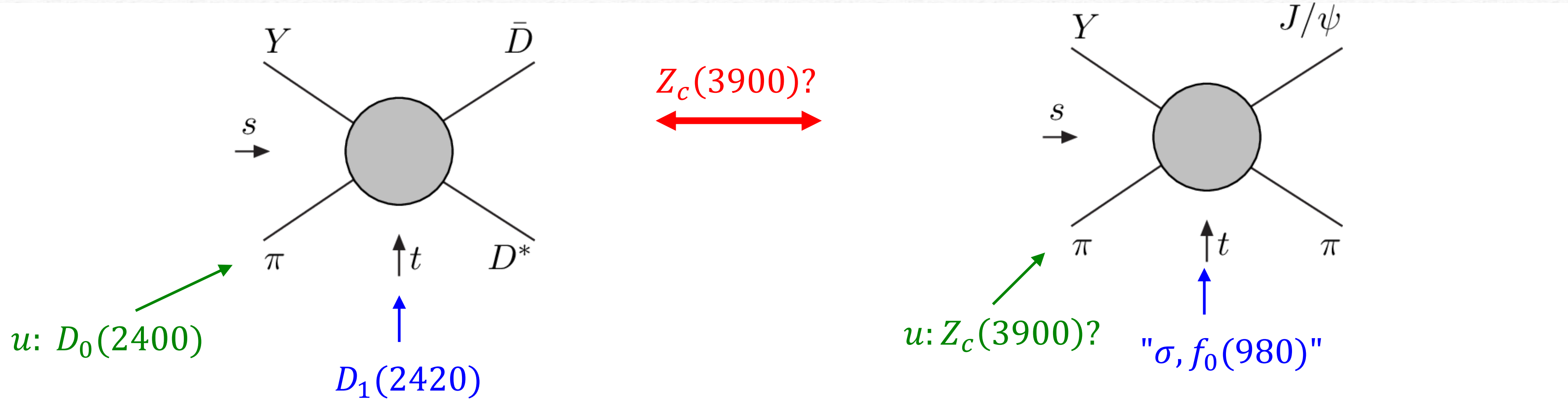


Such a state would require a minimal 4q content and would be manifestly exotic

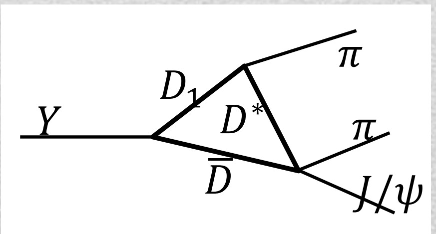
Amplitude analysis for $Z_c(3900)$

One can test different parametrizations of the amplitude, which correspond to
different singularities → different natures

AP *et al.* (JPAC), PLB772, 200

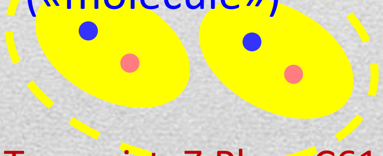


Triangle rescattering,
logarithmic branching point



Szczepaniak, PLB747, 410

(anti)bound state,
II/IV sheet pole
(«molecule»)

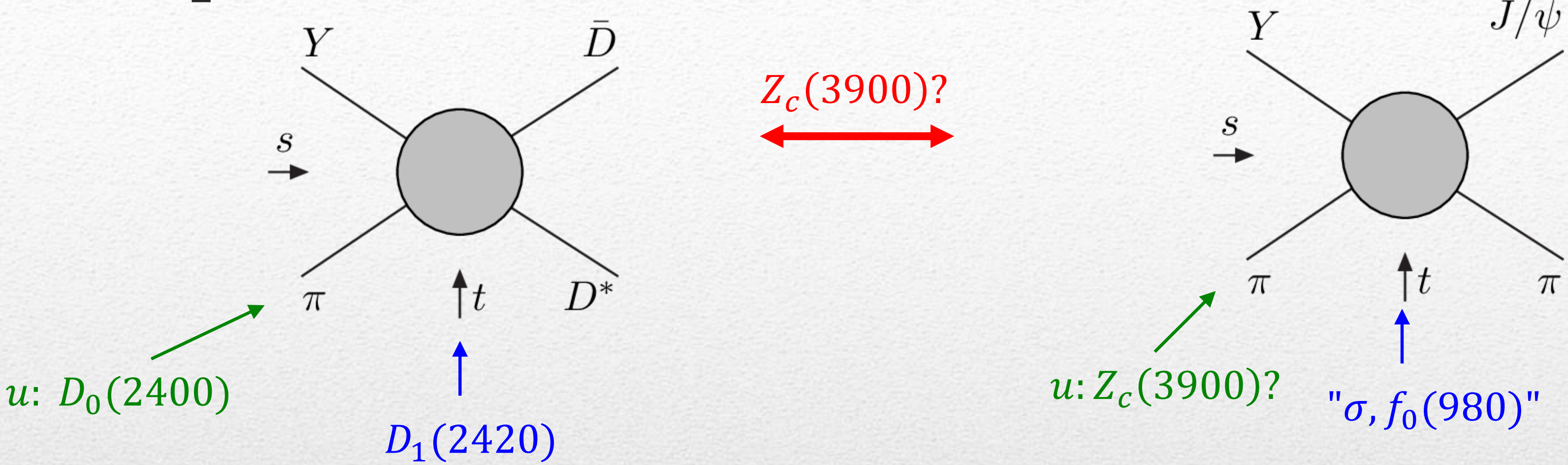


Tornqvist, Z.Phys. C61, 525
Swanson, Phys.Rept. 429
Hanhart *et al.* PRL111, 132003

Resonance,
III sheet pole
(«compact state»)

Maiani *et al.*, PRD71, 014028
Faccini *et al.*, PRD87, 111102
Esposito *et al.*, Phys.Rept. 668

Amplitude model



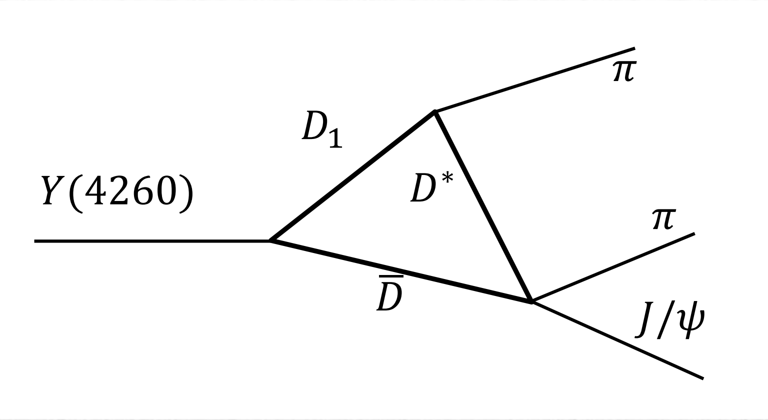
$$f_i(s,t,u) = 16\pi \sum_{l=0}^{L_{\max}} (2l+1)\left(a_{l,i}^{(s)}(s)P_l(z_s) + a_{l,i}^{(t)}(t)P_l(z_t) + a_{l,i}^{(u)}(u)P_l(z_u)\right)$$

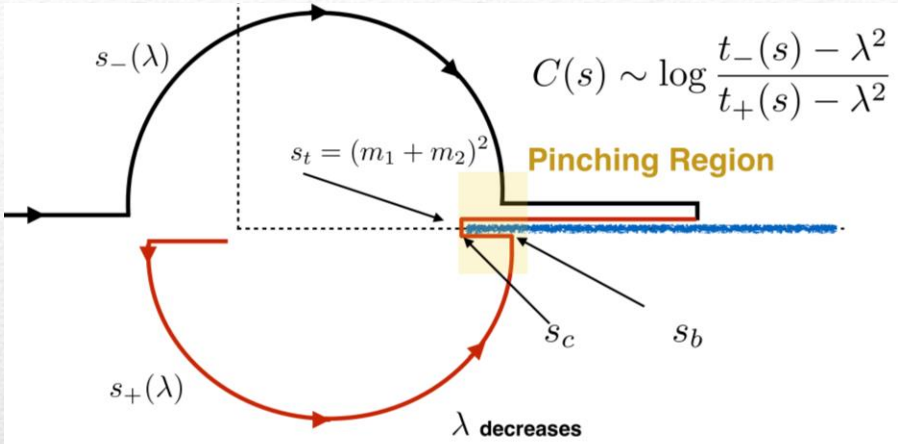
$$f_{0,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, f_i\left(s, t(s,z_s), u(s,z_s)\right) = a_{0,i}^{(s)} + \frac{1}{32\pi}\int_{-1}^{1} dz_s \left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv a_{0,i}^{(s)} + b_{0,i}(s)$$

$$f_{l,i}(s) = \frac{1}{32\pi}\int_{-1}^{1} dz_s\, P_l(z_s)\left(a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u)\right) \equiv b_{l,i}(s) \quad \text{for } l > 0. \quad f_{0,i}(s) = b_{0,i}(s) + \sum_j t_{ij}(s)\frac{1}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'-s},$$

$$f_i(s,t,u) = 16\pi\left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s)\left(c_j + \frac{s}{\pi}\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_{0,j}(s')}{s'\,(s'-s)}\right)\right],$$

Triangle singularity





Logarithmic branch points due to exchanges in the cross channels can simulate a resonant behavior, only in very special kinematical conditions (Coleman and Norton, Nuovo Cim. 38, 438), However, this effects cancels in Dalitz projections, no peaks (Schmid, Phys.Rev. 154, 1363)

$$f_{0,i}(s) = b_{0,i}(s) + \frac{t_{ij}}{\pi} \int_{s_i}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s' - s}$$

...but the cancellation can be spread in different channels, you might still see peaks in other channels only!

Szczepaniak, PLB747, 410-416
Szczepaniak, PLB757, 61-64
Guo, Meissner, Wang, Yang PRD92, 071502

Testing scenarios

- We approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities.
  This also limits the number of free parameters

$$f_i(s,t,u) = 16\pi \left[a_{0,i}^{(t)}(t) + a_{0,i}^{(u)}(u) + \sum_j t_{ij}(s) \left(c_j + \frac{s}{\pi} \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_{0,j}(s')}{s'(s'-s)} \right) \right],$$

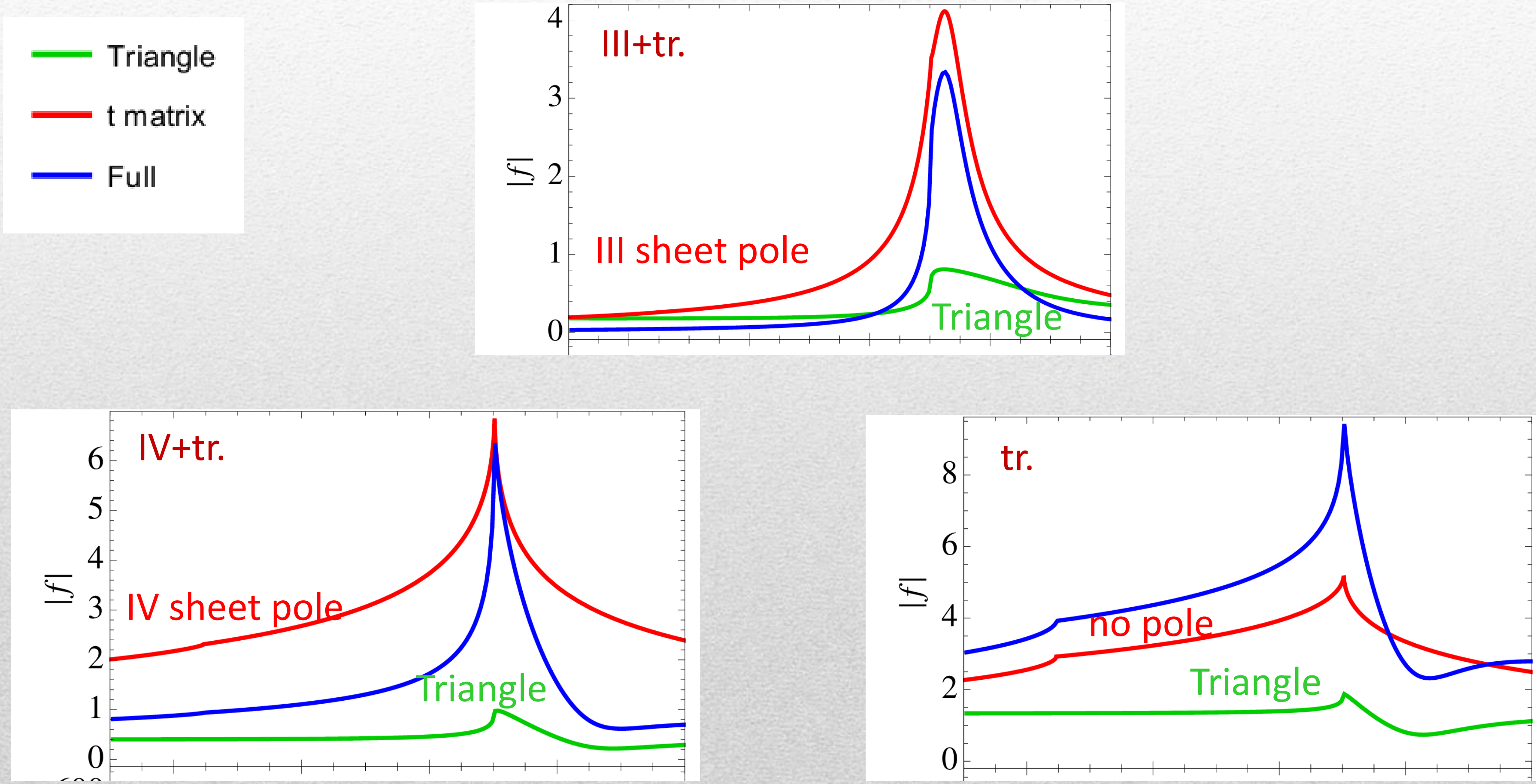
The scattering matrix is parametrized as $(t^{-1})_{ij} = K_{ij} - i\,\rho_i\,\delta_{ij}$

Four different scenarios considered:

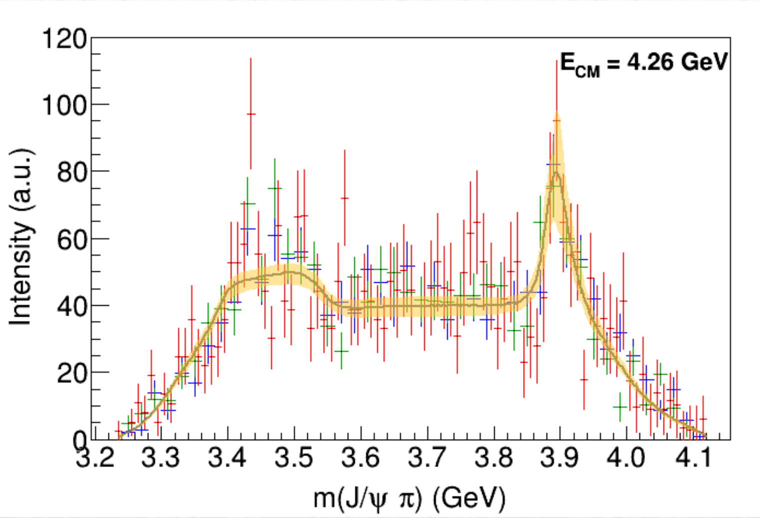
- «III»: the K matrix is $\frac{g_i\,g_j}{M^2-s}$, this generates a pole in the closest unphysical sheet the rescattering integral is set to zero
- «III+tr.»: same, but with the correct value of the rescattering integral
- «IV+tr.»: the K matrix is constant, this generates a pole in the IV sheet
- «tr.»: same, but the pole is pushed far away by adding a penalty in the χ^2

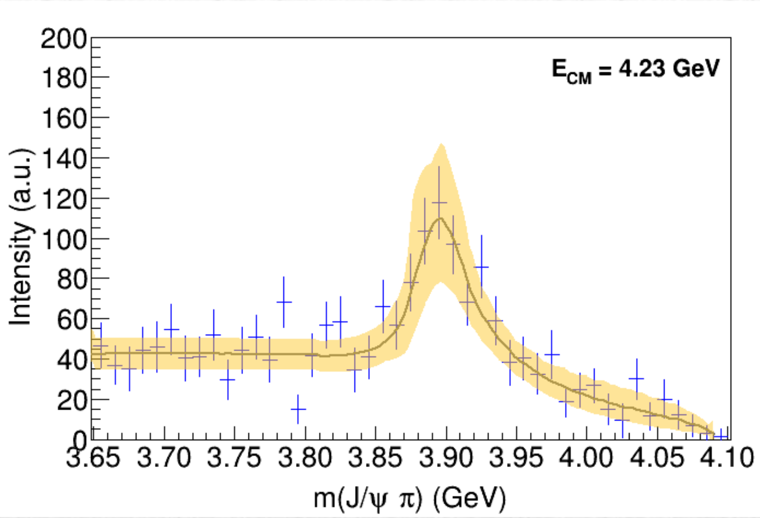
Singularities and lineshapes

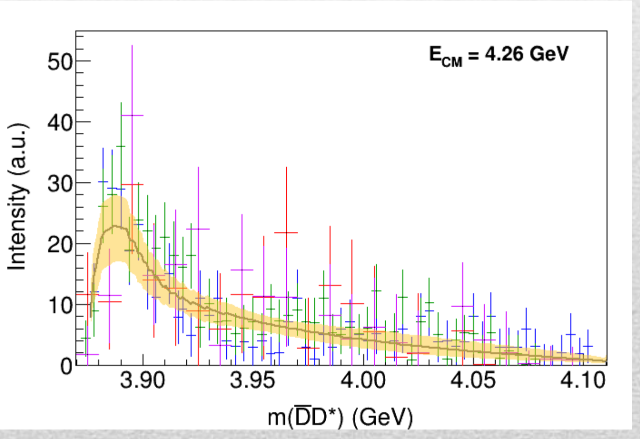
Different lineshapes according to different singularities

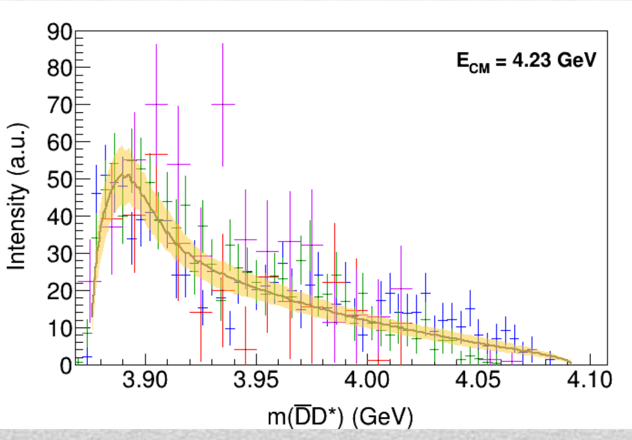


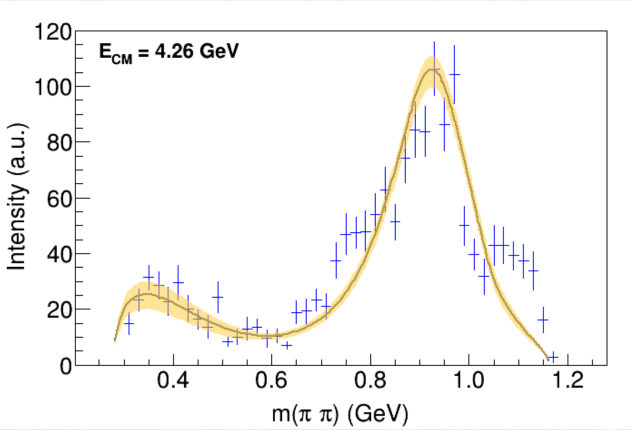
Fit: III


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)

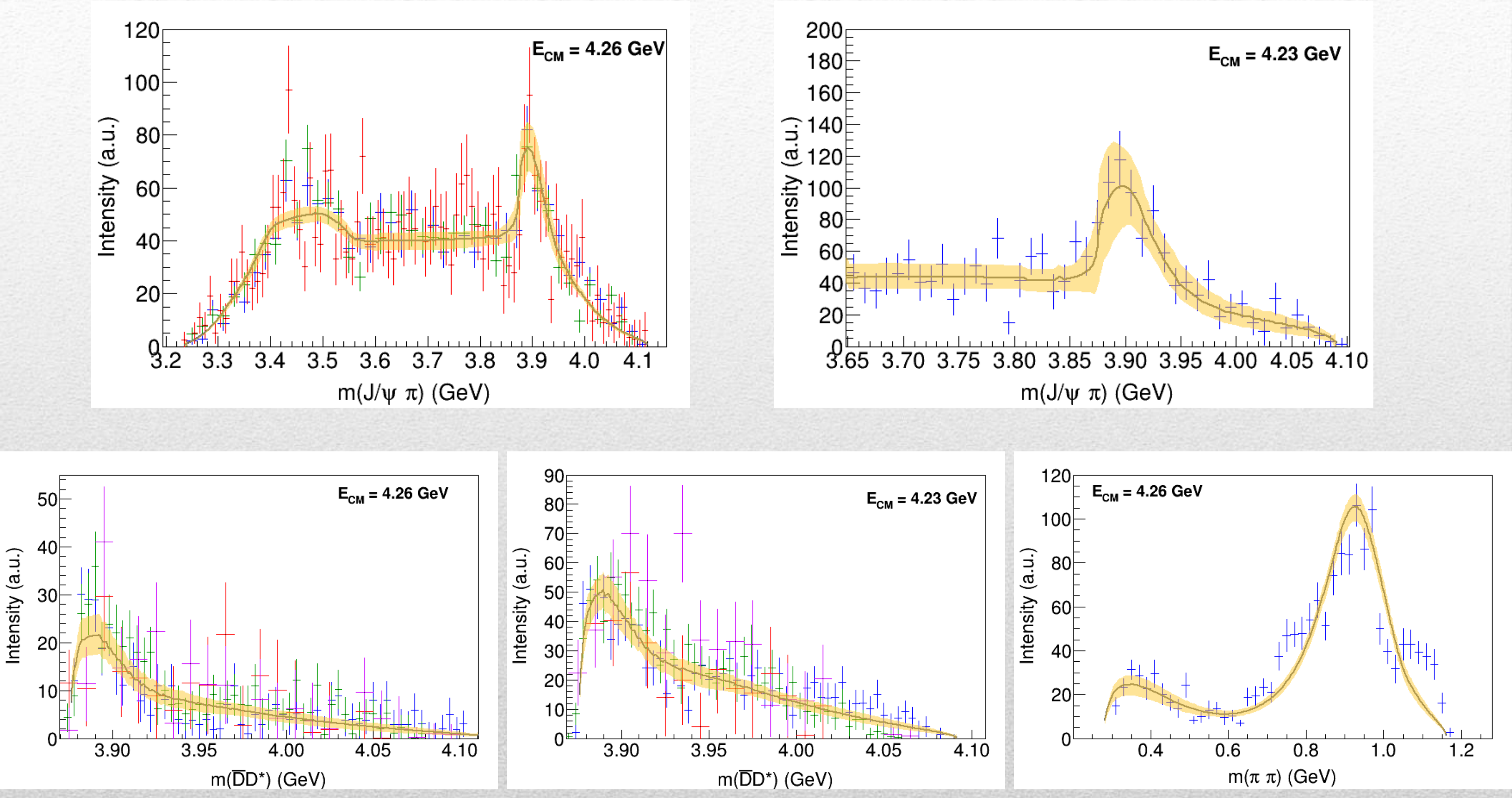

E_{CM} = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)


E_{CM} = 4.26 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)

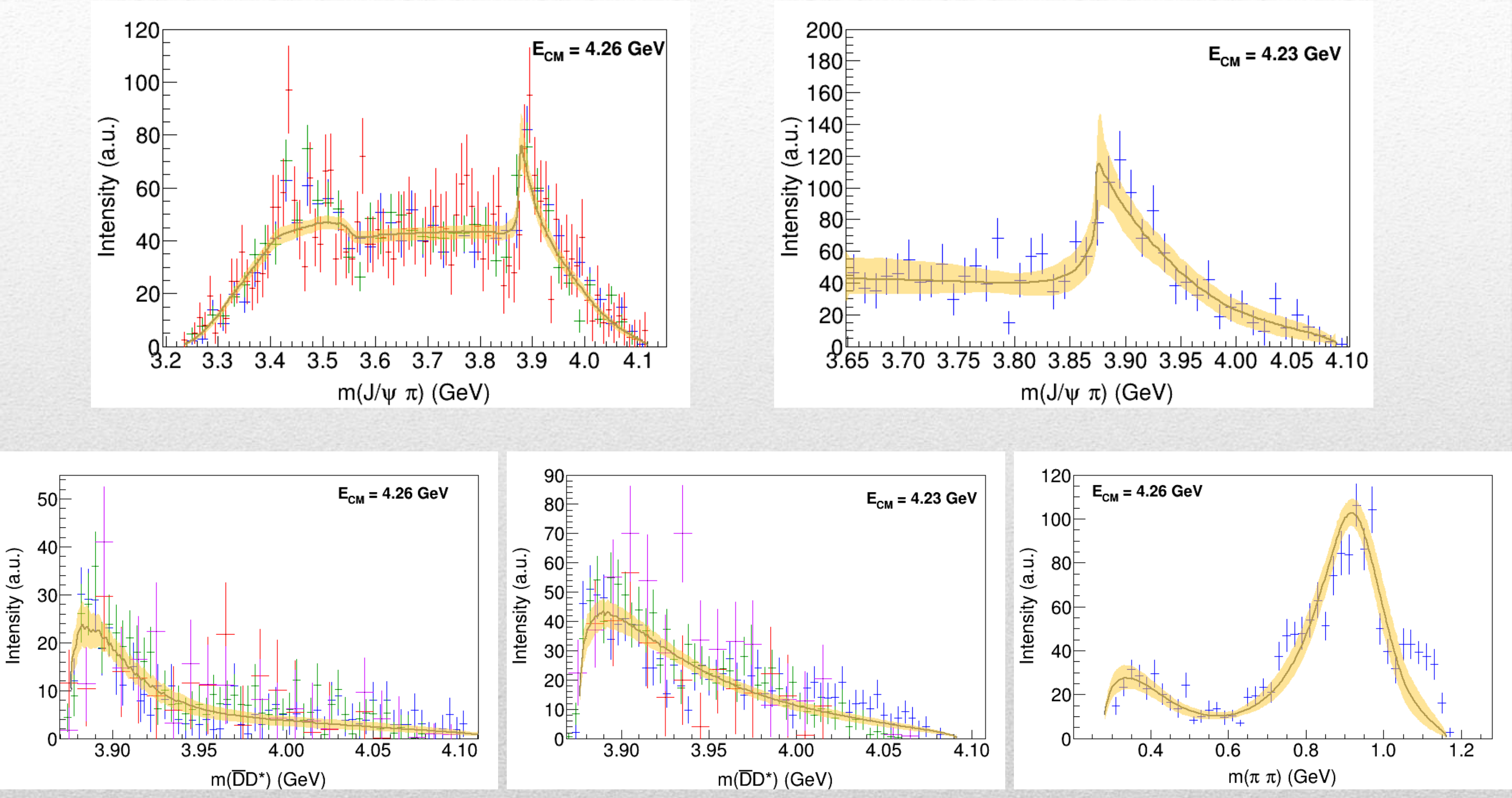

E_{CM} = 4.23 GeV
Intensity (a.u.)
m($\bar{D}D^*$) (GeV)


E_{CM} = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: III+tr.

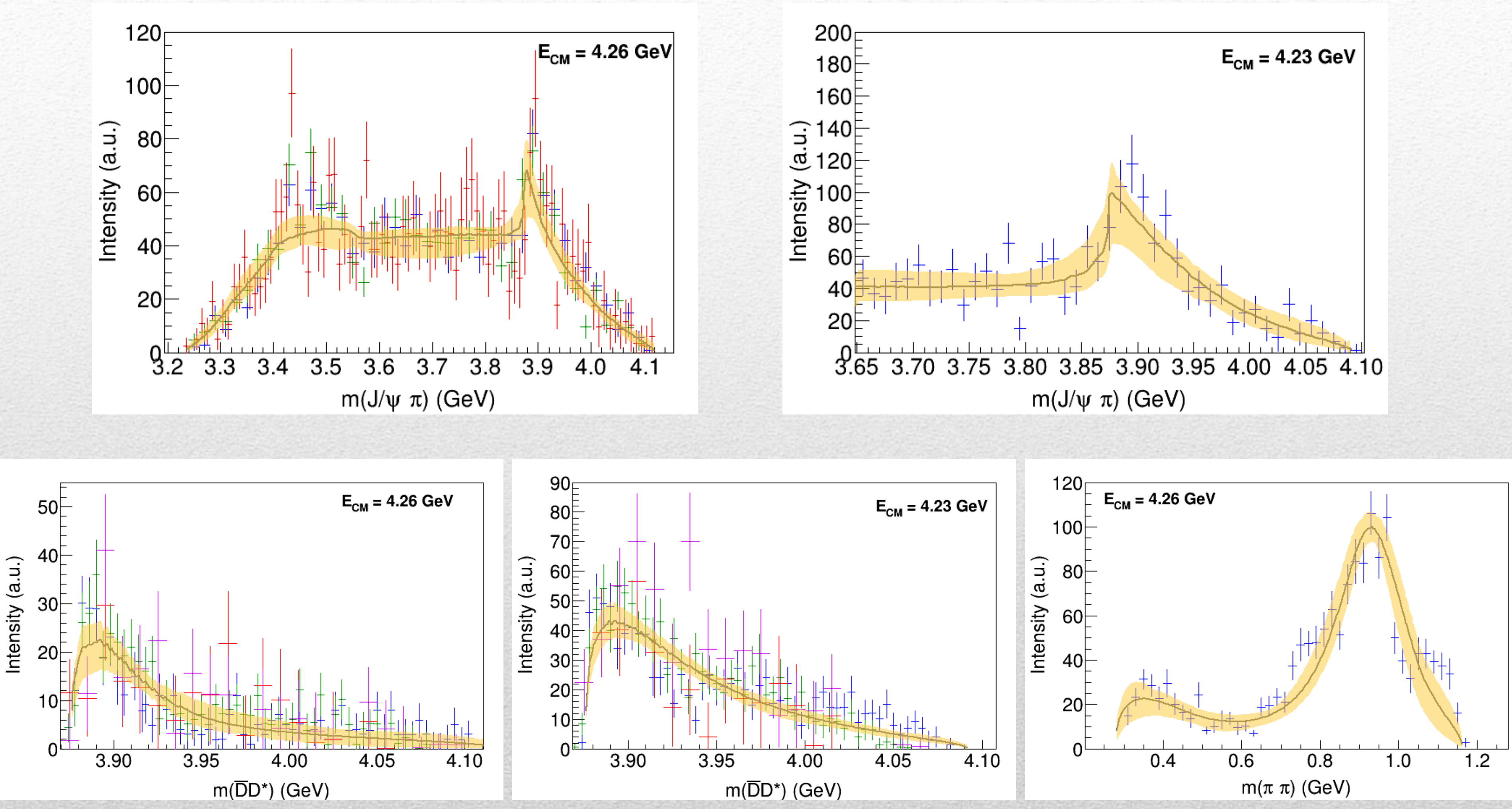

E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E$_{CM}$ = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m($\bar{D}$D*) (GeV)
E$_{CM}$ = 4.23 GeV
Intensity (a.u.)
m($\bar{D}$D*) (GeV)
E$_{CM}$ = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: IV+tr.



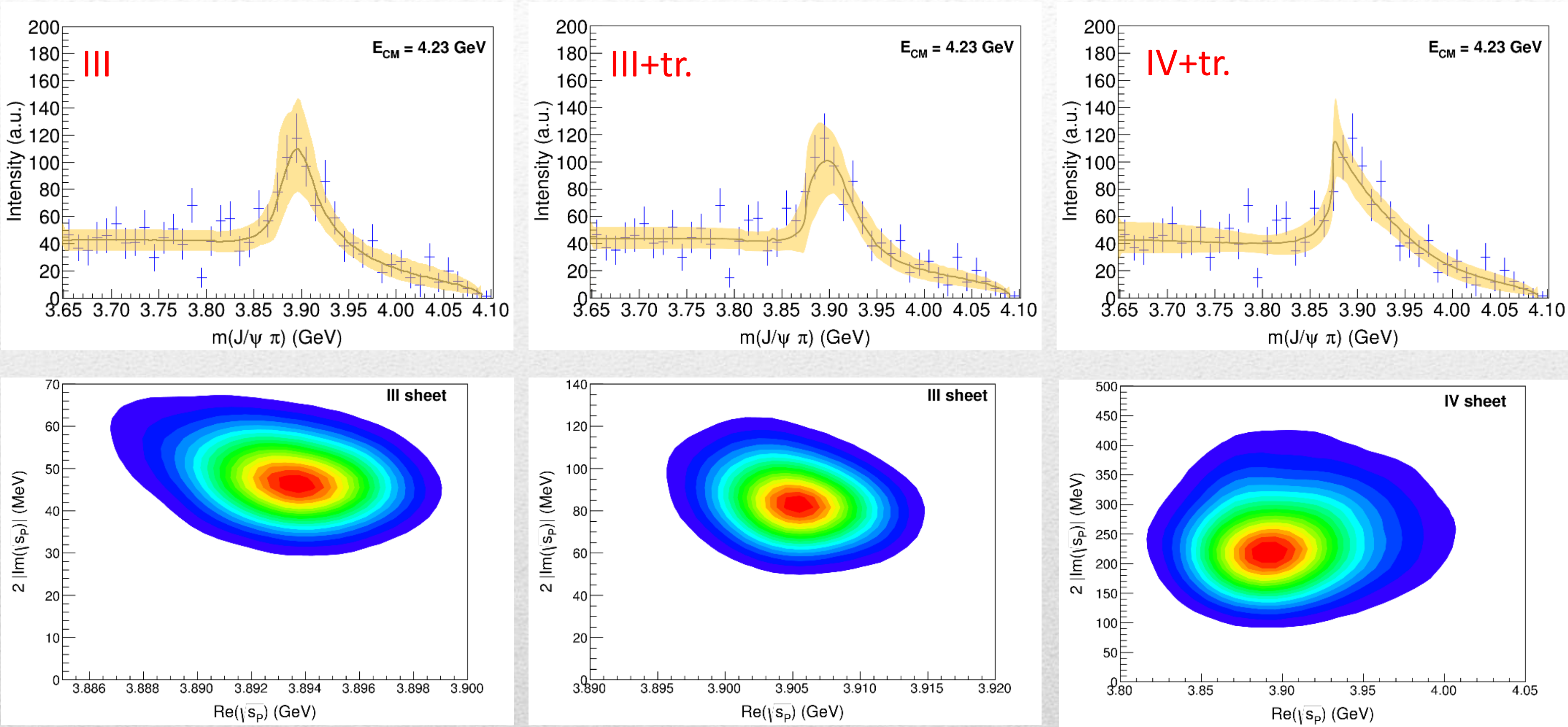
E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Fit: tr.



E_CM = 4.26 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(J/ψ π) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.23 GeV
Intensity (a.u.)
m(D̄D*) (GeV)
E_CM = 4.26 GeV
Intensity (a.u.)
m(π π) (GeV)

Pole extraction



Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.	–	–	"2.1σ" ("0.9σ")

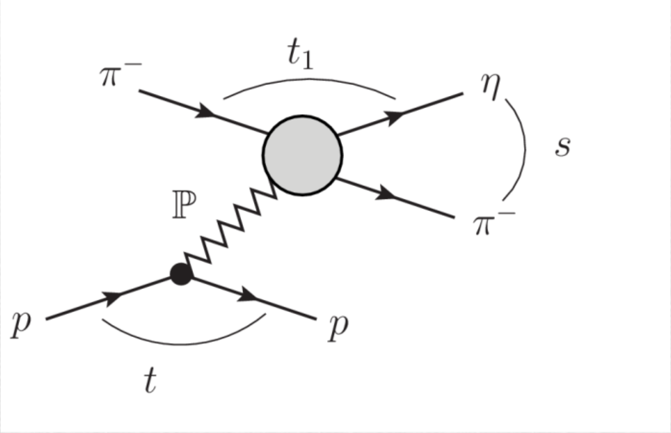
Not conclusive at this stage

	III	III+tr.	IV+tr.
M (MeV)	$3893.2^{+5.5}_{-7.7}$	3905^{+11}_{-9}	3900^{+140}_{-90}
Γ (MeV)	48^{+19}_{-14}	85^{+45}_{-26}	240^{+230}_{-130}

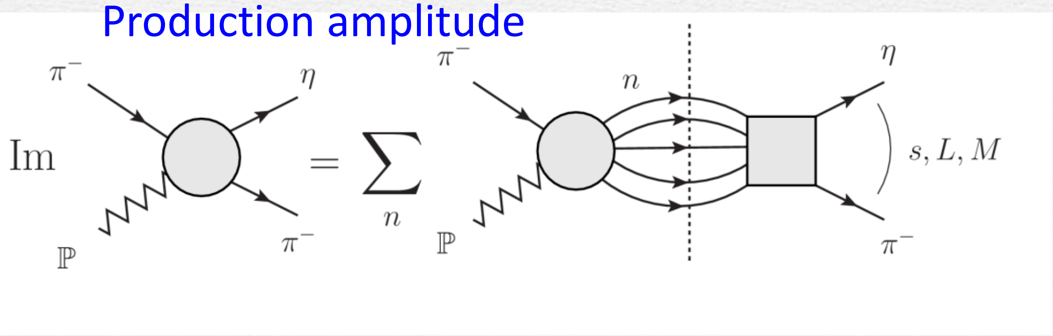
Searching for resonances in $\eta\pi$

- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the N/D method
- We test against the D-wave data, where the a_2 and the a_2' show up

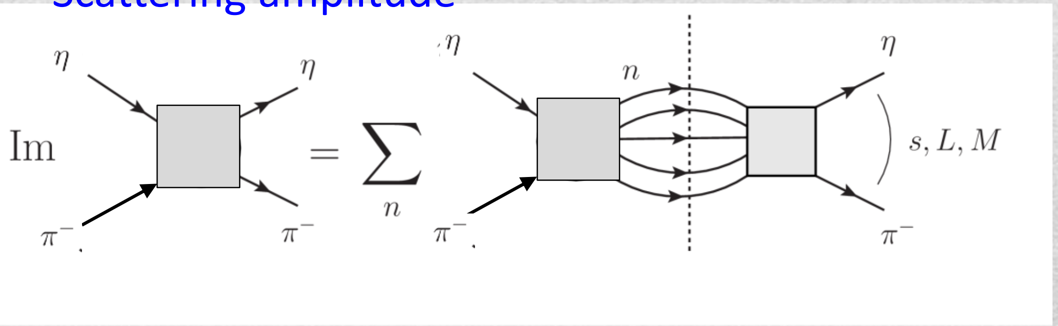
A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472



Production amplitude



Scattering amplitude



Searching for resonances in $\eta\pi$

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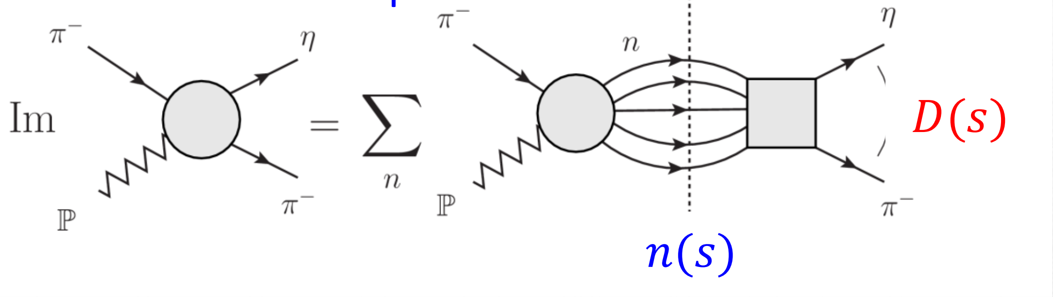
A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472

$$t(s) = \frac{N(s)}{D(s)}$$

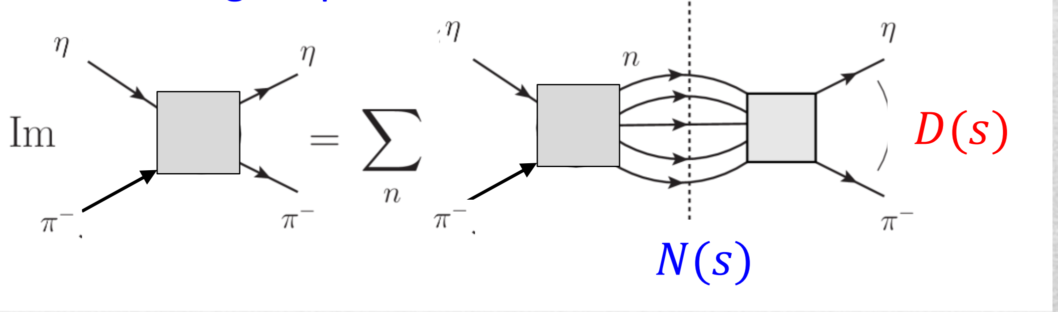
The $D(s)$ has only right hand cuts;
it contains all the Final State Interactions constrained by unitarity → universal

$$\text{Im}\, D(s) = -\rho\, N(s)$$

Production amplitude



Scattering amplitude



Searching for resonances in $\eta\pi$

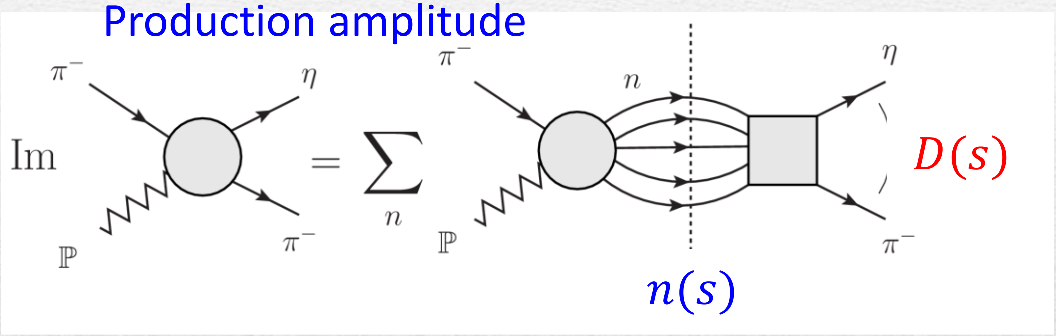
- The $\eta\pi$ system is one of the golden modes for hunting hybrid mesons
- We build the partial wave amplitudes according to the N/D method
- We test against the D-wave data, where the a_2 and the a_2' show up

A. Jackura, M. Mikhasenko, AP *et al.* (JPAC & COMPASS), PLB779, 464-472

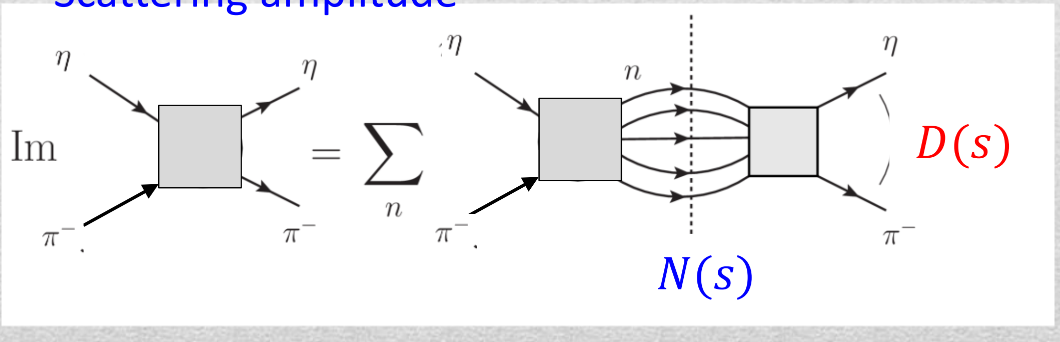
$$t(s) = \frac{N(s)}{D(s)}$$

The $n(s), N(s)$ have left hand cuts only, they depend on the exchanges → process-dependent, smooth

Production amplitude



Scattering amplitude



Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'\,(s'-s)}ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s}$$

OR

$$K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

$$\rho(s)N(s) = g\frac{\lambda^{(2l+1)/2}\left(s, m_\pi^2, m_\eta^2\right)}{\left(s+s_R\right)^7}$$

Searching for resonances in $\eta\pi$

The denominator $D(s)$ contains all the FSI constrained by unitarity → universal

$$D(s) = (K^{-1})(s) - \frac{s}{\pi}\int_{s_i}^{\infty} \frac{\rho(s')N(s')}{s'\,(s'-s)} ds'$$

$$K(s) = \sum_R \frac{g_R^2}{M_R^2 - s}$$

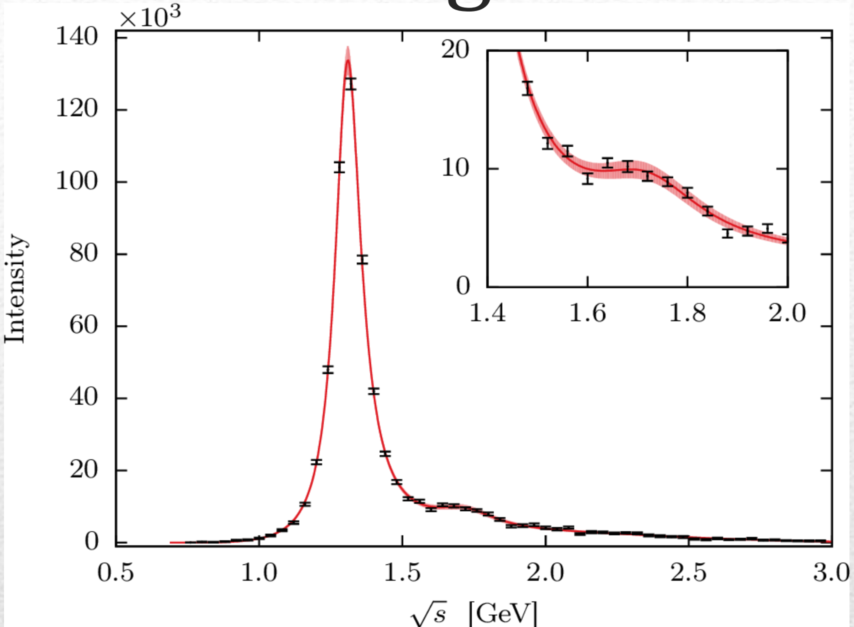
OR

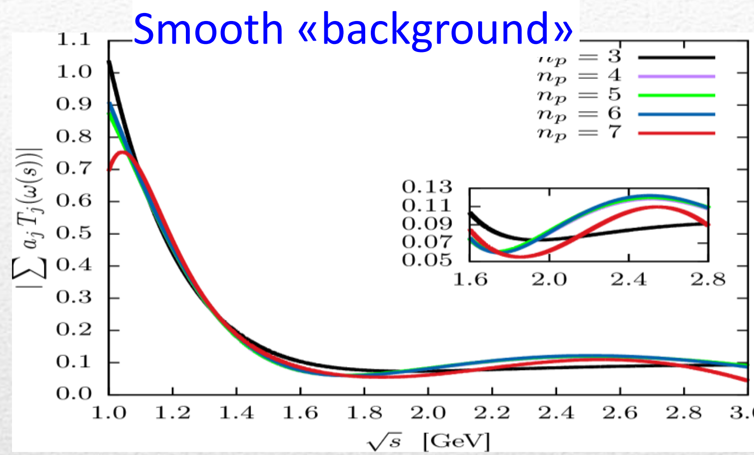
$$K^{-1}(s) = c_0 - c_1 s + \sum_i \frac{c_i}{M_i^2 - s}$$

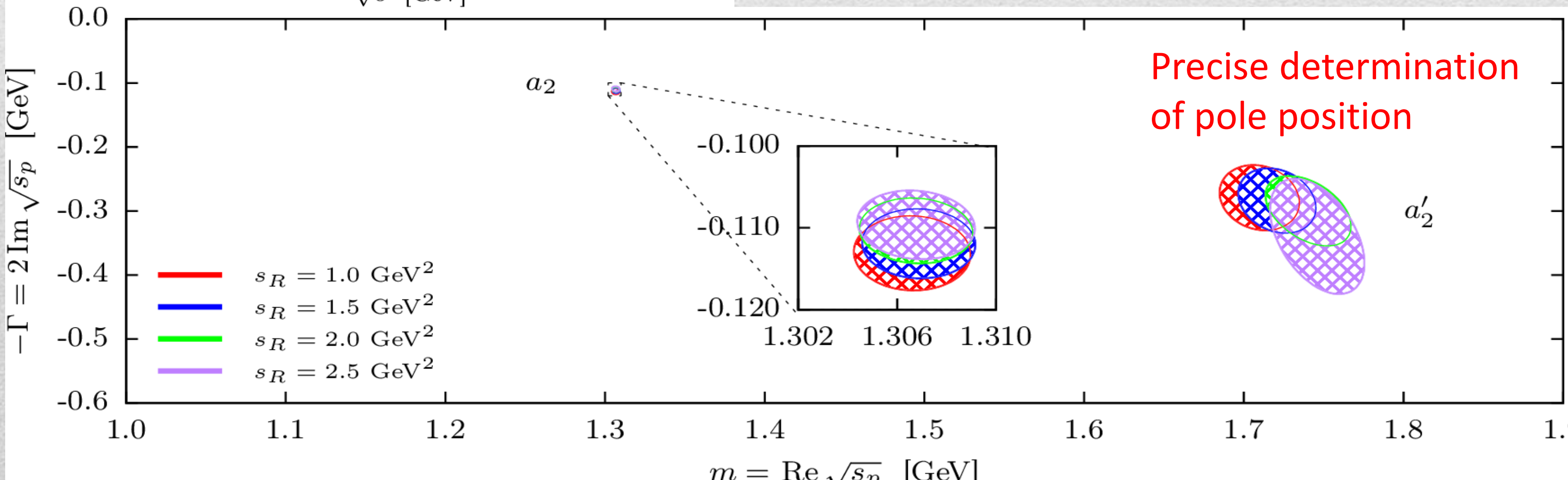
The $n(s)$ is process-dependent, smooth

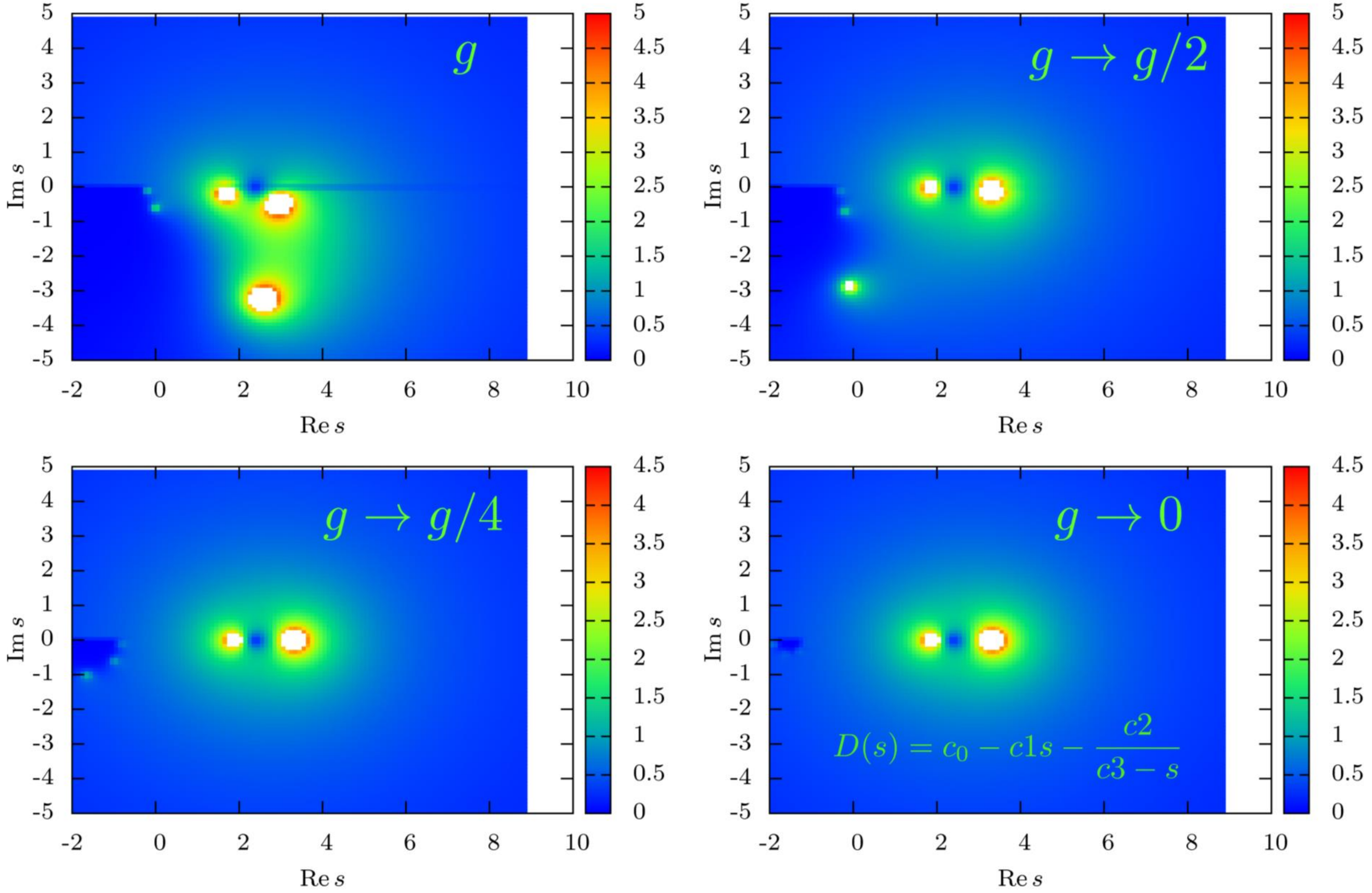
$$n(s) = \sum_j a_j T_j\left(\omega(s)\right) \qquad \omega(s) = \frac{s}{s+s_0}$$

Searching for resonances in $\eta\pi$







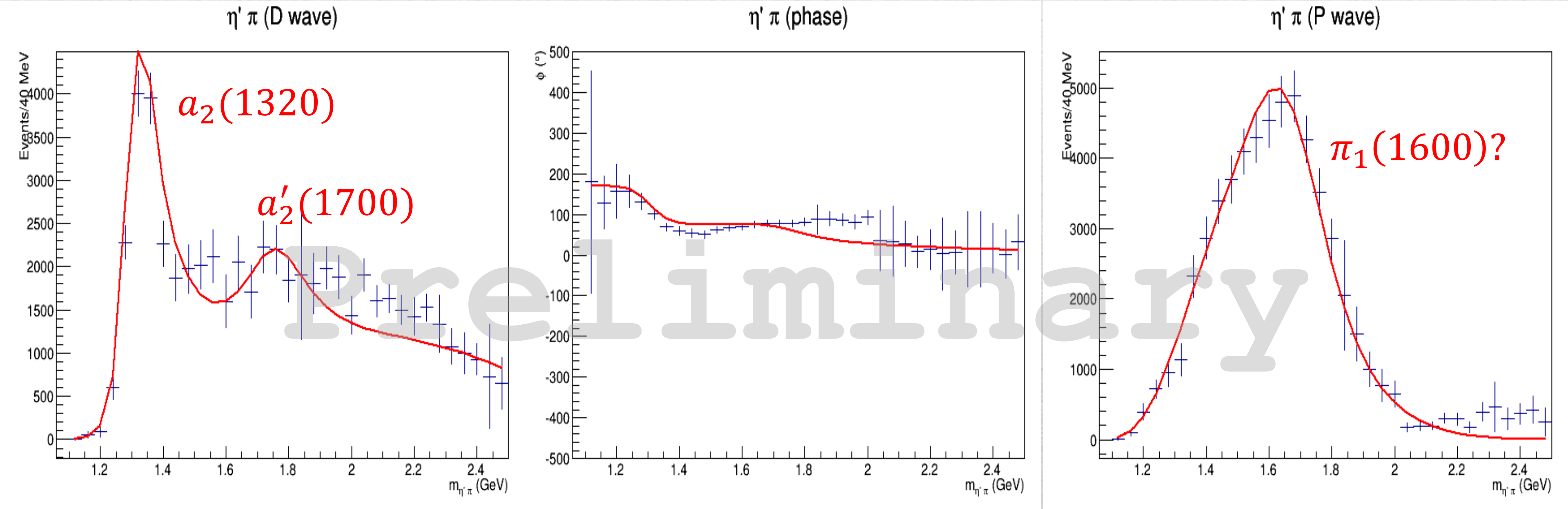


g
$g \to g/2$
$g \to g/4$
$g \to 0$
$D(s) = c_0 - c1s - \frac{c2}{c3 - s}$
Im s
Re s

Searching for resonances in $\eta\pi$

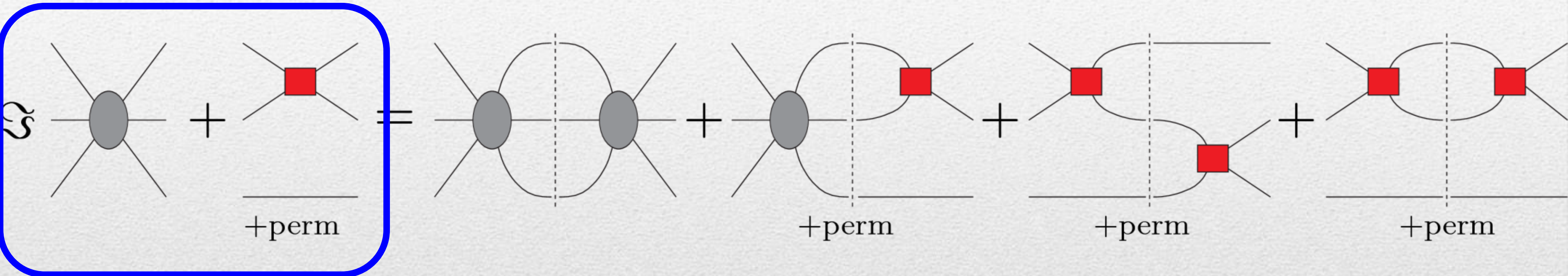
$$m(a_2) = (1307 \pm 1 \pm 6)\ \text{MeV} \qquad m(a_2') = (1720 \pm 10 \pm 60)\ \text{MeV}$$
$$\Gamma(a_2) = (112 \pm 1 \pm 8)\ \text{MeV} \qquad \Gamma(a_2') = (280 \pm 10 \pm 70)\ \text{MeV}$$

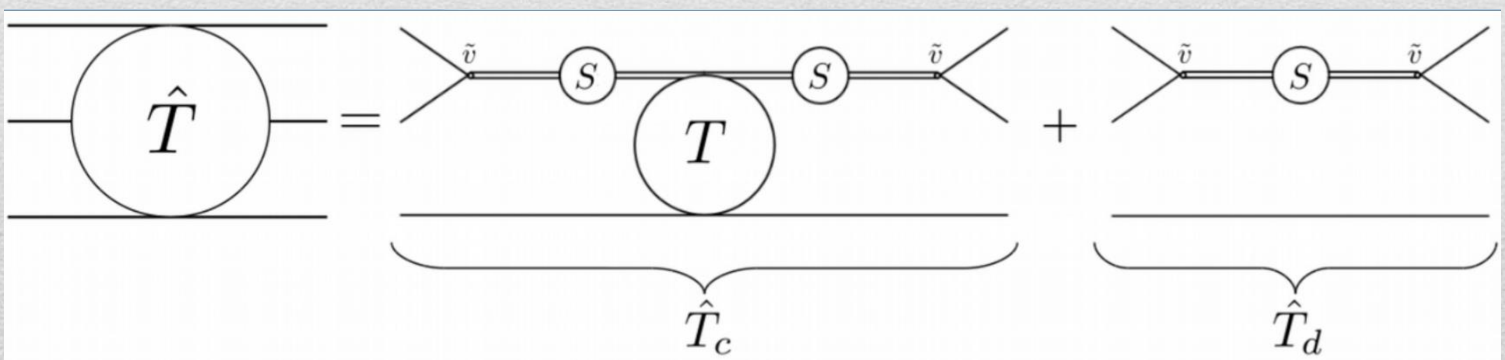
- The coupled channel analysis involving the exotic P-wave is ongoing, as well as the extention to the GlueX production mechanism and kinematics



Three-Body Unitarity

Amado, Aaron, Young (1968)
Mai, Hu, Doring, AP, Szczepaniak, EPJA53, 9, 177





Three-Body Unitarity

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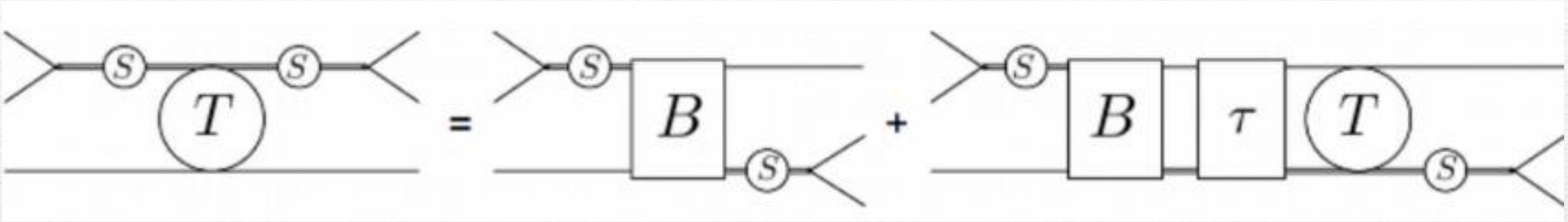
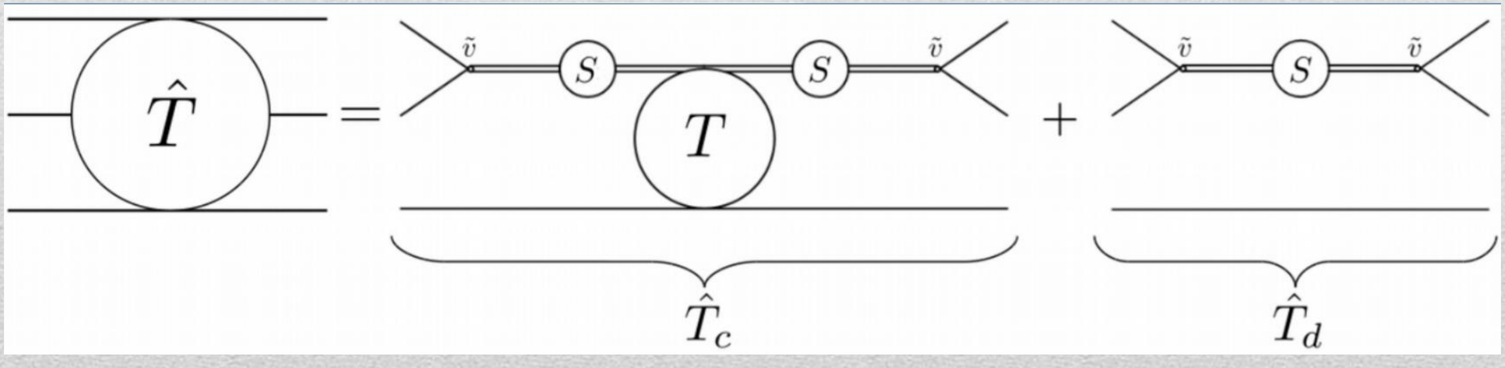


BS ansatz

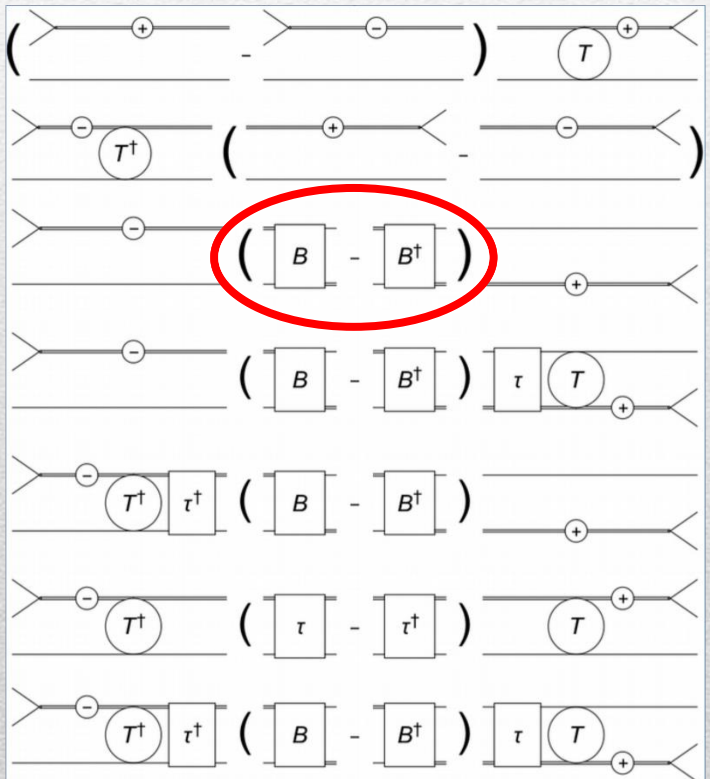


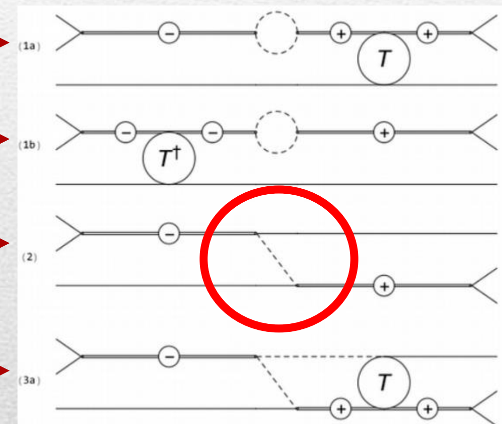
Three-Body Unitarity

$$\langle q_1, q_2, q_3|(\hat{T}-\hat{T}^\dagger)|p_1,p_2,p_3\rangle \quad = \quad i\int_P \langle q_1,q_2,q_3|\hat{T}^\dagger|k_1,k_2,k_3\rangle\langle k_1,k_2,k_3|\hat{T}|p_1,p_2,p_3\rangle$$

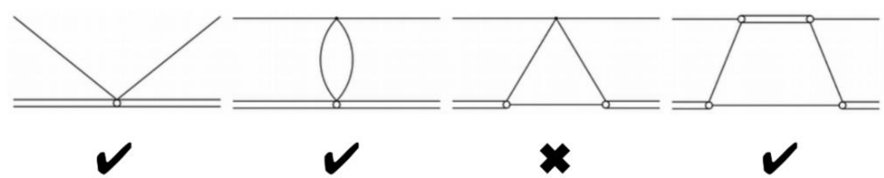
BS ansatz

Product of disconnected terms are source for the connected amplitude





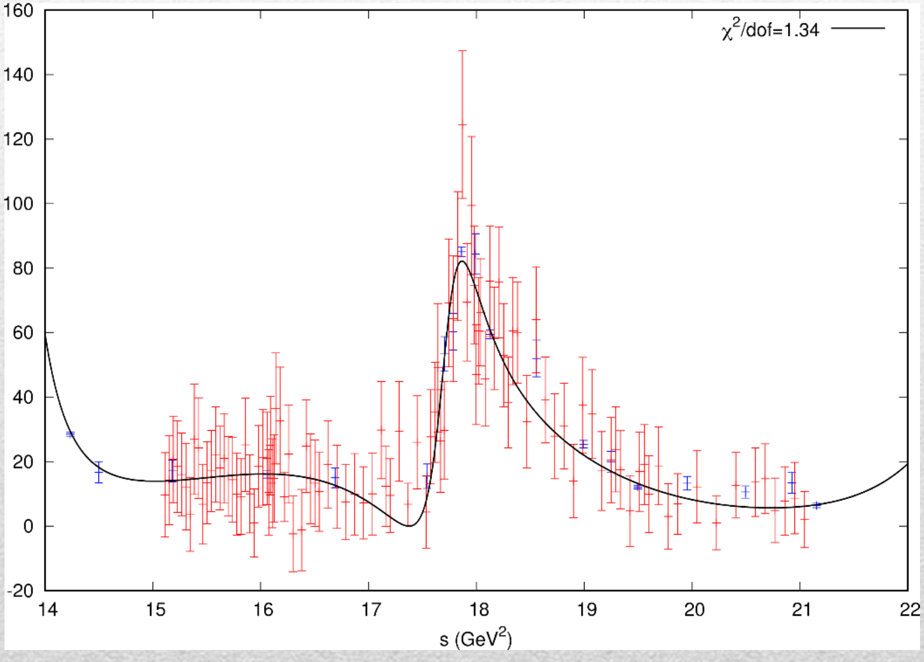
$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q-\sqrt{m^2+\mathbf{Q}^2}+i\epsilon\right)}$$

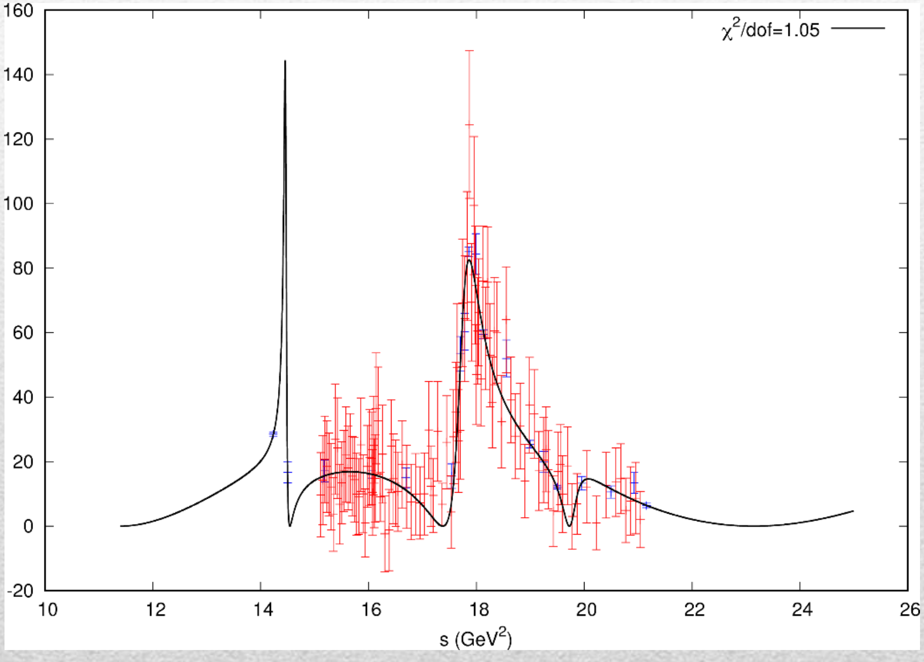


The $Y(4260)$

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in preparation

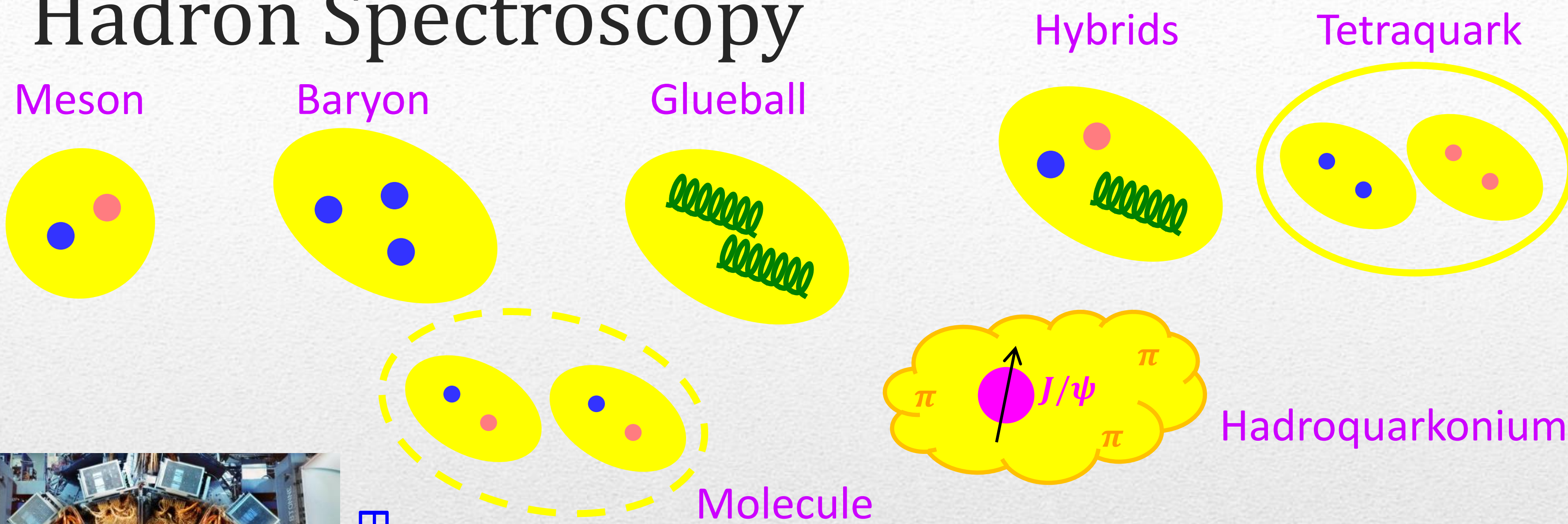
We start analyzing the single channel $e^+e^- \to J/\psi\, \pi\pi$
We consider the amplitude in the elastic, quasi two-body approximation



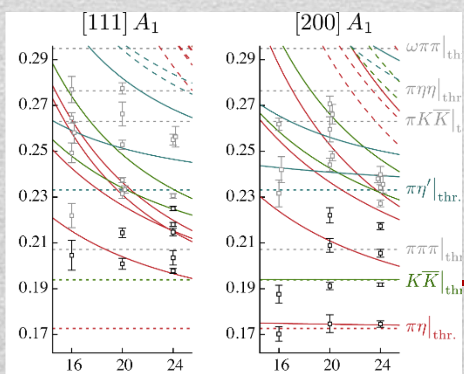


LASSO method (linear penalization in the χ^2) is helpful in constraining the number of resonances and parameters in the numerator

Hadron Spectroscopy



Experiment



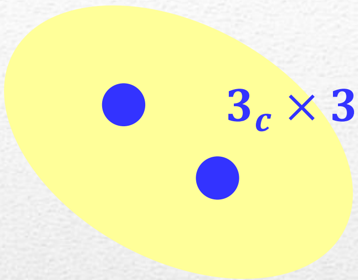
Lattice QCD

Data → Amplitude analysis → Properties, Model building

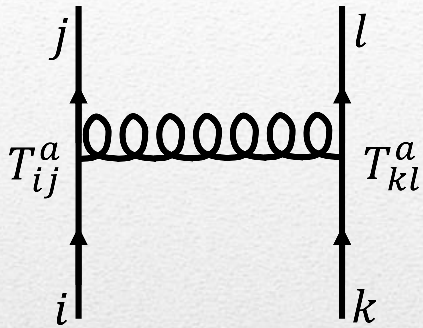
Interpretations on the spectrum leads to understanding fundamental laws of nature

Diquarks

Attraction and repulsion in 1-gluon exchange approximation is given by



$\mathbf{3}_c \times \mathbf{3}_c \in \bar{\mathbf{3}}_c$



$$R = \frac{1}{2}\left(C_2(R_{12}) - C_2(R_1) - C_2(R_2)\right)$$

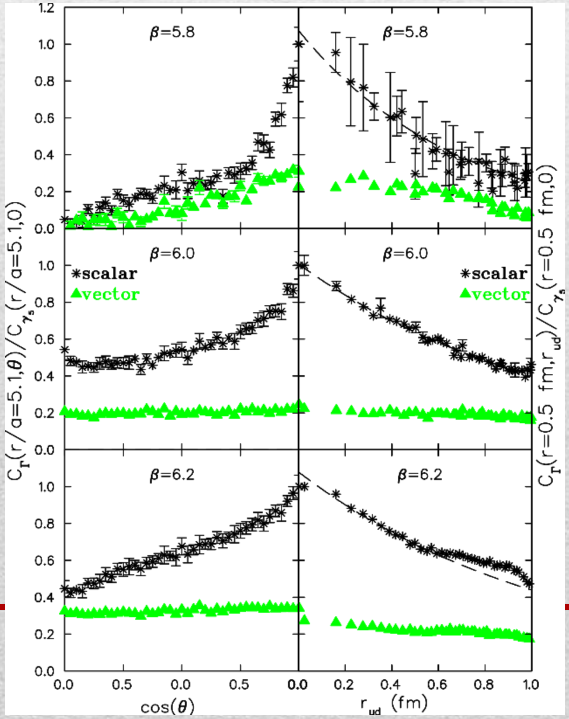
$$R_1 = -\frac{4}{3}, R_8 = +\frac{1}{6}$$

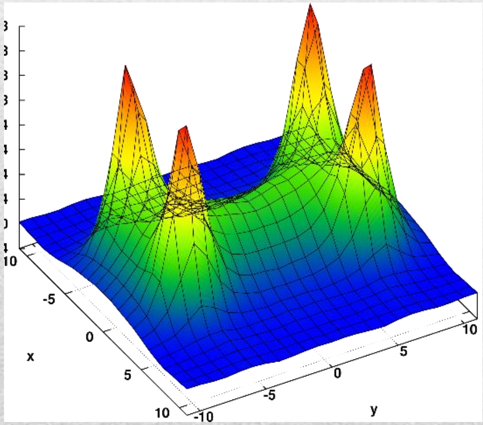
$$R_3 = -\frac{2}{3}, R_6 = +\frac{1}{3}$$

The singlet $\mathbf{1}_c$ is attractive

A diquark in $\bar{\mathbf{3}}_c$ is attractive

Evidence (?) of diquarks in LQCD, Alexandrou, de Forcrand, Lucini, PRL 97, 222002





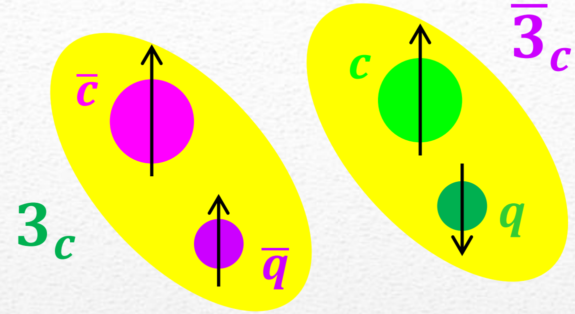
H-shape with a 4 quark system

Cardoso, Cardoso, Bicudo, PRD84, 054508

Tetraquark

In a constituent (di)quark model, we can think of a **diquark-antidiquark compact state**

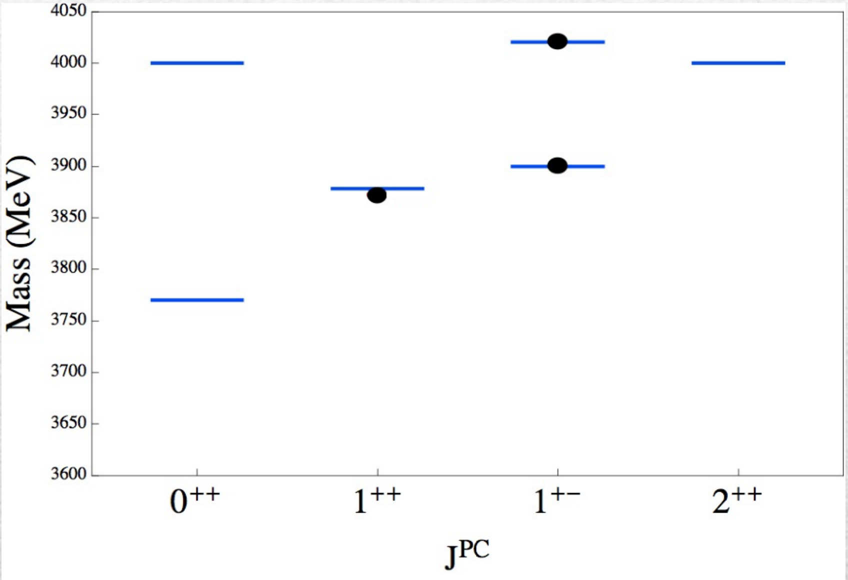
$$[cq]_{S=0}[\bar{c}\bar{q}]_{S=1} + h.c.$$



Maiani, Piccinini, Polosa, Riquer PRD71 014028
Faccini, Maiani, Piccinini, AP, Polosa, Riquer PRD87 111102
Maiani, Piccinini, Polosa, Riquer PRD89 114010

Spectrum according to color-spin hamiltonian (all the terms of the Breit-Fermi hamiltonian are absorbed into a constant diquark mass):

$$H = \sum_{dq} m_{dq} + 2 \sum_{i<j} \kappa_{ij}\, \vec{S_i} \cdot \vec{S_j}\, \frac{\lambda_i^a}{2} \frac{\lambda_j^a}{2}$$



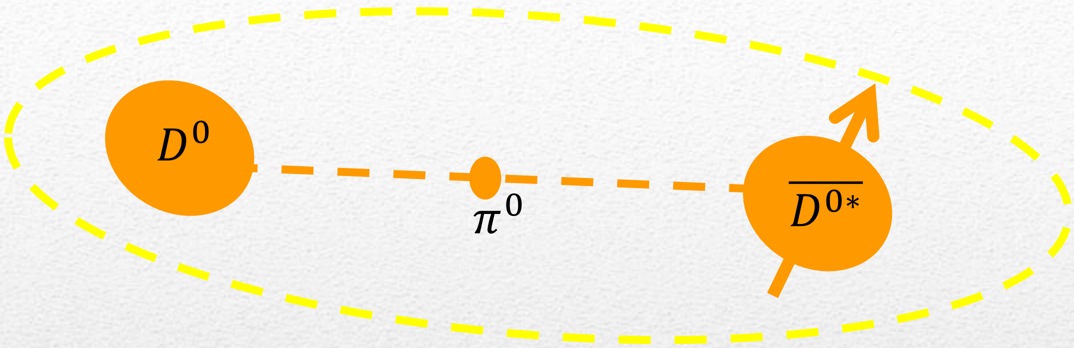
Decay pattern mostly driven by HQSS ✓
Fair understanding of existing spectrum ✓
A full nonet for each level is expected ✗

New ansatz: the diquarks are compact objects spacially separated from each other, only $\kappa_{cq} \neq 0$

Existing spectrum is fitted if $\kappa_{cq} = 67$ MeV

Other models: Molecule

Tornqvist, Z.Phys. C61, 525
Braaten and Kusunoki, PRD69 074005
Swanson, Phys.Rept. 429 243-305



$X(3872) \sim \bar{D}^0 D^{*0}$
$Z_c(3900) \sim \bar{D}^0 D^{*+}$
$Z_c'(4020) \sim \bar{D}^{*0} D^{*+}$
$Y(4260) \sim \bar{D} D_1$

A deuteron-like meson pair, the interaction is mediated by the exchange of light mesons

- Some model-independent relations (Weinberg's theorem) ✓
- Good description of decay patterns (mostly to constituents) and $X(3872)$ isospin violation ✓
- States appear close to thresholds ✓ (but $Z(4430)$ ✗)
- Lifetime of costituents has to be $\gg 1/m_\pi$
- Binding energy varies from -70 to -0.1 MeV, or even positive (repulsive interaction) ✗
- Unclear spectrum (a state for each threshold?) – depends on potential models ✗

$$V_\pi(r) = \frac{g_{\pi N}^2}{3} (\vec{\tau_1} \cdot \vec{\tau_2}) \left\{ [3(\vec{\sigma_1} \cdot \hat{r})(\vec{\sigma_2} \cdot \hat{r}) - (\vec{\sigma_1} \cdot \vec{\sigma_2})] \left(1 + \frac{3}{(m_\pi r)^2} + \frac{3}{m_\pi r} \right) + (\vec{\sigma_1} \cdot \vec{\sigma_2}) \right\} \frac{e^{-m_\pi r}}{r}$$

Needs regularization, cutoff dependence

Weinberg theorem

Resonant scattering amplitude

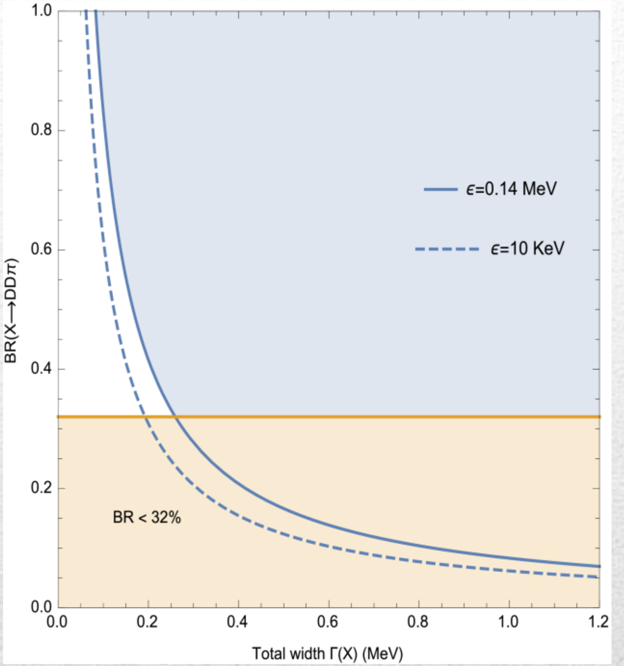
$$f(ab \to c \to ab) = -\frac{1}{8\pi\, E_{CM}} g^2 \frac{1}{(p_a + p_b)^2 - m_c^2}$$

with $m_c = m_a + m_b - B$, and $B, T \ll m_{a,b}$

$$f(ab \to c \to ab) = -\frac{1}{16\pi(m_a + m_b)^2} g^2 \frac{1}{B + T}$$

This has to be compared with the potential scattering for slow particles ($kR \ll 1$, being $R \sim 1/m_\pi$ the range of interaction) in an attractive potential U with a superficial level at $-B$

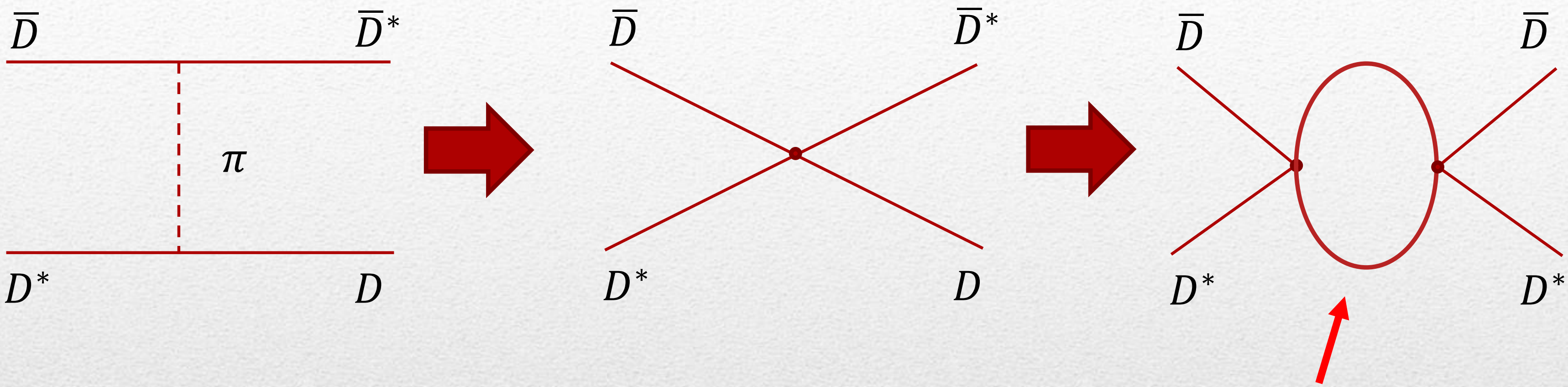
$$f(ab \to ab) = -\frac{1}{\sqrt{2\mu}} \frac{\sqrt{B} - i\sqrt{T}}{B + T}, B = \frac{g^4}{512\pi^2} \frac{\mu^5}{(m_a m_b)^2}$$



This corresponds to the pure molecular interpretation of the $X(3872)$

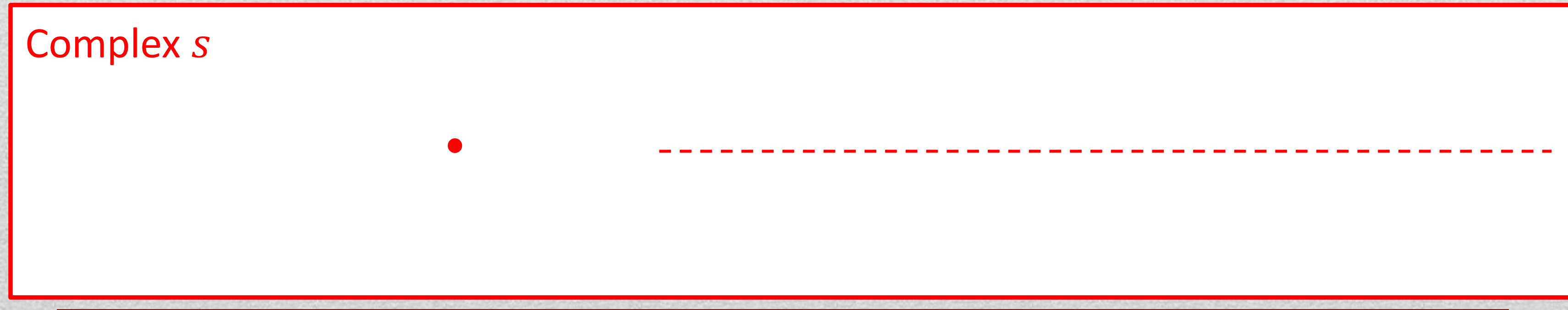
Weinberg, PR 130, 776
Weinberg, PR 137, B672
Polosa, PLB 746, 248

Weinberg and amplitudes



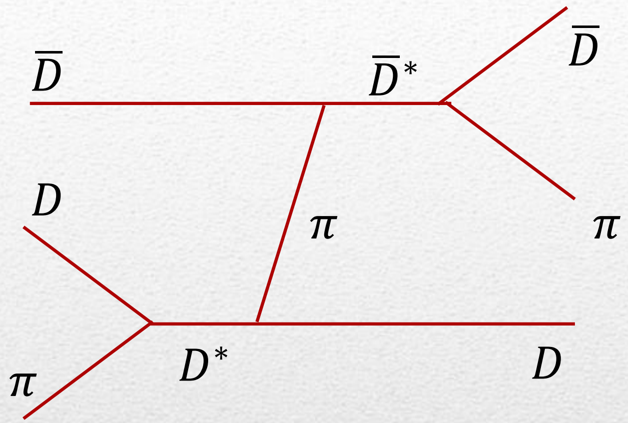
This means that IF you can consider the pion exchange as a contact interaction, the amplitude is determined by the pole close to threshold

This loop is now divergent,
I need to renormalize the integral
I can put the pole where I want



Weinberg and amplitudes

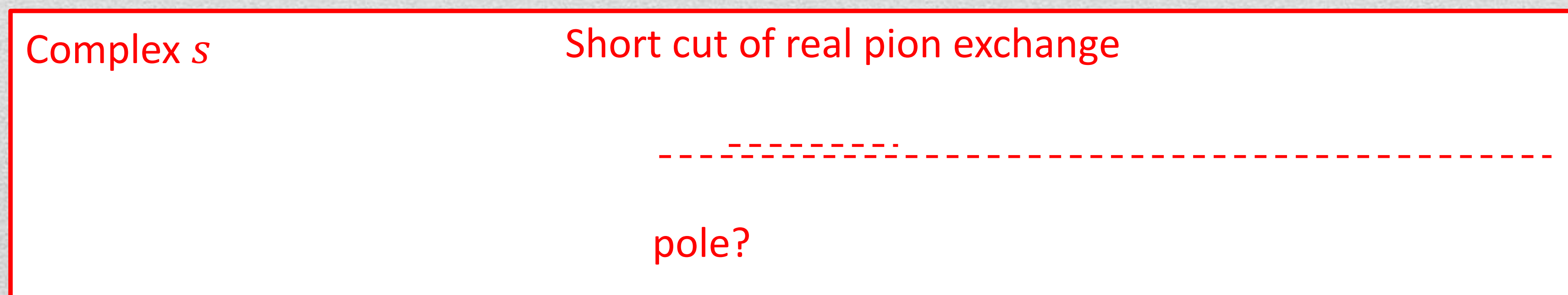
A. Jackura, AP et al., in progress



BUT the D^* actually decays into $D\pi$ and the system is constrained by 3-body unitarity

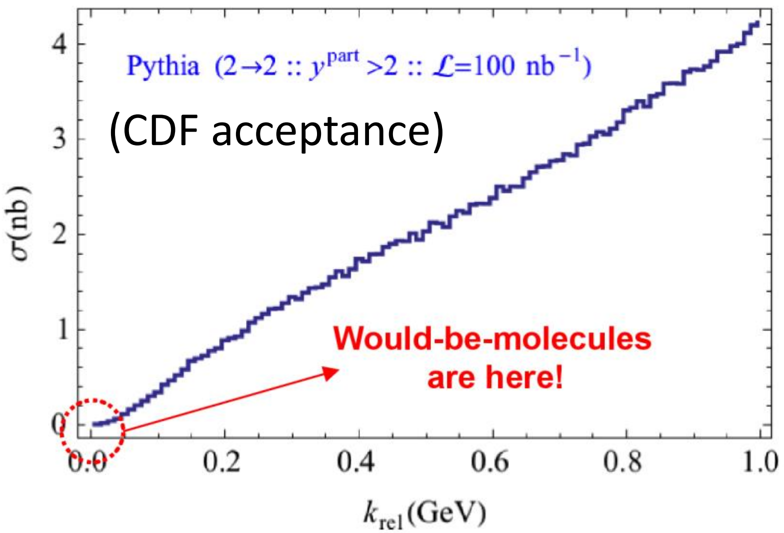
The position of the pole can be calculated given a model for the simple pion exchange

The simplest model leads to a convergent dispersion relation, the pole position is determined
One can check whether this purely molecular amplitude is consistent or not with data



Prompt production of $X(3872)$

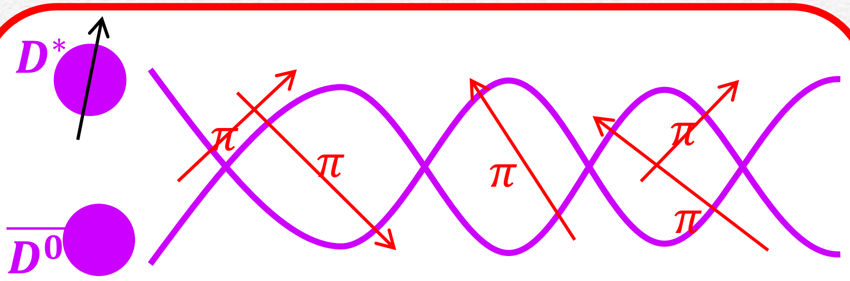
$X(3872)$ is the Queen of exotic resonances, the most popular interpretation is a $D^0\bar{D}^{0*}$ molecule (bound state, pole in the 1st Riemann sheet?) but it is copiously promptly produced at hadron colliders



$$\sigma_{MC}(p\bar{p} \to DD^*|k < k_{max}) \approx 0.1 \text{ nb}$$

$$\sigma_{exp}(p\bar{p} \to X(3872)) \approx 30 - 70 \text{ nb!!!}$$

Bignamini *et al.* PRL103 (2009) 162001



A solution can be FSI (rescattering of DD^*), which allow k_{max} to be as large as $5m_\pi$, $\sigma(p\bar{p} \to DD^*|k < k_{max}) \approx 230$ nb

Artoisenet and Braaten, PRD81, 114018

However, the rescattering is flawed by the presence of pions that interfere with DD^* propagation. Estimating the effect of these pions increases σ, but not enough

Bignamini *et al.* PLB684, 228-230
Esposito, Piccinini, AP, Polosa, JMP 4, 1569
Guerrieri, Piccinini, AP, Polosa, PRD90, 034003

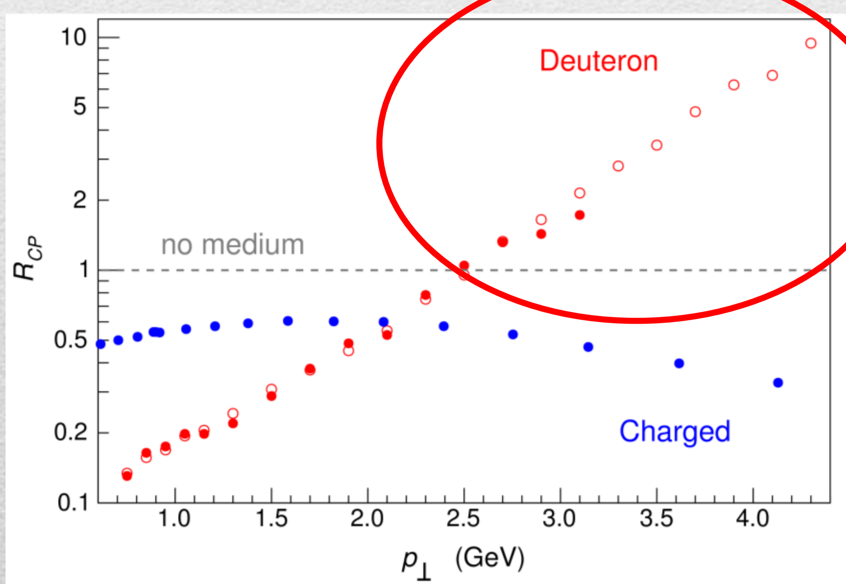
Nuclear modification factors

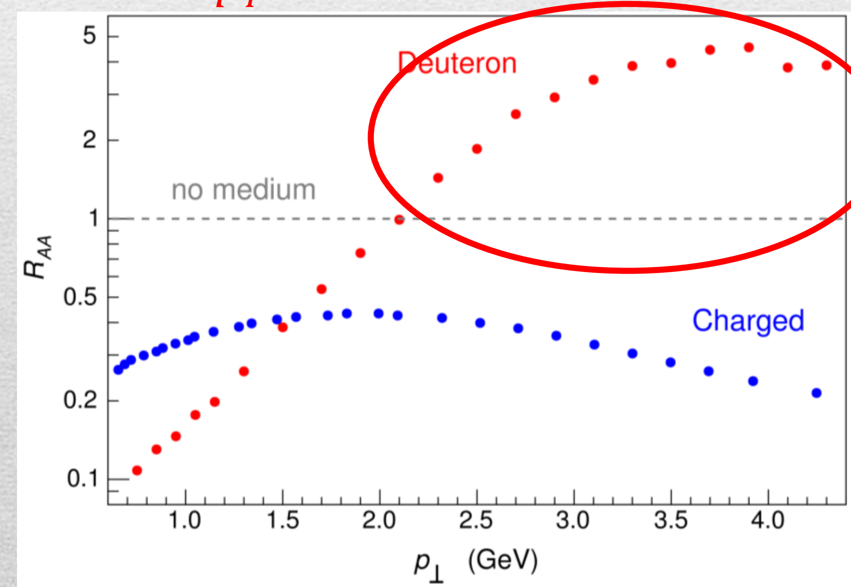
What happens to molecules in heavy ion collisions?
We can use deuteron data to extract the values of the nuclear modification factors

$$R_{CP} = \frac{N_{coll}^{P}\left(\frac{dN}{dp_T}\right)_C}{N_{coll}^{C}\left(\frac{dN}{dp_T}\right)_P}$$

$$R_{AA} = \frac{\left(\frac{dN}{dp_T}\right)_{\text{Pb}-\text{Pb}}}{N_{coll}\left(\frac{dN}{dp_T}\right)_{pp}}$$

Larger than 1 at $p_T > 2.5$ GeV



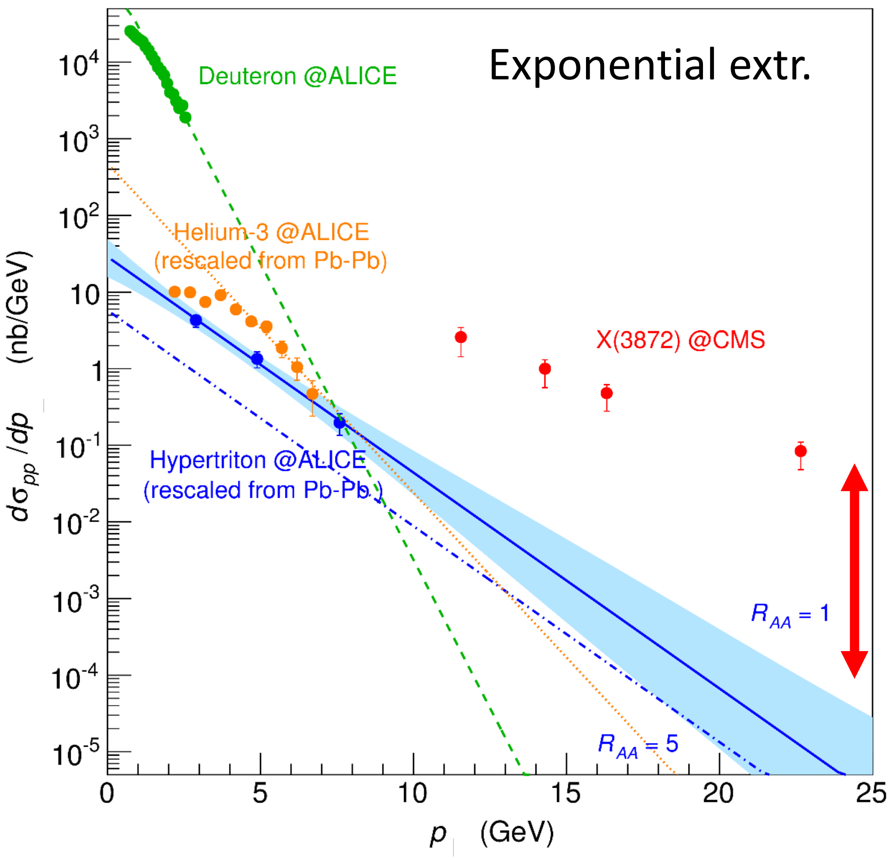


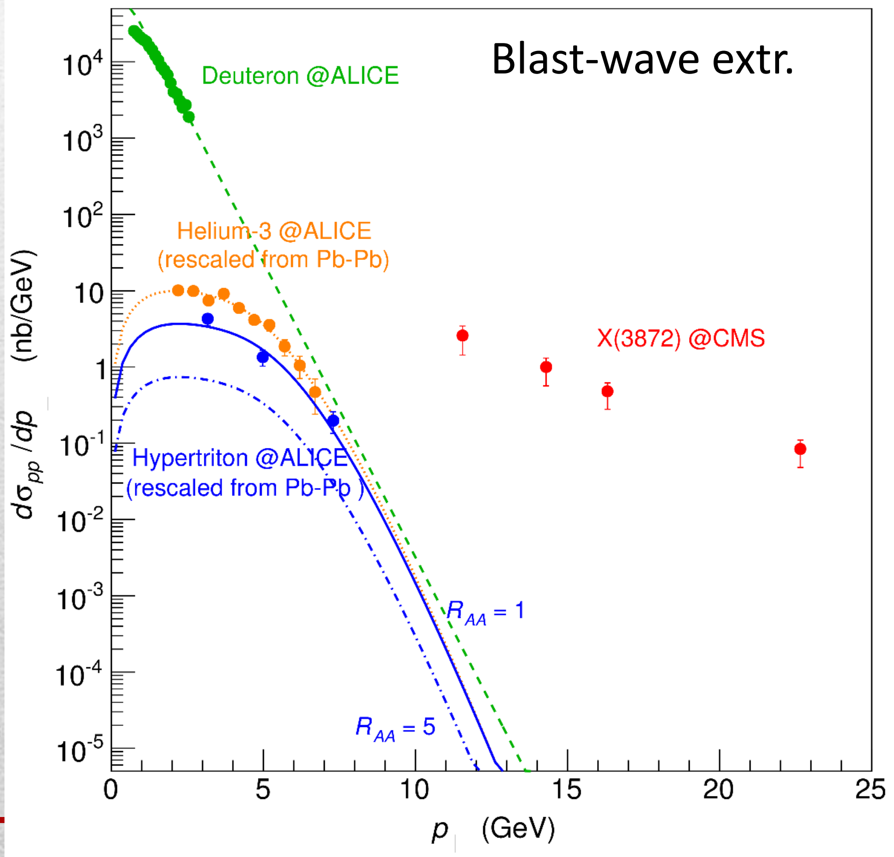
Light nuclei at ALICE vs. $X(3872)$

Esposito, Guerrieri, Maiani, Piccinini, AP, Polosa, Riquer, PRD92, 034028

We assume a pure Glauber model (RAA = 1) and a value RAA = 5 to rescale Pb-Pb data to pp

The $X(3872)$ is way larger than the extrapolated cross section





Production of $Y(4260)$ and $P_c(4450)$

Given the new lineshape by BESIII, we need to rethink the binding energy of the $Y(4260)$

J. Nys and AP, to appear

	Constituents	Bind. Energy	Bind. Mom.	Mediator
$X(3872)$	$\bar{D}^0 D^{*0}$	$\sim$100 keV	$\sim$50 MeV	1π ($\sim$300 MeV)
$Y(4260)$	$\bar{D} D_1$	$\sim$70 MeV	$\sim$400 MeV	2π ($\sim$600 MeV)
$P_c(4450)$	$\bar{D}^* \Sigma_c$	$\sim$10 MeV	$\sim$150 MeV	1π ($\sim$300 MeV)

If the states are purely hadron molecule, all the properties depend on the position of the pole with respect to threshold – all the features are universal

What does the production of $X(3872)$ implies for the other states?

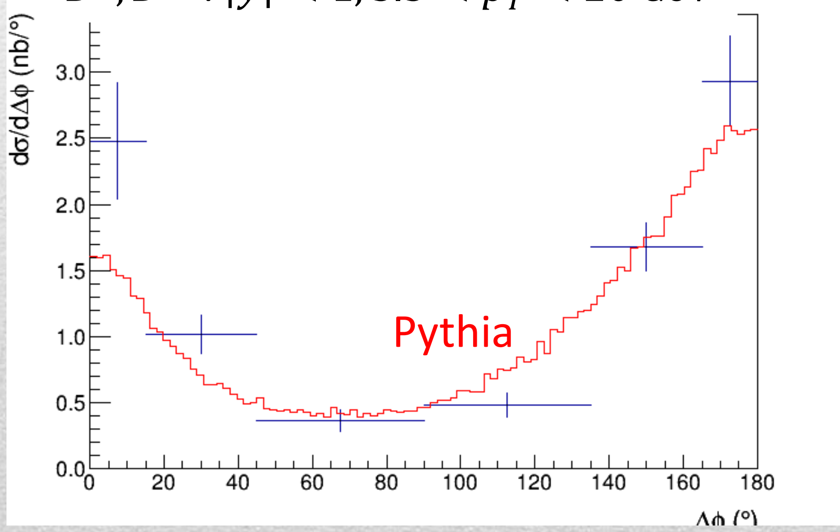
Production of $Y(4260)$ and $P_c(4450)$

We can use Pythia to simulate the production of event, and calculate the relative production of $Y(4260)$ and $P_c(4450)$ with respect to the $X(3872)$

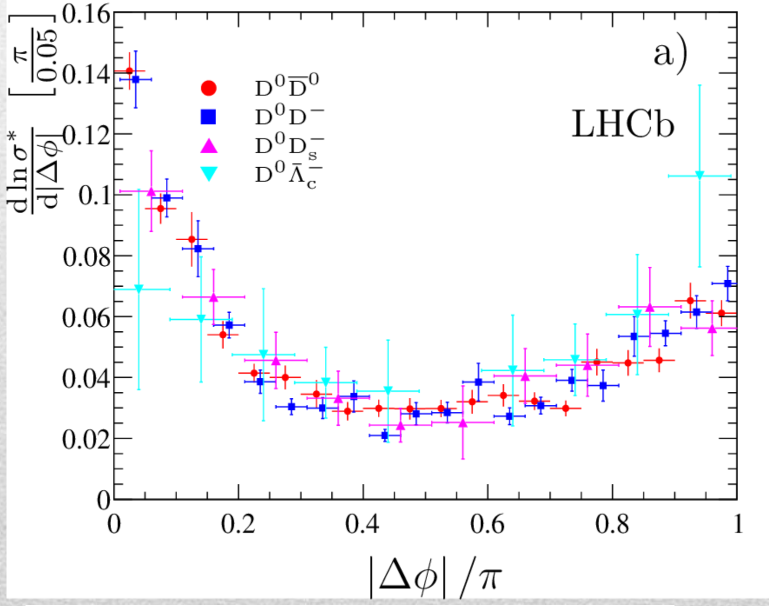
J. Nys and AP, to appear

We tune our MC on charm pair production

CDF data, $\sqrt{s} = 1.96$ TeV
D^0, D^{*-}: $|y| < 1, 5.5 < p_T < 20$ GeV



For baryons we can double check with LHCb data



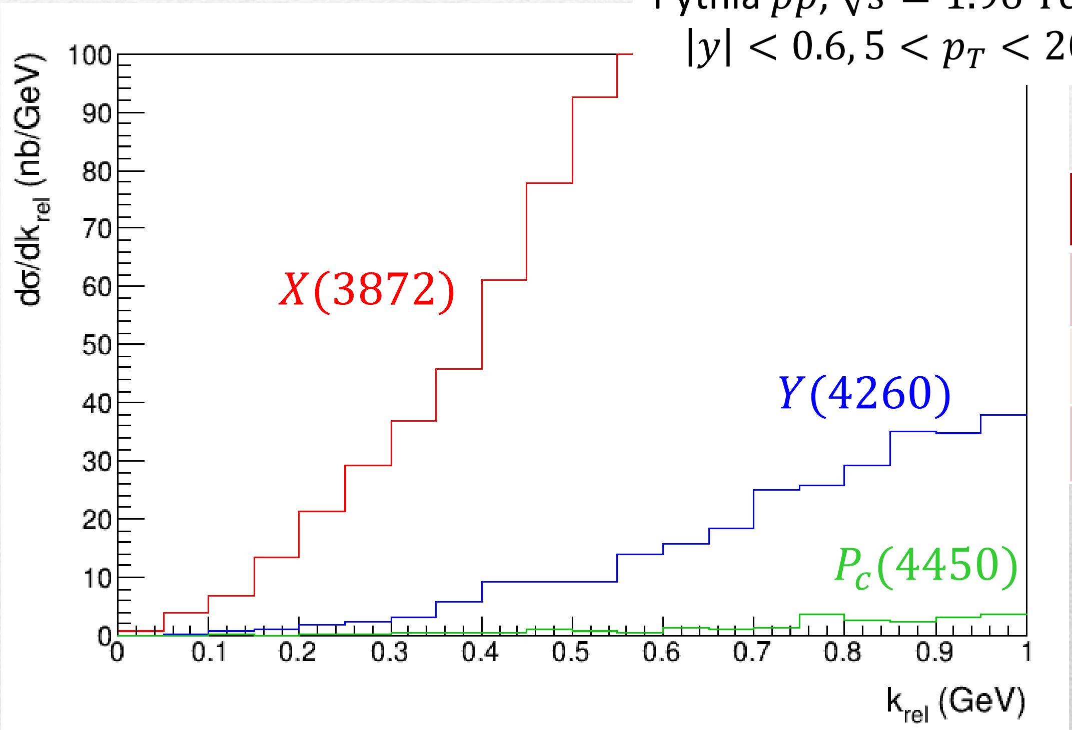
LHCb, $\sqrt{s} = 7$ TeV, JHEP 1206, 141
all: $2 < y < 4, 3 < p_T < 12$ GeV

Production of $Y(4260)$ and $P_c(4450)$

Naively, the fragmentation function of the D_1 is 1/10 of the D^*, but the cross section scales as k_{max}^3

J. Nys and AP, to appear

Pythia $p\bar{p}$, $\sqrt{s} = 1.96$ TeV
$|y| < 0.6, 5 < p_T < 20$ GeV



	No FSI	With FSI
$Y(4260)/X$	23	0.75
$P_c(4450)$/X	1.0	0.01

The production of $Y(4260)$ is expected to be at worse comparable with the $X(3872)$

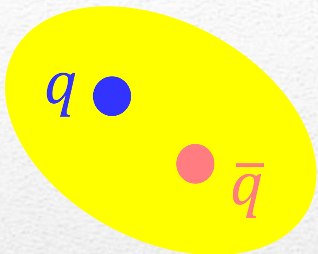
Conclusions & prospects

- The discovery of exotic states has challenged the well established Charmonium framework
- Experiments are (too) prolific! Constant feedback on predictions
- Thorough amplitude anlyses might shed some light on the microscopic nature of the new states
- The implementation of 3-body unitarity will be a major step to understand several of these phenomena
- Some fantasy needed, many phenomenological models introduced.
- Nuclei observation at hadron colliders can give an unexpected help in testing some phenomenological hypotheses for the XYZP states
- Search for exotic states in prompt production is a necessary step to improve our understanding of the sector

Thank you

BACKUP

Dictionary – Quark model



L = orbital angular momentum
S = spin $q + \bar{q}$

J = total angular momentum
= exp. measured spin

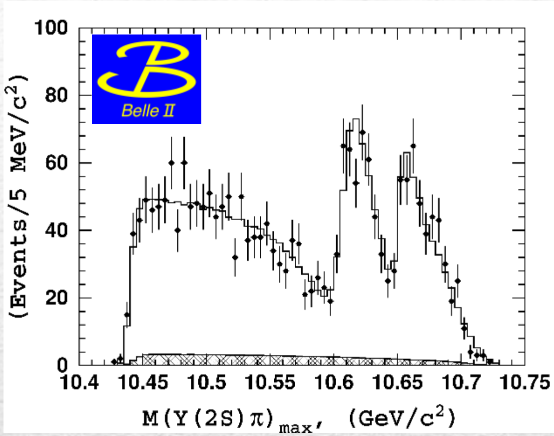
I = isospin = 0 for quarkonia

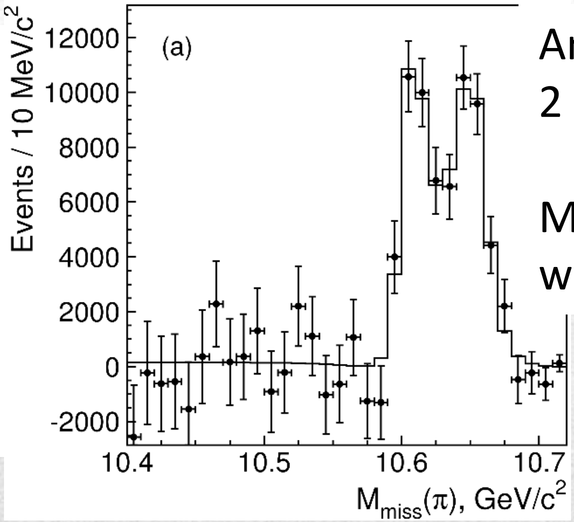
$$L - S \leq J \leq L + S$$
$$P = (-1)^{L+1}, C = (-1)^{L+S}$$
$$G = (-1)^{L+S+I}$$

J^{PC}	L	S	Charmonium ($c\bar{c}$)	Bottomonium ($b\bar{b}$)
0^{-+}	0 (S-wave)	0	$\eta_c(nS)$	$\eta_b(nS)$
1^{--}		1	$\psi(nS)$	$\Upsilon(nS)$
1^{+-}	1 (P-wave)	0	$h_c(nP)$	$h_b(nP)$
0^{++}		1	$\chi_{c0}(nP)$	$\chi_{b0}(nP)$
1^{++}		1	$\chi_{c1}(nP)$	$\chi_{b1}(nP)$
2^{++}		1	$\chi_{c2}(nP)$	$\chi_{b2}(nP)$

But $J/\psi = \psi(1S)$, $\psi' = \psi(2S)$

Charged Z states: $Z_b(10610), Z_b'(10650)$





Anomalous dipion width in $\Upsilon(5S)$,
2 orders of magnitude larger than $\Upsilon(nS)$

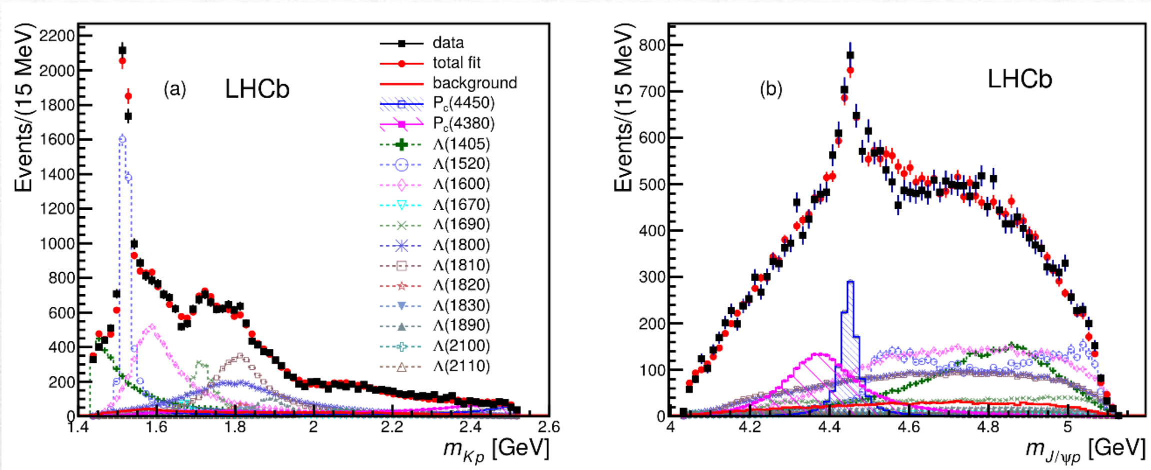
Moreover, observed $\Upsilon(5S) \to h_b(nP)\pi\pi$
which violates HQSS

2 twin resonances!

$\Upsilon(5S) \to Z_b(10610)^+\pi^- \to \Upsilon(nS)\,\pi^+\pi^-, h_b(nP)\,\pi^+\pi^-$
and $\to (BB^*)^+\pi^-$
$M = 10607.2 \pm 2.0$ MeV, $\Gamma = 18.4 \pm 2.4$ MeV

$\Upsilon(5S) \to Z_b'(10650)^+\pi^- \to \Upsilon(nS)\,\pi^+\pi^-, h_b(nP)\,\pi^+\pi^-$
and $\to \bar{B}^{*0}B^{*+}\pi^-$
$M = 10652.2 \pm 1.5$ MeV, $\Gamma = 11.5 \pm 2.2$ MeV

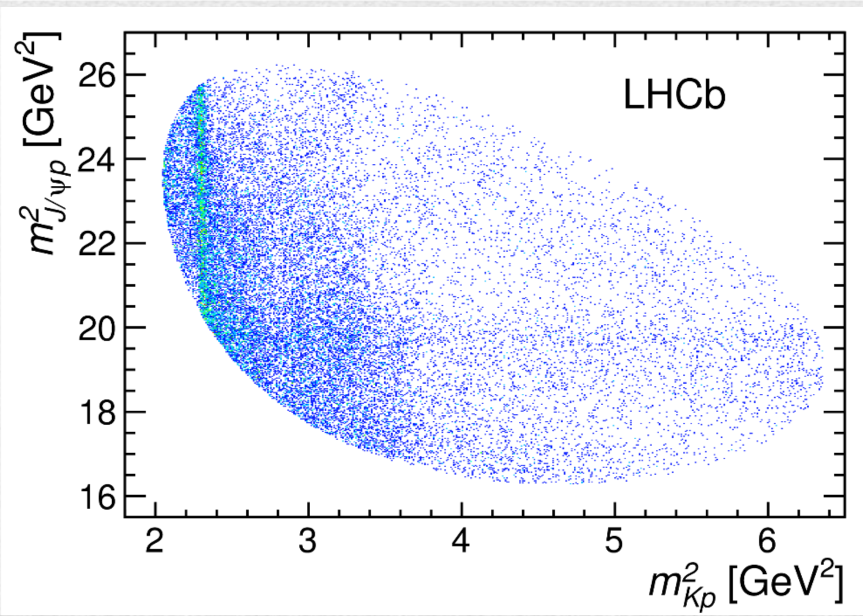
Pentaquarks!



LHCb, PRL 115, 072001
LHCb, PRL 117, 082003

Two states seen in $\Lambda_b \to (J/\psi\, p)\, K^-$, evidence in $\Lambda_b \to (J/\psi\, p)\, \pi^-$

$$M_1 = 4380 \pm 8 \pm 29 \text{ MeV}$$
$$\Gamma_1 = 205 \pm 18 \pm 86 \text{ MeV}$$
$$M_2 = 4449.8 \pm 1.7 \pm 2.5 \text{ MeV}$$
$$\Gamma_2 = 39 \pm 5 \pm 19 \text{ MeV}$$



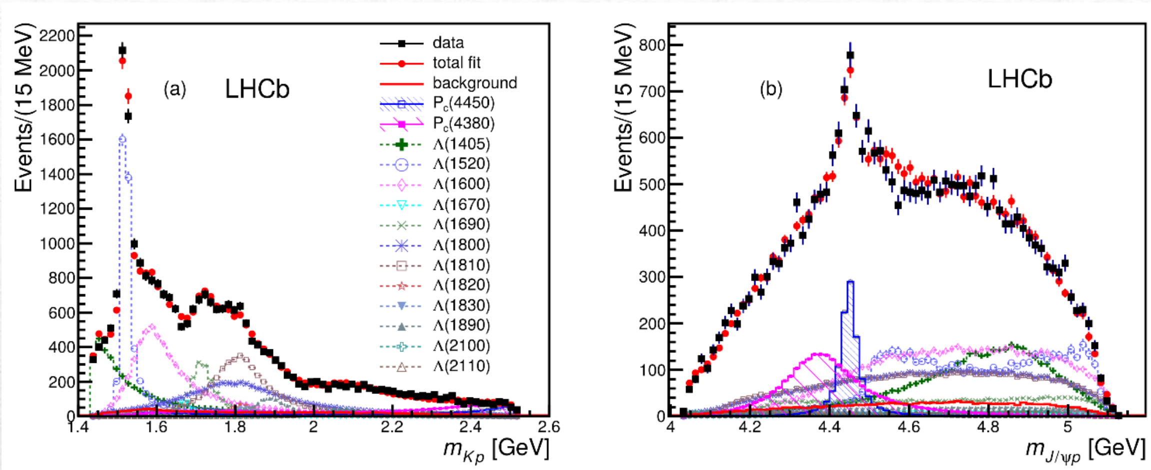
Quantum numbers

$$J^P = \left(\frac{3}{2}^-, \frac{5}{2}^+\right) \text{ or } \left(\frac{3}{2}^+, \frac{5}{2}^-\right) \text{ or } \left(\frac{5}{2}^+, \frac{3}{2}^-\right)$$

Opposite parities needed for the interference to correctly describe angular distributions, low mass region contaminated by Λ^* (model dependence?)

No obvious threshold nearby

Pentaquarks!



LHCb, PRL 115, 072001
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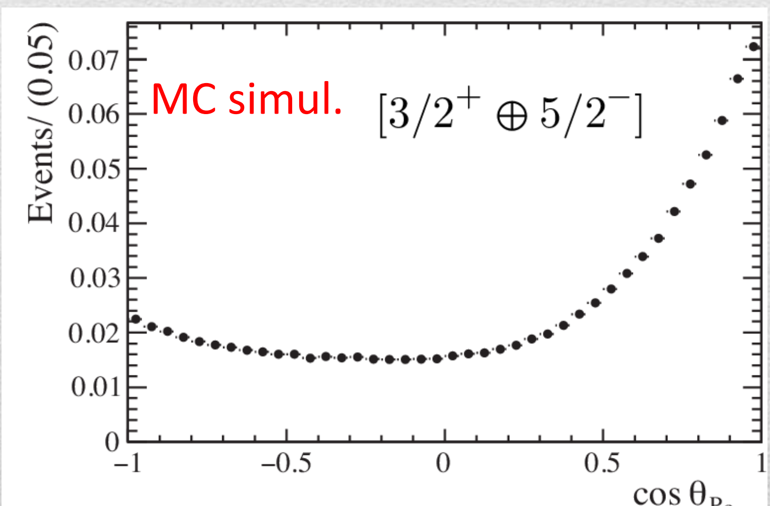

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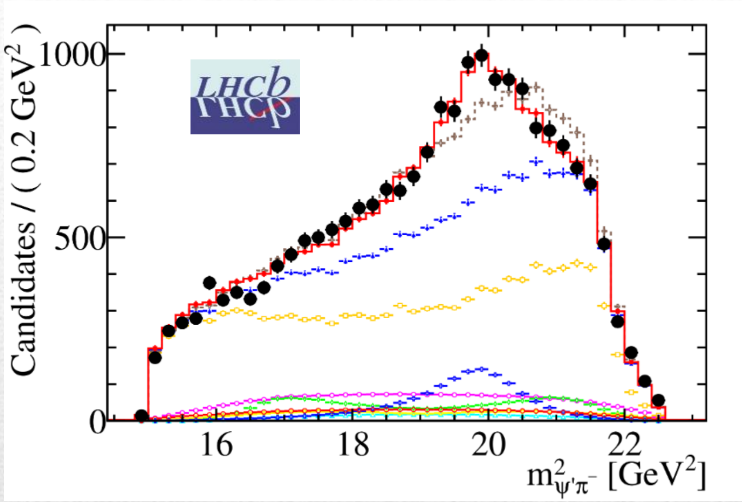
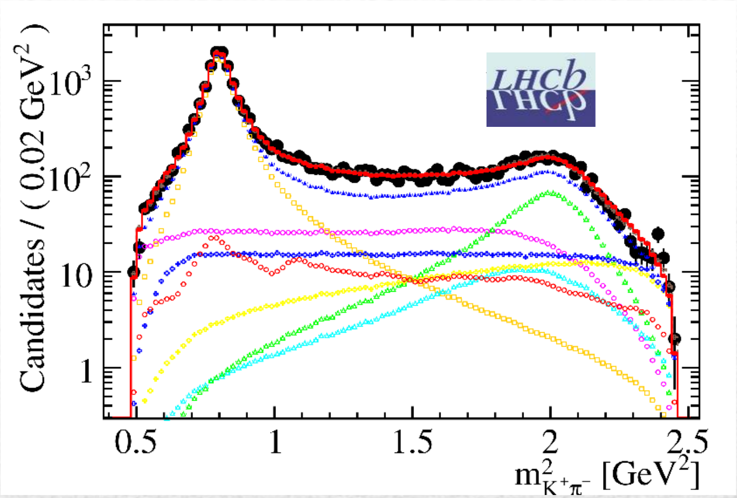
No obvious threshold nearby

Charged Z states: $Z(4430)$

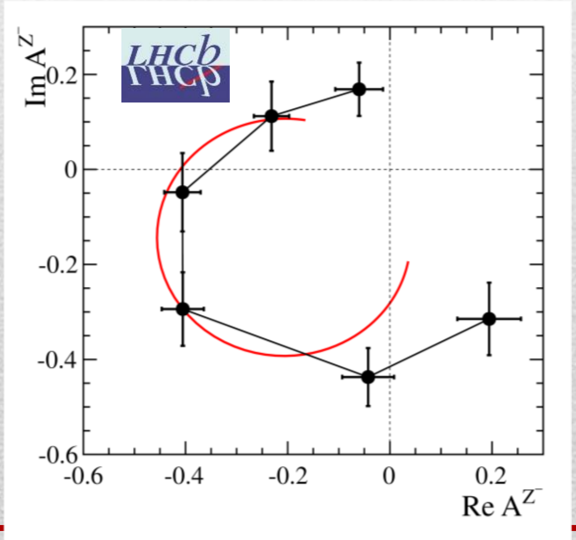




$Z(4430)^+ \to \psi(2S)\,\pi^+$
$I^G J^{PC} = 1^+ 1^{+-}$

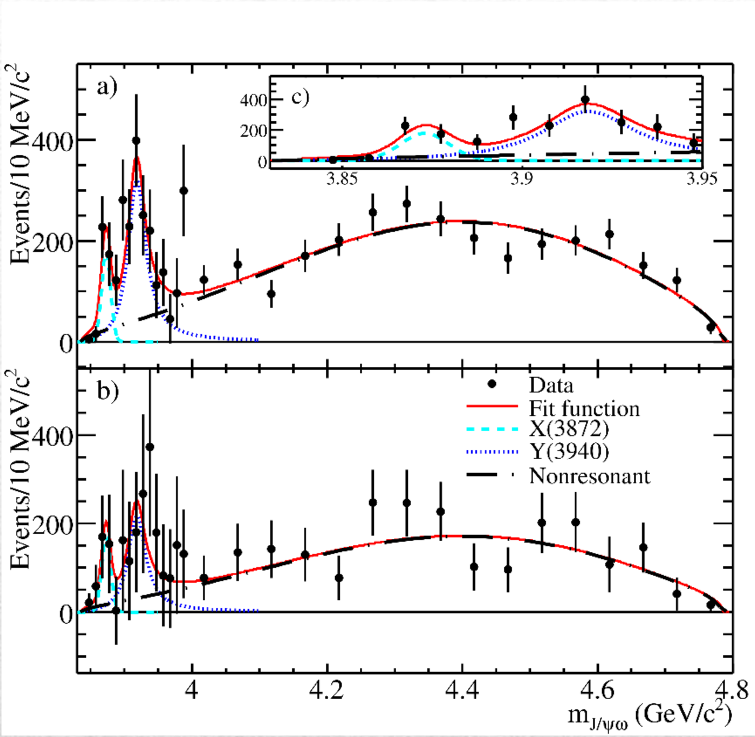
$M = 4475 \pm 7^{+15}_{-25}$ MeV
$\Gamma = 172 \pm 13^{+37}_{-34}$ MeV

Far from open charm thresholds

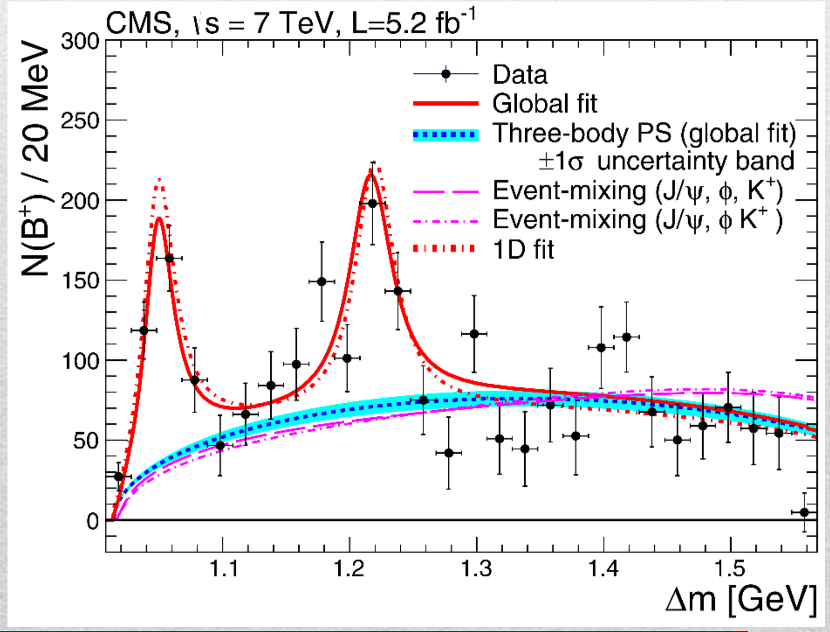


If the amplitude is a free complex number, in each bin of $m^2_{\psi'\pi^-}$, the resonant behaviour appears as well

Other beasts



$X(3915)$, seen in $B \to X\ K \to J/\psi\ \omega$
and $\gamma\gamma \to X \to J/\psi\ \omega$
$J^{PC} = 0^{++}$, candidate for $\chi_{c0}(2P)$
But $X(3915) \not\to D\bar{D}$ as expected,
and the hyperfine splitting
$\mathrm{M}(2^{++}) - \mathrm{M}(0^{++})$ too small

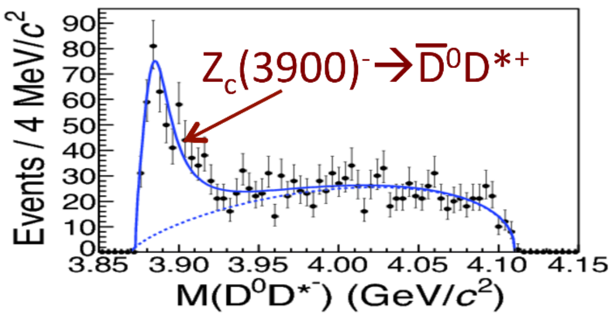


One/two peaks seen in $B \to XK \to J/\psi\ \phi\ K$, close to threshold

$Y(4260) \to \bar{D} D_1$?

$e^+e^- \to Y(4260) \to \pi^- \bar{D}^0 D^{*+}$

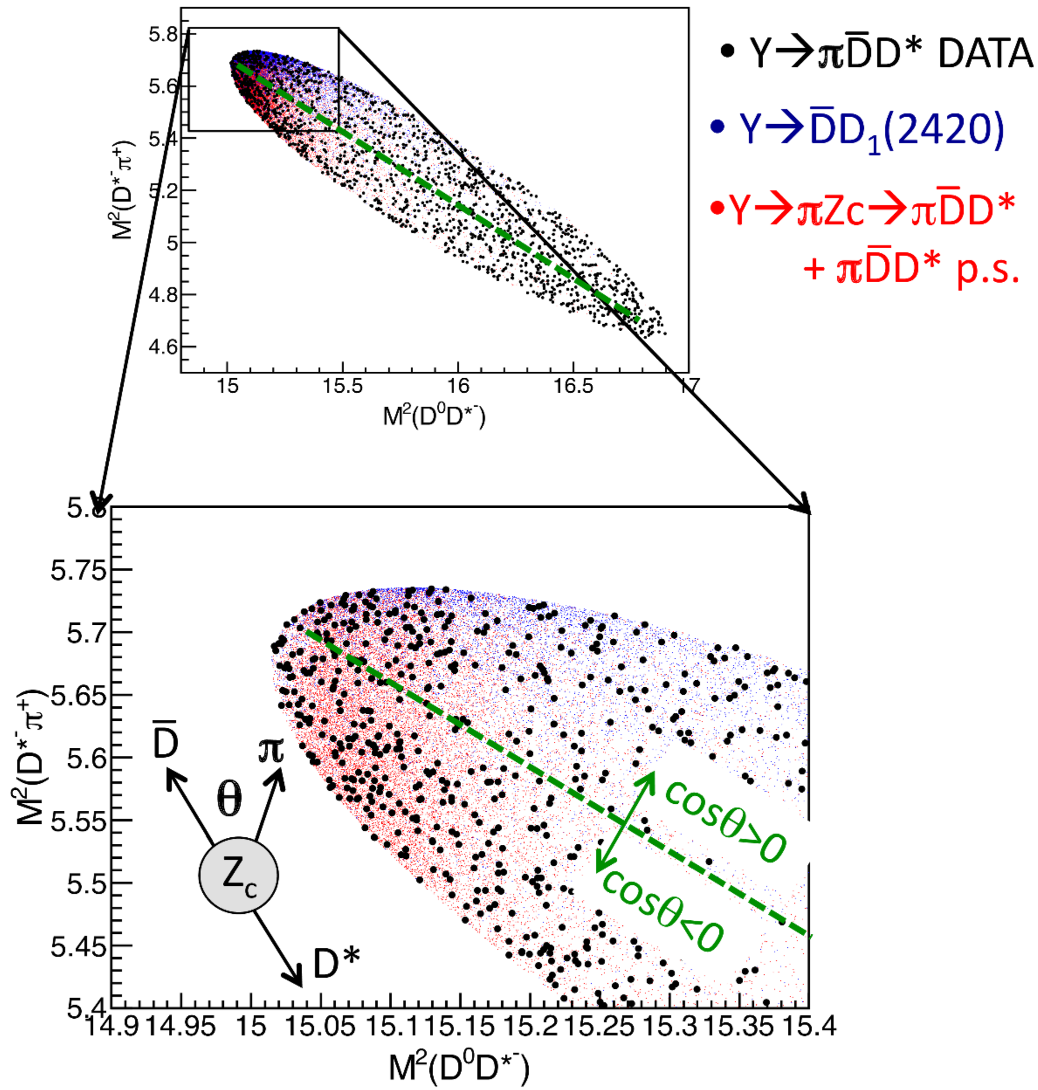
BESIII PRL 112, 022001



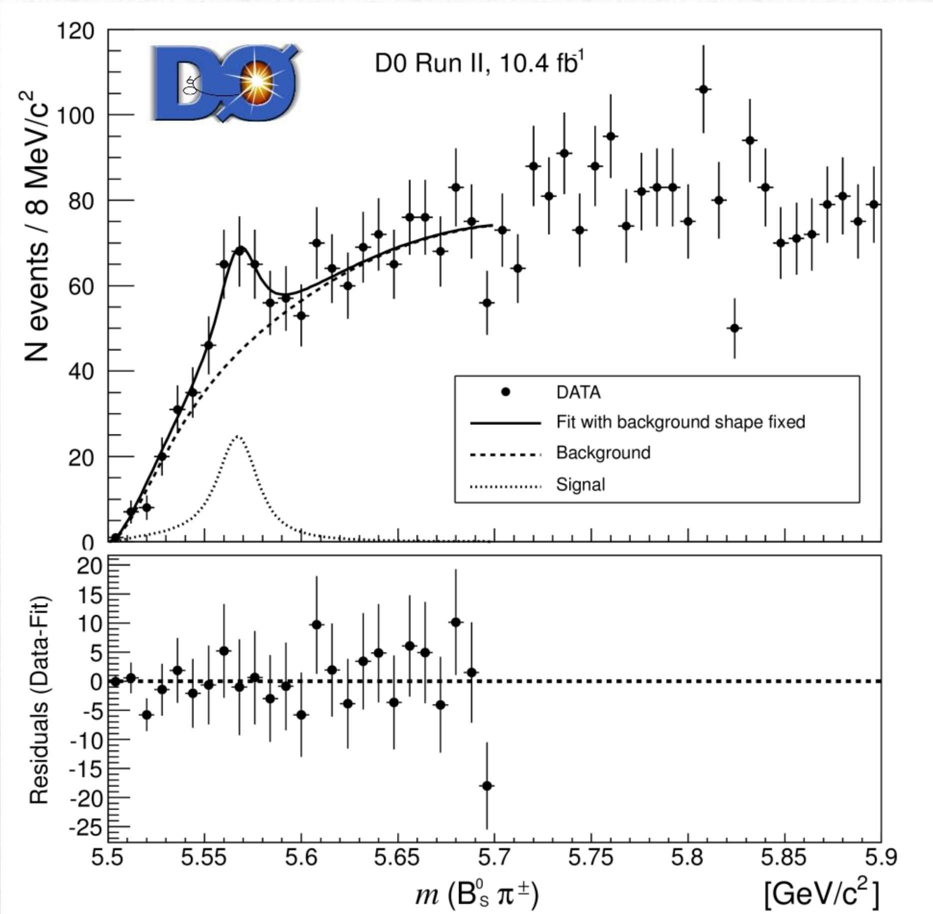
$$\mathcal{A} = \frac{N_{|cos\theta|>0.5} - N_{|cos\theta|<0.5}}{N_{|cos\theta|>0.5} + N_{|cos\theta|<0.5}}$$

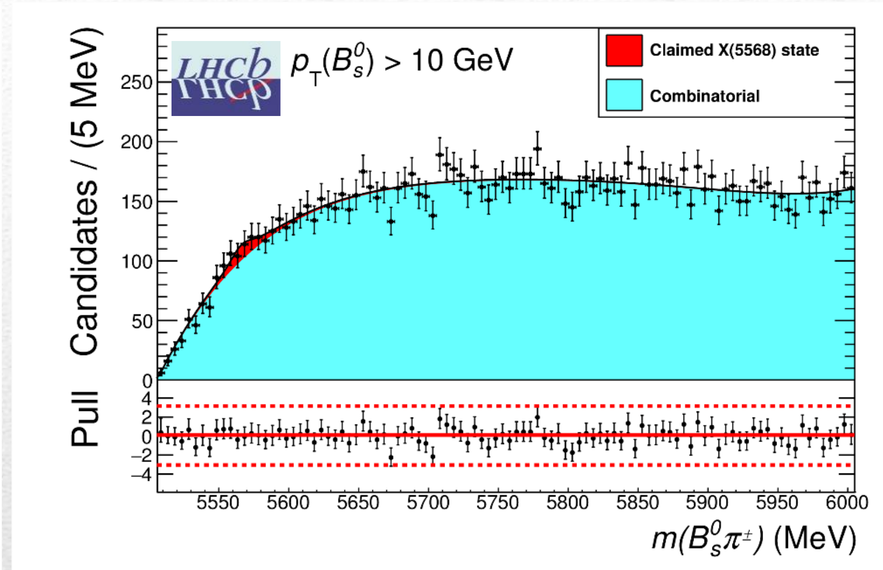
	DD_1 MC	Z_c+ps MC	data
$\mathcal{A}$	0.43±0.04	0.02±0.02	0.12+0.06

Not a lot of room for $\bar{D}D_1(2410)$



Flavored $X(5568)$

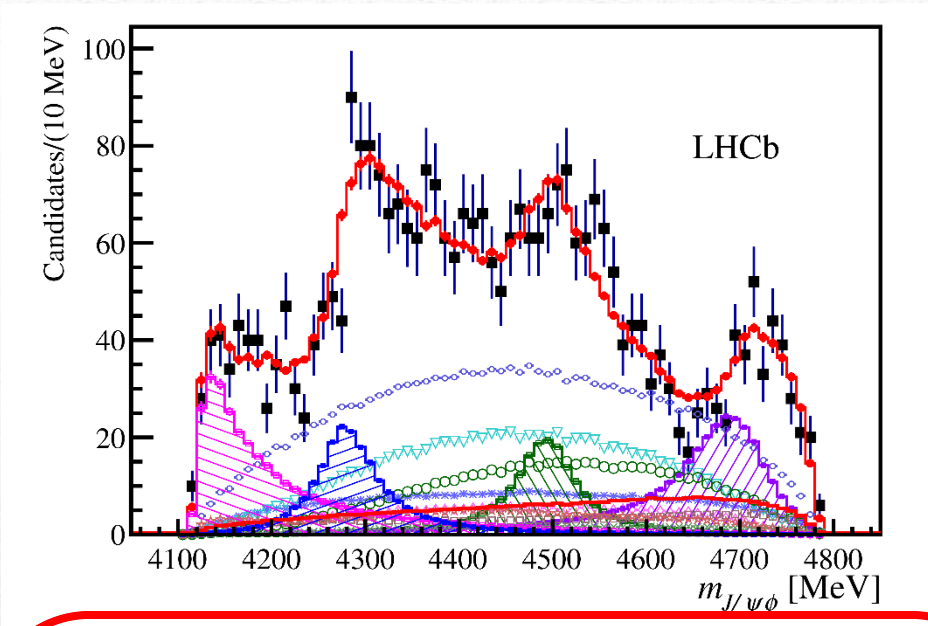


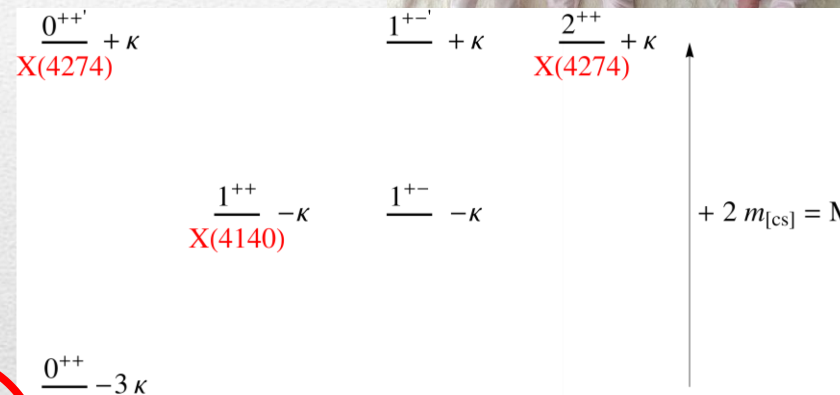


- A flavored state seen in $B_s^0\,\pi$ invariant mass by D0 (both $B_s^0 \to J/\psi\,\phi$ and $\to D_s\mu\nu$),
- not confermed by LHCb or CMS
- (different kinematics? Compare differential distributions)

Controversy to be solved

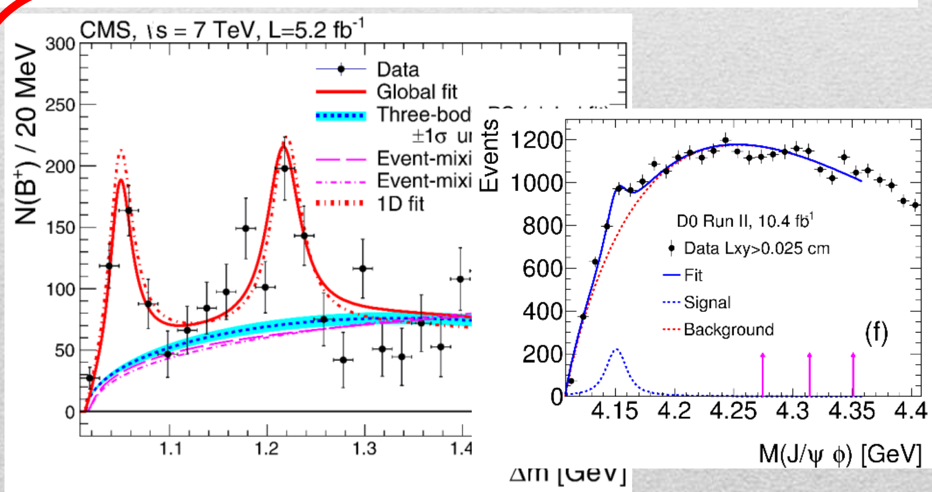
Tetraquark: the $c\bar{c}s\bar{s}$ states





Good description of the spectrum **but** one has to assume the axial assignment for the $X(4274)$ to be incorrect (two unresolved states with 0^{++} and 2^{++})

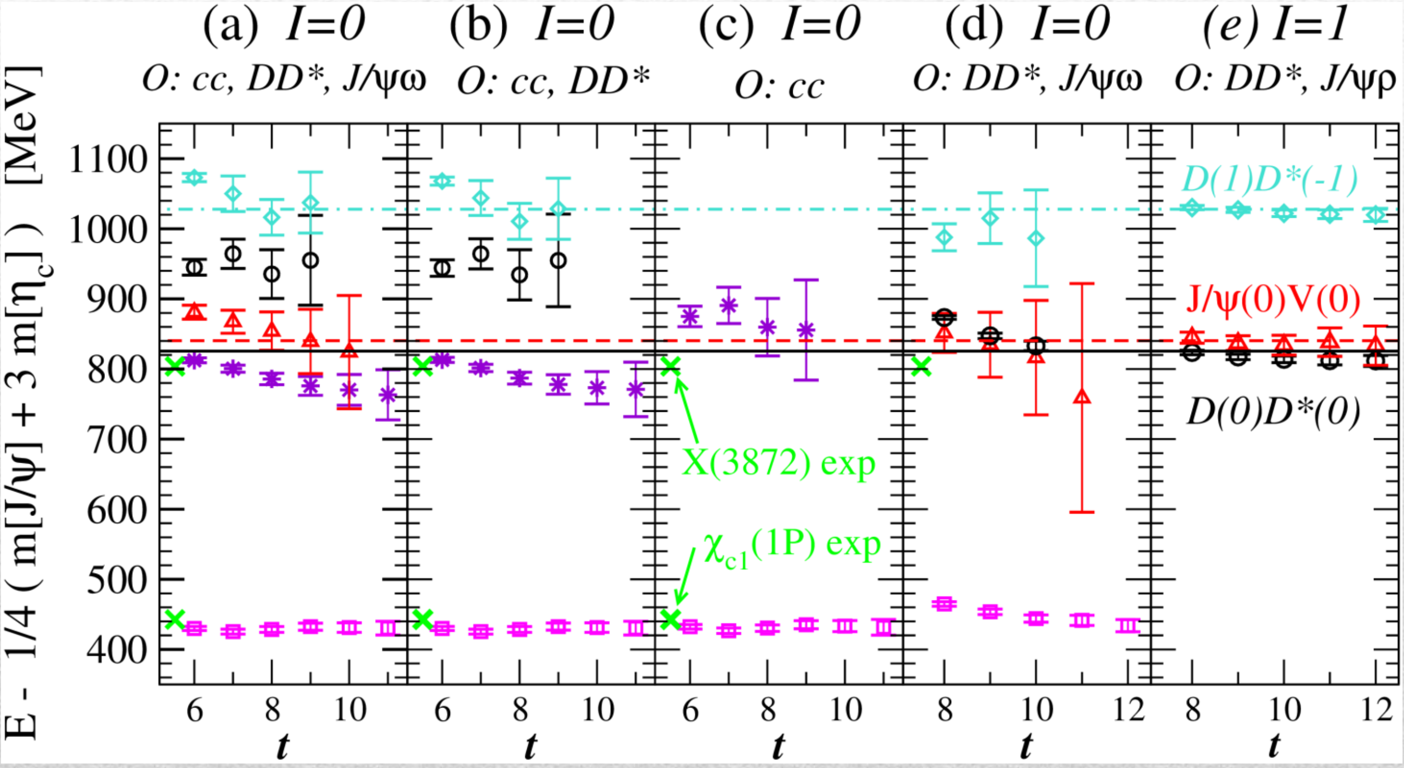
Maiani, Polosa and Riquer, PRD 94, 054026



Much narrower than LHCb! Look for prompt!

$X(3872)$ on the lattice

There is only evidence (?) for the $X(3872)$ in the $I^G J^{PC} = 0^+ 1^{++}$ channel



Caveats:

- Small lattices, large artifacts
- Three body dynamics may play a role
- Interpretation of the overlap coefficients is questionable

Status of other XYZ on the lattice is even less clear

S. Prelovsek, L. Leskovec, PRL111, 192001

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$X(3823)$	3823.1 ± 1.9	< 24	$?^{?-}$	$B \to K(\chi_{c1}\gamma)$	Belle[23] (4.0)
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle[24,25] (>10), *BABAR*[26] (8.6)
				$p\bar{p} \to (\pi^+\pi^- J/\psi)\ldots$	CDF[27,28] (11.6), D0[29] (5.2)
				$pp \to (\pi^+\pi^- J/\psi)\ldots$	LHCb[30,31] (np)
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle[32] (4.3), *BABAR*[33] (4.0)
				$B \to K(\gamma\, J/\psi)$	Belle[34] (5.5), *BABAR*[35] (3.5)
					LHCb[36] (> 10)
				$B \to K(\gamma\, \psi(2S))$	*BABAR*[35] (3.6), Belle[34] (0.2)
					LHCb[36] (4.4)
				$B \to K(D\bar{D}^*)$	Belle[37] (6.4), *BABAR*[38] (4.9)
$Z_c(3900)^+$	3888.7 ± 3.4	35 ± 7	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III[39] (np)
				$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III[40] (8), Belle[41] (5.2)
					CLEO data[42] (>5)
$Z_c(4020)^+$	4023.9 ± 2.4	10 ± 6	1^{+-}	$Y(4260) \to \pi^-(\pi^+ h_c)$	BES III[43] (8.9)
				$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III[44] (10)
$Y(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \to K(\omega J/\psi)$	Belle[45] (8), *BABAR*[33,46] (19)
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle[47] (7.7), *BABAR*[48] (7.6)
$Z(3930)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle[49] (5.3), *BABAR*[50] (5.8)
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle[51,52] (6)
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle[41,53] (7.4)
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (1.1)
$Y(4140)$	4145.6 ± 3.6	14.3 ± 5.9	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF[56,57] (5.0), Belle[58] (1.9),
					LHCb[59] (1.4), CMS[60] (>5)
					DØ[61] (3.1)
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle[52] (5.5)
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (7.2)

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)
$Y(4220)$	4196^{+35}_{-30}	39 ± 32	1^{--}	$e^+e^- \to (\pi^+\pi^- h_c)$	BES III data[63,64] (4.5)
$Y(4230)$	4230 ± 8	38 ± 12	1^{--}	$e^+e^- \to (\chi_{c0}\omega)$	BES III[65] (>9)
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle[54] (5.0), *BABAR*[55] (2.0)
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	*BABAR*[66,67] (8), CLEO[68,69] (11)
					Belle[41,53] (15), BES III[40] (np)
				$e^+e^- \to (f_0(980) J/\psi)$	*BABAR*[67] (np), Belle[41] (np)
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III[40] (8), Belle[41] (5.2)
				$e^+e^- \to (\gamma\, X(3872))$	BES III[70] (5.3)
$Y(4290)$	4293 ± 9	222 ± 67	1^{--}	$e^+e^- \to (\pi^+\pi^- h_c)$	BES III data[63,64] (np)
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle[58] (3.2)
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^- \psi(2S))$	Belle[71] (8), *BABAR*[72] (np)
$Z(4430)^+$	4478 ± 17	180 ± 31	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle[73,74] (6.4), *BABAR*[75] (2.4)
					LHCb[76] (13.9)
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle[62] (4.0)
$Y(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle[77] (8.2)
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^- \psi(2S))$	Belle[71] (5.8), *BABAR*[72] (5)
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(5S) \to \pi(\pi\Upsilon(nS))$	Belle[78,79] (>10)
				$\Upsilon(5S) \to \pi^-(\pi^+ h_b(nP))$	Belle[78] (16)
				$\Upsilon(5S) \to \pi^-(B\bar{B}^*)^+$	Belle[80] (8)
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(5S) \to \pi^-(\pi^+\Upsilon(nS))$	Belle[78] (>10)
				$\Upsilon(5S) \to \pi^-(\pi^+ h_b(nP))$	Belle[78] (16)
				$\Upsilon(5S) \to \pi^-(B^*\bar{B}^*)^+$	Belle[80] (6.8)

Esposito, AP, Polosa, Phys.Rept. 668

Guerrieri, AP, Piccinini, Polosa, IJMPA 30, 1530002

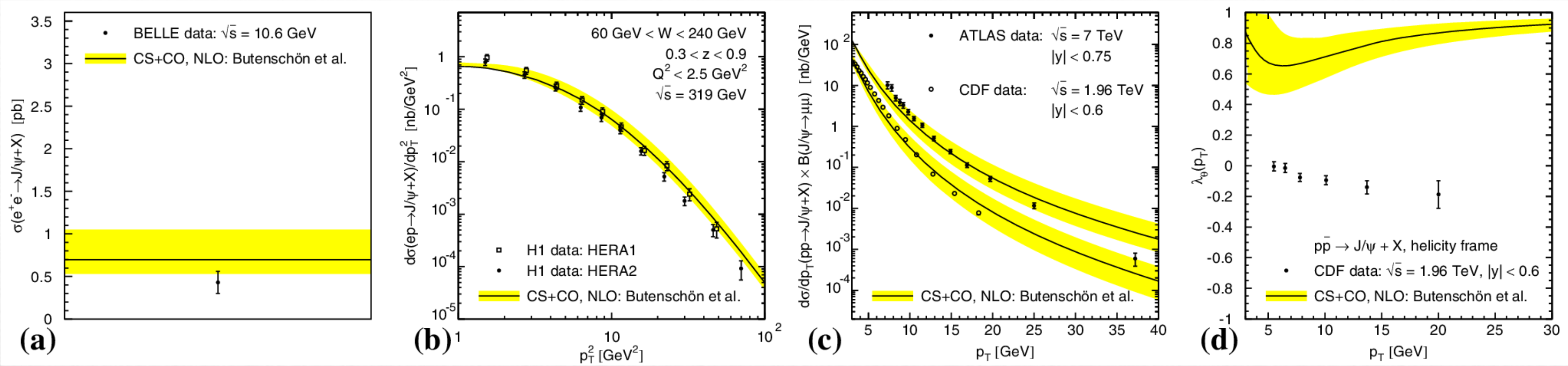
Multiscale system

Systematically integrate out the heavy scale, $m_Q \gg \Lambda_{QCD}$

$$m_Q \gg m_Q v \gg m_Q v^2$$

Full QCD → NRQCD → pNRQCD

$m_b \sim 5\text{ GeV}, m_c \sim 1.5\text{ GeV}$
$v_b^2 \sim 0.1, v_c^2 \sim 0.3$

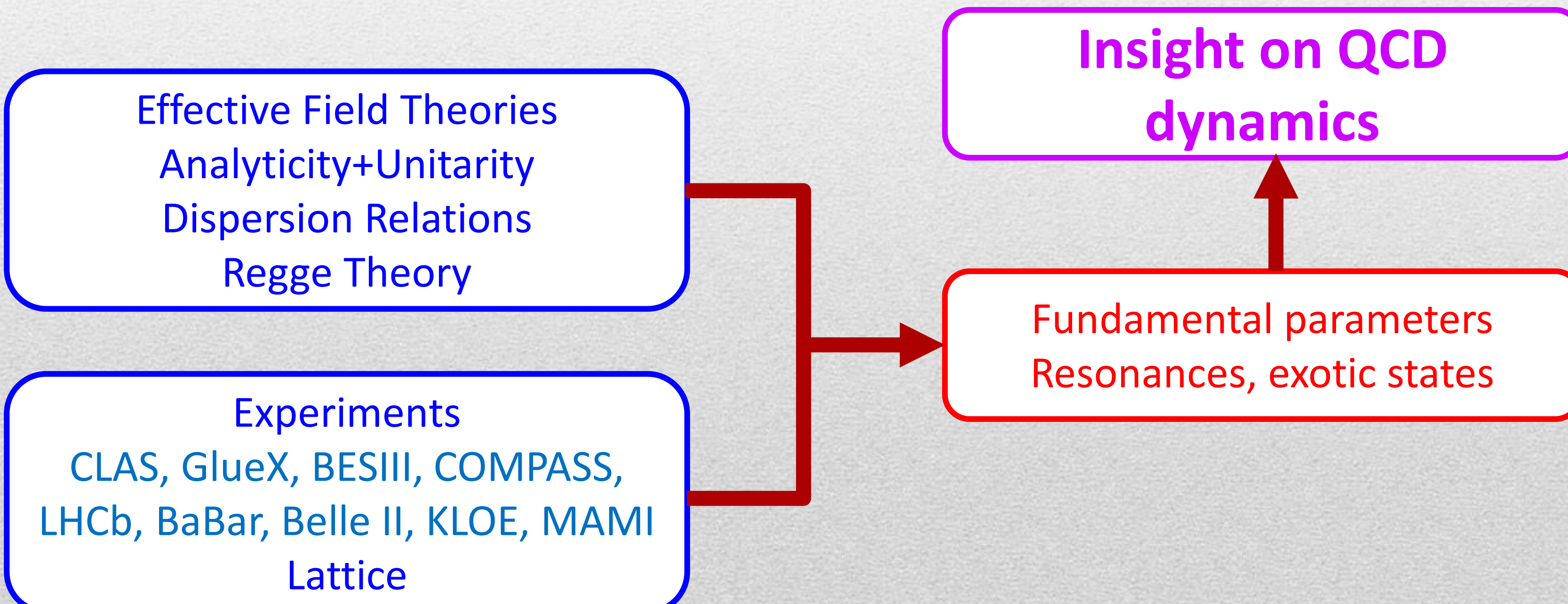


Factorization (to be proved) of universal LDMEs

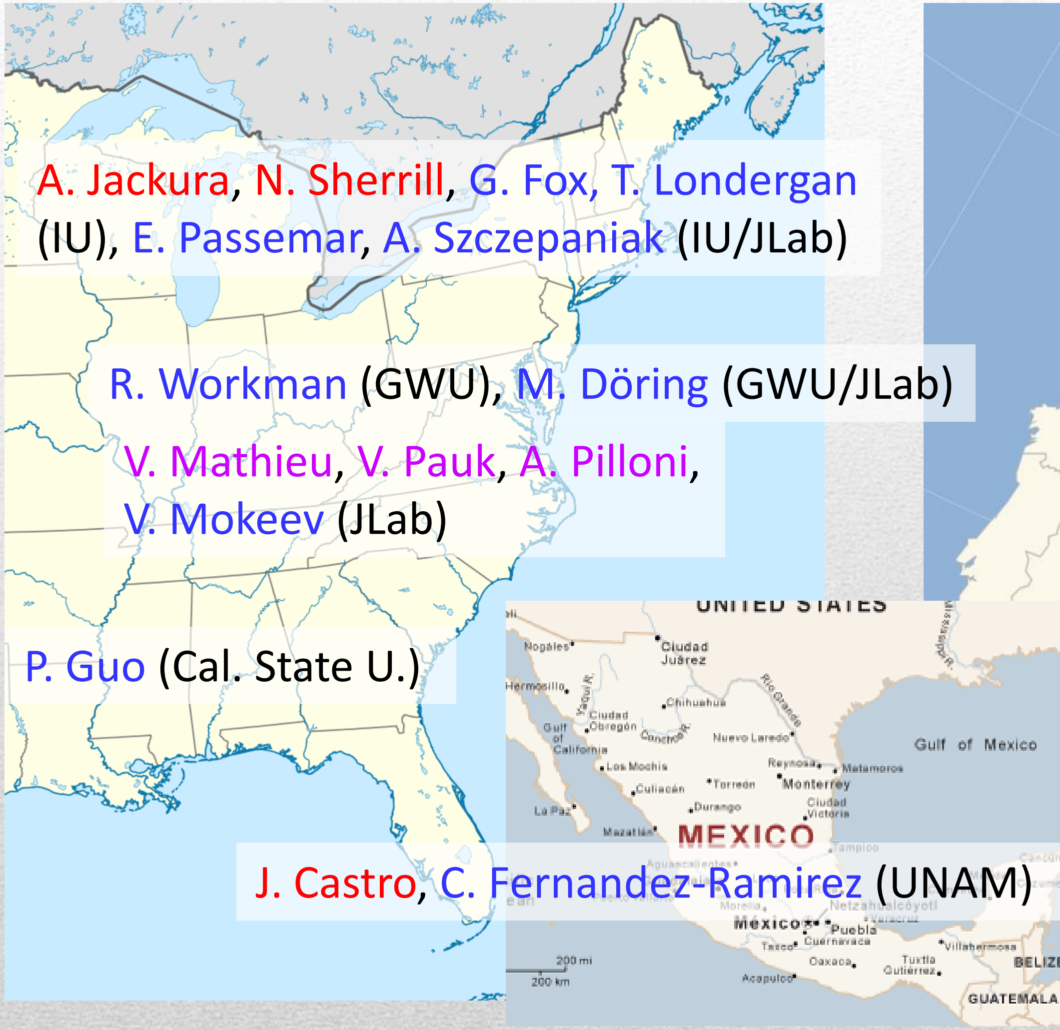
Good description of many production channels, some known puzzles (polarizations)

Joint Physics Analysis Center

- Joint effort between theorists and experimentalists to work together to make the best use of the next generation of very precise data taken at JLab and in the world
- Created in 2013 by JLab & IU agreement
- It is engaged in education of further generations of hadron physics practitioners



Joint Physics Analysis Center



A. Jackura, N. Sherrill, G. Fox, T. Londergan (IU), E. Passemar, A. Szczepaniak (IU/JLab)

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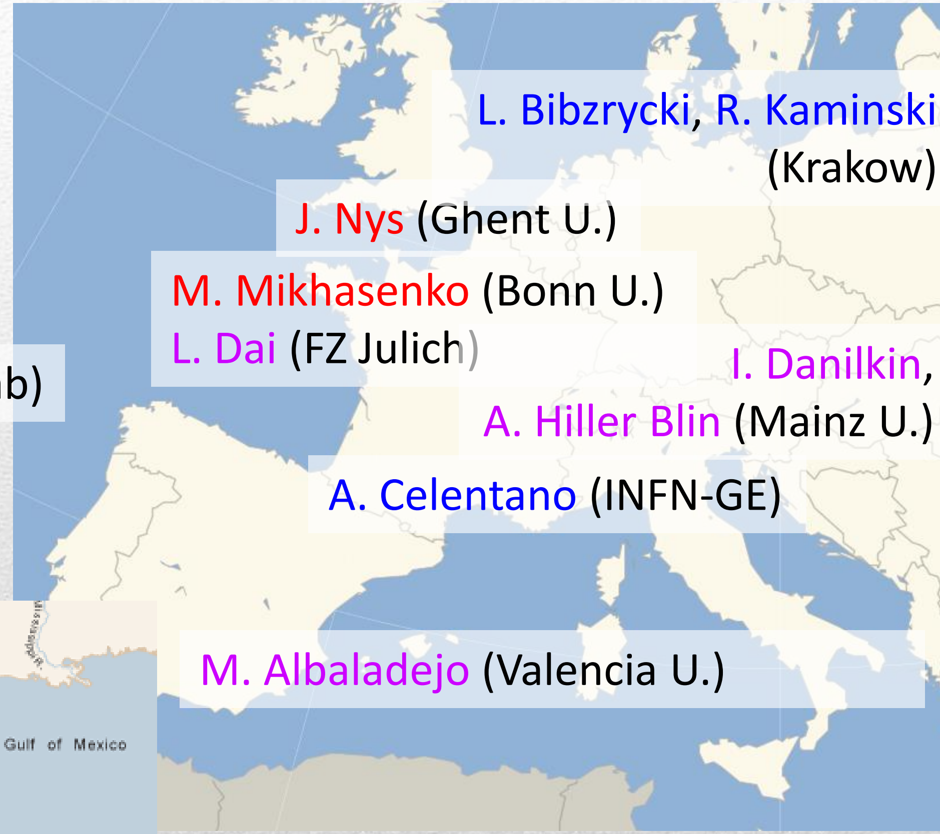
V. Mathieu, V. Pauk, A. Pilloni, V. Mokeev (JLab)

P. Guo (Cal. State U.)

J. Castro, C. Fernandez-Ramirez (UNAM)



L. Bibzrycki, R. Kaminski (Krakow)

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L. Dai (FZ Julich)

I. Danilkin, A. Hiller Blin (Mainz U.)

A. Celentano (INFN-GE)

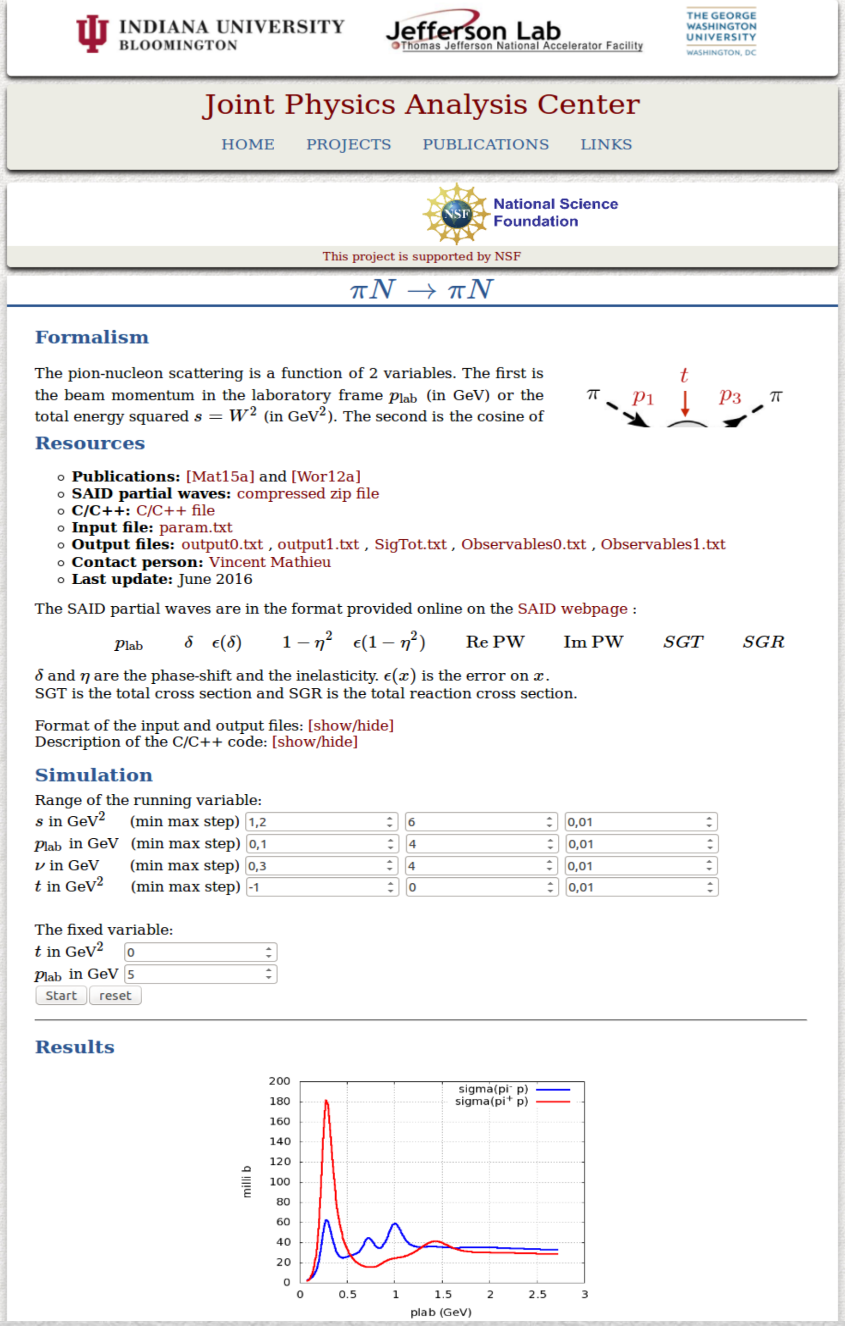
M. Albaladejo (Valencia U.)

Students, Postdocs, Faculties

Interactive tools

- Completed projects are fully documented on interactive portals
- These include description on physics, conventions, formalism, etc.
- The web pages contain source codes with detailed explanation how to use them. Users can run codes online, change parameters, display results.

http://www.indiana.edu/~jpac/



Strategy

AP *et al.* (JPAC), arXiv:1612.06490

- We fit the following invariant mass distributions:
  - BESIII PRL110, 252001 $J/\psi\,\pi^+, J/\psi\,\pi^-, \pi^+\pi^-$ at $E_{CM} = 4.26$ GeV
  - BESIII PRL110, 252001 $J/\psi\,\pi^0$ at $E_{CM} = 4.23, 4.26,$ ~~4.36~~ GeV
  - BESIII PRD92, 092006 $\overline{D^0}D^{*+}, \overline{D^{*0}}D^+$ (double tag) at $E_{CM} = 4.23, 4.26$ GeV
  - BESIII PRL115, 222002 $\overline{D^0}D^{*0}, \overline{D^{*0}}D^0$ at $E_{CM} = 4.23, 4.26$ GeV
  - ~~BESIII PRL112, 022001 $\overline{D^0}D^{*+}, \overline{D^{*0}}D^+$ (single tag) at $E_{CM} = 4.26$ GeV~~
  - ~~Belle PRL110, 252002 $J/\psi\,\pi^\pm$ at $E_{CM} = 4.26$ GeV~~
  - ~~CLEO-c data PLB727, 366 $J/\psi\,\pi^\pm, J/\psi\,\pi^0$ at at $E_{CM} = 4.17$ GeV~~

- Published data are not efficiency/acceptance corrected,
  → we are not able to give the absolute normalization of the amplitudes

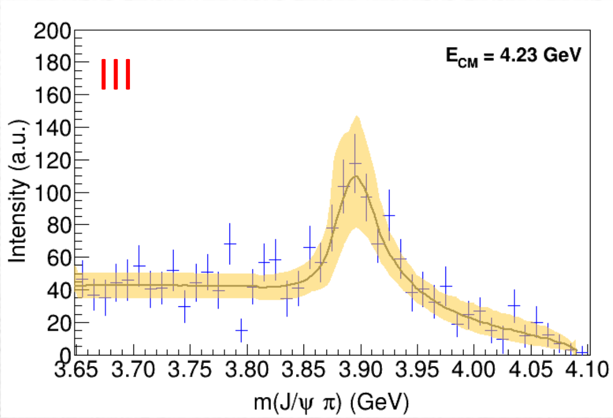
- No given dependence on E_{CM} is assumed – the couplings at different E_{CM} are independent parameters

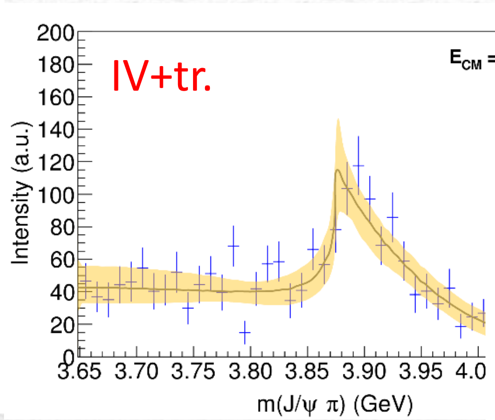
Strategy

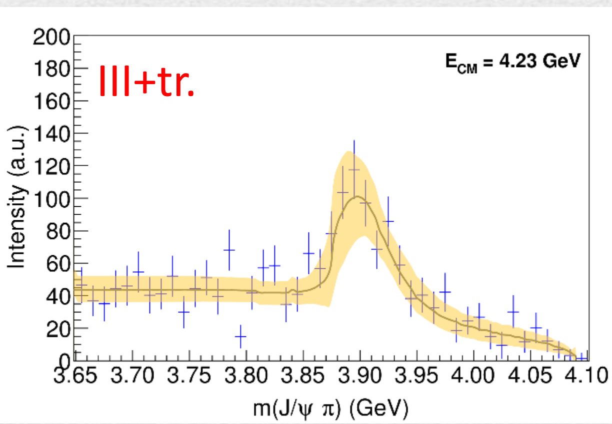
AP *et al.* (JPAC), arXiv:1612.06490

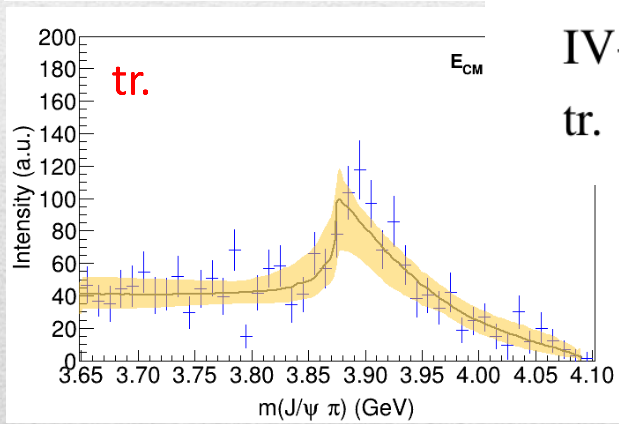
- Reducible (incoherent) backgrounds are pretty flat and do not influence the analysis, except the peaking background in $\overline{D^0}D^{*0}$, $\overline{D^{*0}}D^0$ (subtracted)
- Some information about angular distributions has been published, but it's not constraining enough → we do not include in the fit
- Because of that, we approximate all the particles to be scalar – this affects the value of couplings, which are not normalized anyway – but not the position of singularities. This also limits the number of free parameters

Fit summary







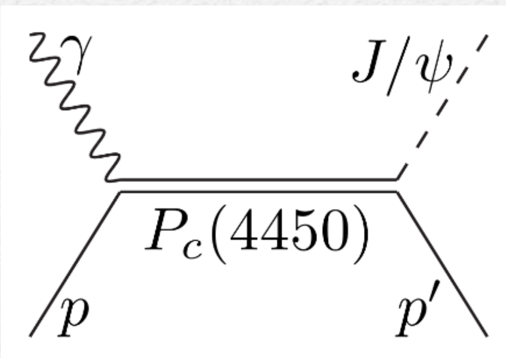


Scenario	χ^2	DOF	χ^2/DOF
III	644	532	1.21
III+tr.	642	532	1.21
IV+tr.	666	532	1.25
tr.	695	532	1.31

Naive loglikelihood ratio test give a $\sim 4\sigma$ significance of the scenario III+tr. over IV+tr., looking at plots it looks too much – better using some more solid test

P_c photoproduction

To exclude any rescattering mechanism, we propose to search the $P_c(4450)$ state in photoproduction.



Hadronic vertex   EM vertex

$$\langle \lambda_\psi \lambda_{p'} | \, T_r \, | \lambda_\gamma \lambda_p \rangle = \frac{\langle \lambda_\psi \lambda_{p'} | \, T_{\mathrm{dec}} \, | \lambda_R \rangle \langle \lambda_R | \, T^\dagger_{\mathrm{em}} \, | \lambda_\gamma \lambda_p \rangle}{M_r^2 - W^2 - \mathrm{i}\Gamma_r M_r}$$

Hadronic part

- 3 independent helicity couplings,
  → approx. equal, $g_{\lambda_\psi, \lambda_{p'}} \sim g$
- g extracted from total width and (unknown) branching ratio

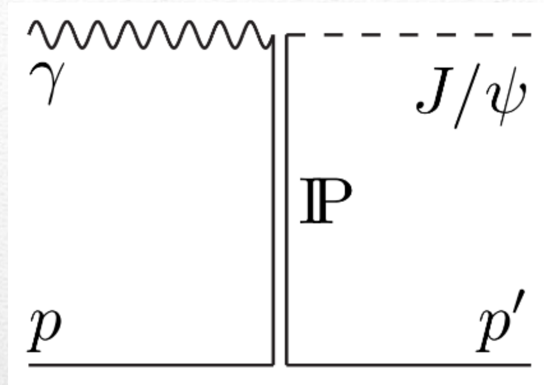
Vector meson dominance relates the radiative width to the hadronic width

$$\Gamma_\gamma = 4\pi\alpha \, \Gamma_{\psi p} \left(\frac{f_\psi}{M_\psi} \right)^2 \left(\frac{\bar{p}_i}{\bar{p}_f} \right)^{2\ell+1} \times \frac{4}{6}$$

Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

Background parameterization

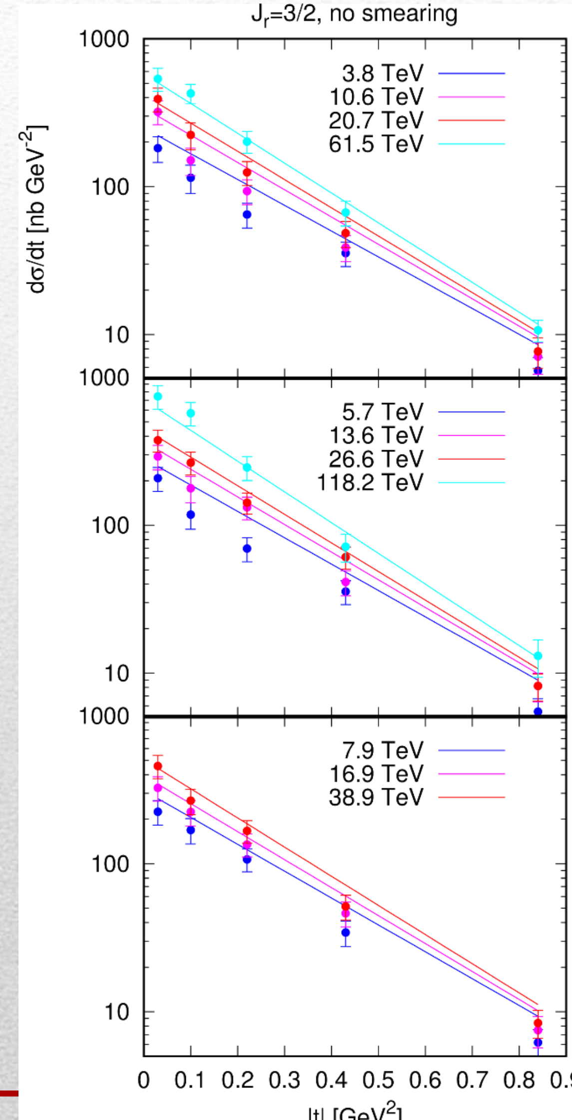
The background is described via an Effective Pomeron, whose parameters are fitted to high energy data from Hera



$$\langle \lambda_\psi \lambda_{p'} | T_P | \lambda_\gamma \lambda_p \rangle = iA \left(\frac{s - s_t}{s_0} \right)^{\alpha(t)} e^{b_0 (t - t_{\min})} \delta_{\lambda_p \lambda_{p'}} \delta_{\lambda_\psi \lambda_\gamma}$$

Asymptotic + Effective threshold

Helicity conservation

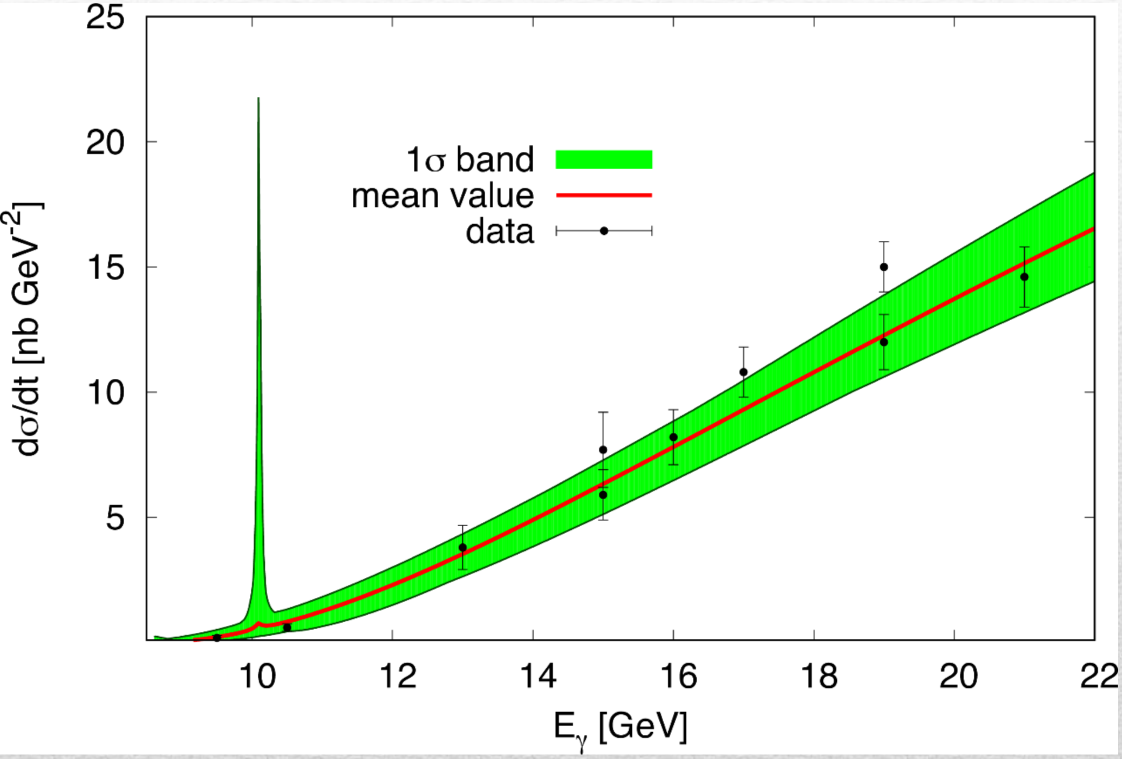


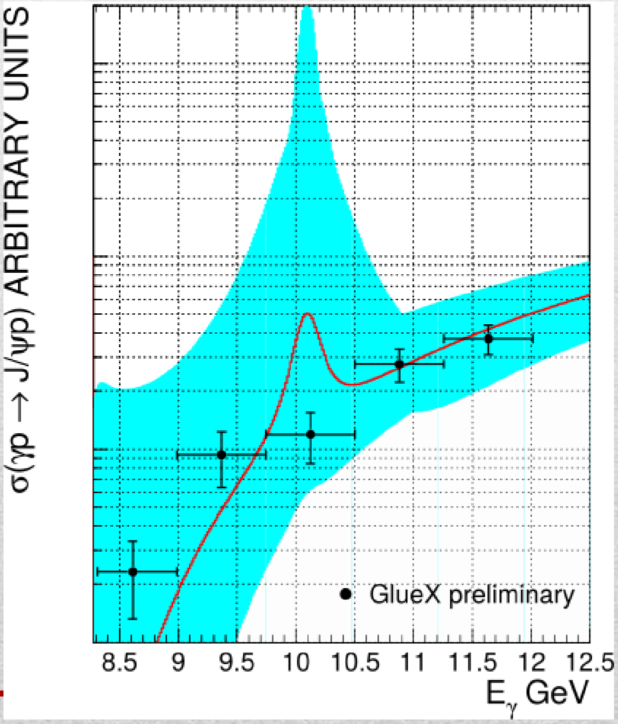
Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

Pentaquark photoproduction

$J^P = (3/2)^-$

σ_s (MeV)	0	60
A	$0.156^{+0.029}_{-0.020}$	$0.157^{+0.039}_{-0.021}$
α_0	$1.151^{+0.018}_{-0.020}$	$1.150^{+0.018}_{-0.026}$
α' (GeV^{-2})	$0.112^{+0.033}_{-0.054}$	$0.111^{+0.037}_{-0.064}$
s_t (GeV^2)	$16.8^{+1.7}_{-0.9}$	$16.9^{+2.0}_{-1.6}$
b_0 (GeV^{-2})	$1.01^{+0.47}_{-0.29}$	$1.02^{+0.61}_{-0.32}$
$\mathcal{B}_{\psi p}$ (95% CL)	≤ 29 %	≤ 30 %





Hiller Blin, AP *et al.* (JPAC), PRD94, 034002

Lineshapes at 4260

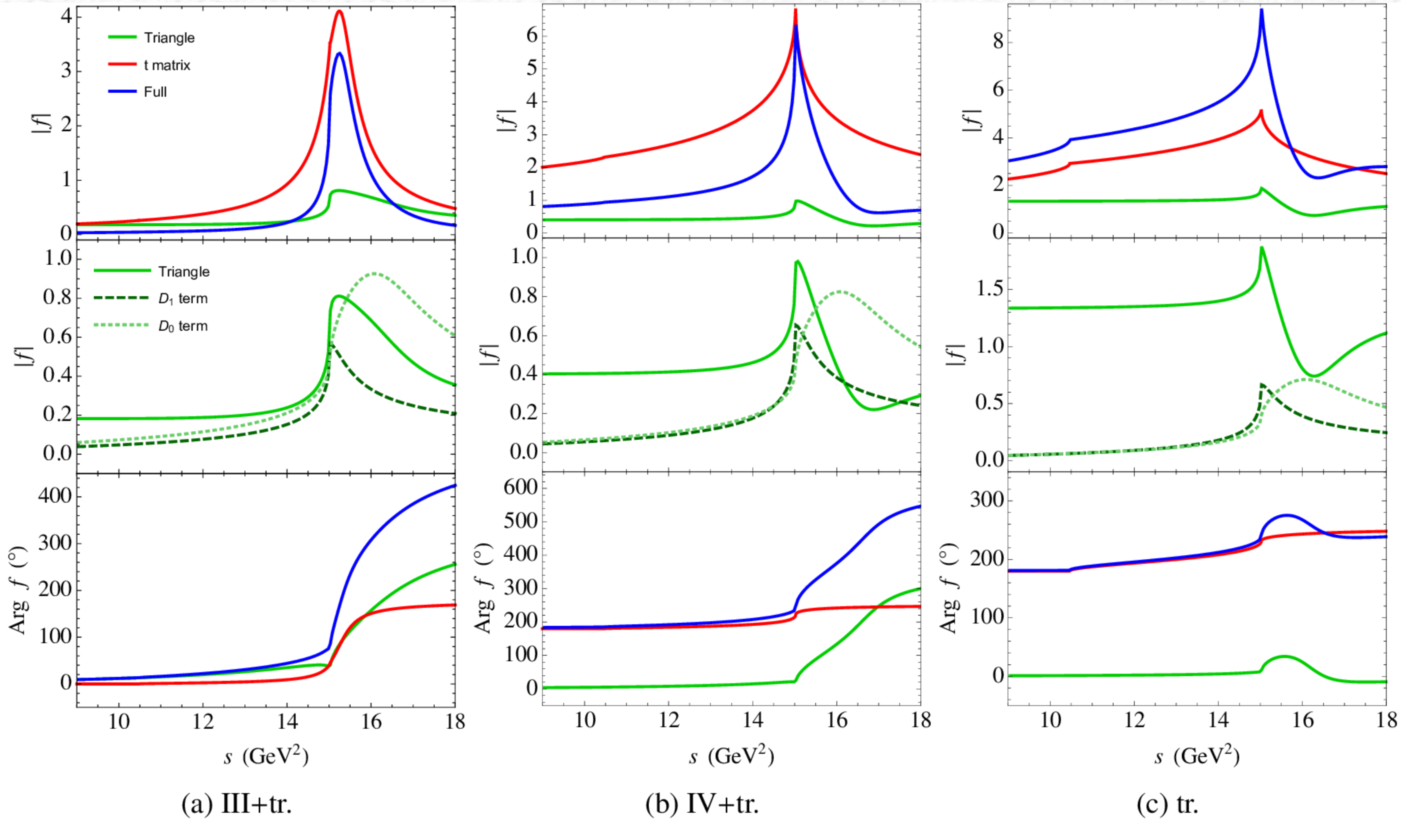


Figure 7: Interplay of scattering amplitude poles and triangle singularity to reconstruct the peak. We focus on the $J/\psi\,\pi$ channel, at $E_{CM} = 4.26$ GeV. The red curve is the t_{12} scattering amplitude, the green curve is the $c_1 + H(s, D_1) + +H(s, D_0)$ term in Eq. (9), and the blue curve is the product of the two. The upper plots show the magnitudes of these terms, the lower plots the phases. The middle row shows the contributions to the unitarized term due to the D_1 (dashed) and the D_0 (dotted). Only for D_1 the singularity is close enough to the physical region to generate a large peak. (a) The pole on the III sheet generates a narrow Breit-Wigner-like peak. The contribution of the triangle is not particularly relevant. (b) The sharp cusp in the scattering amplitude is due to the IV sheet pole close by; the triangle contributes to make the peak sharper. (c) The scattering amplitude has a small cusp due to the threshold factor, and the triangle is needed to make it sharp enough to fit the data.

Lineshapes at 4230

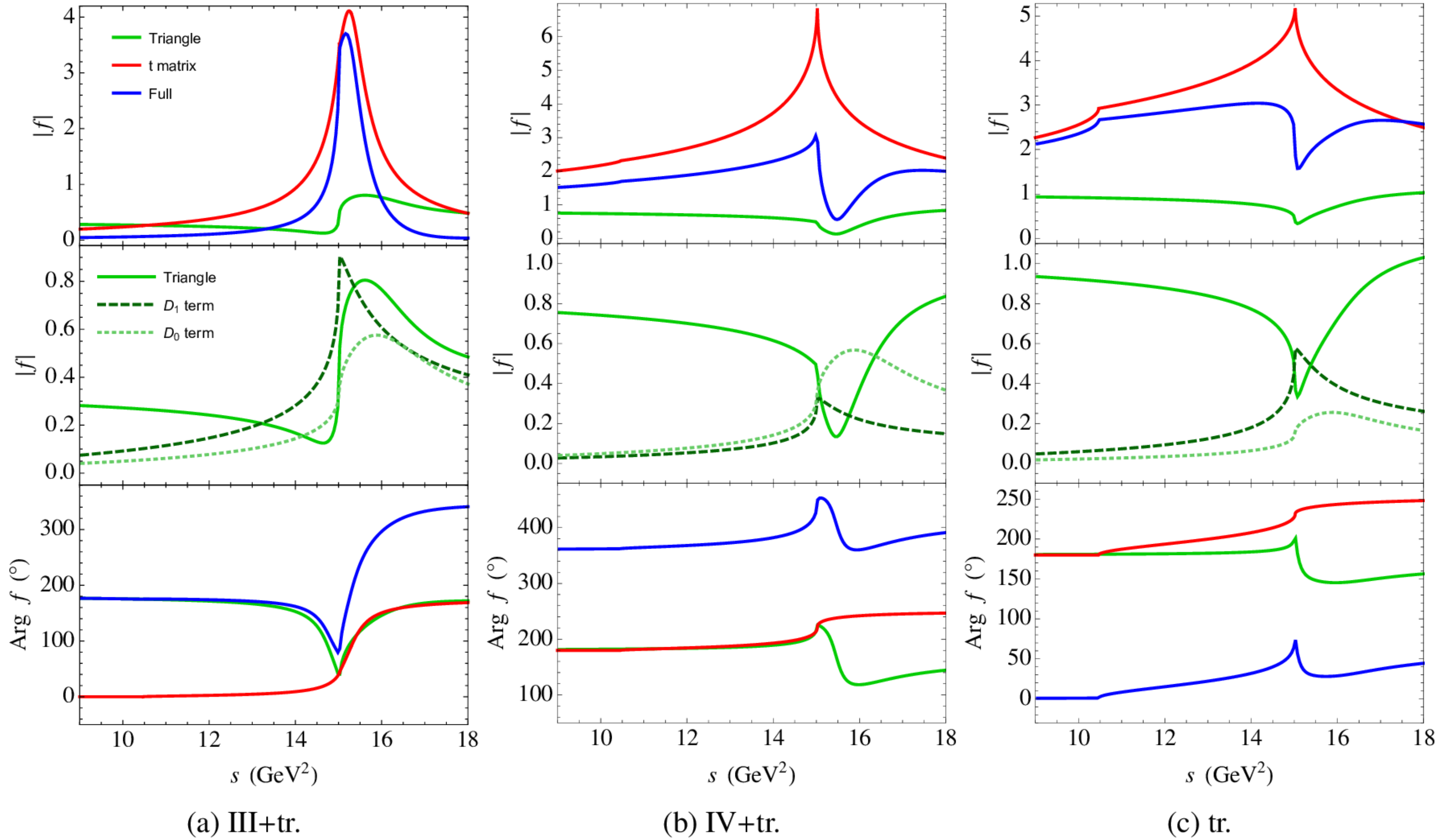
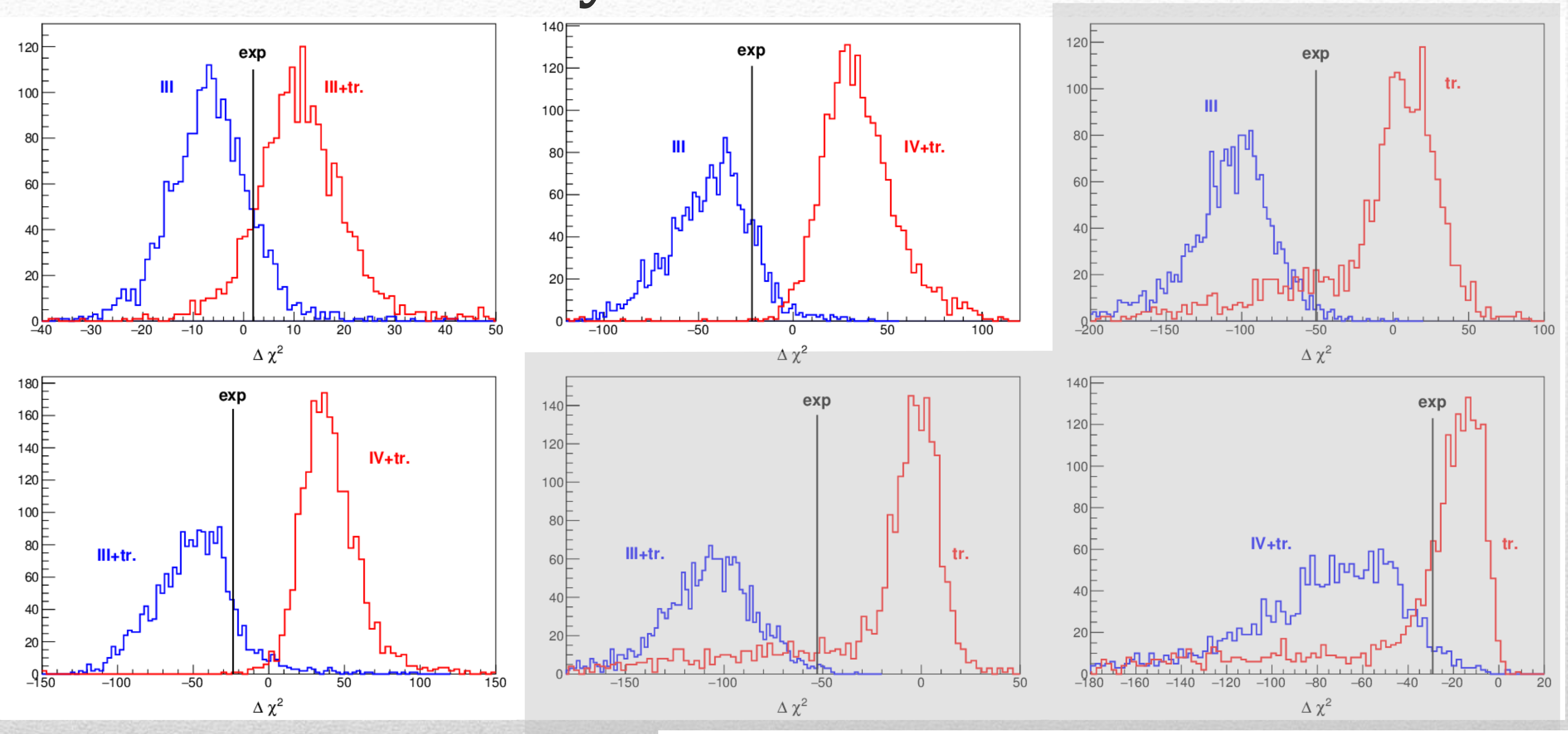


Figure 8: Same as Figure 7, but for E_{CM} = 4.23 GeV.

Statistical analysis



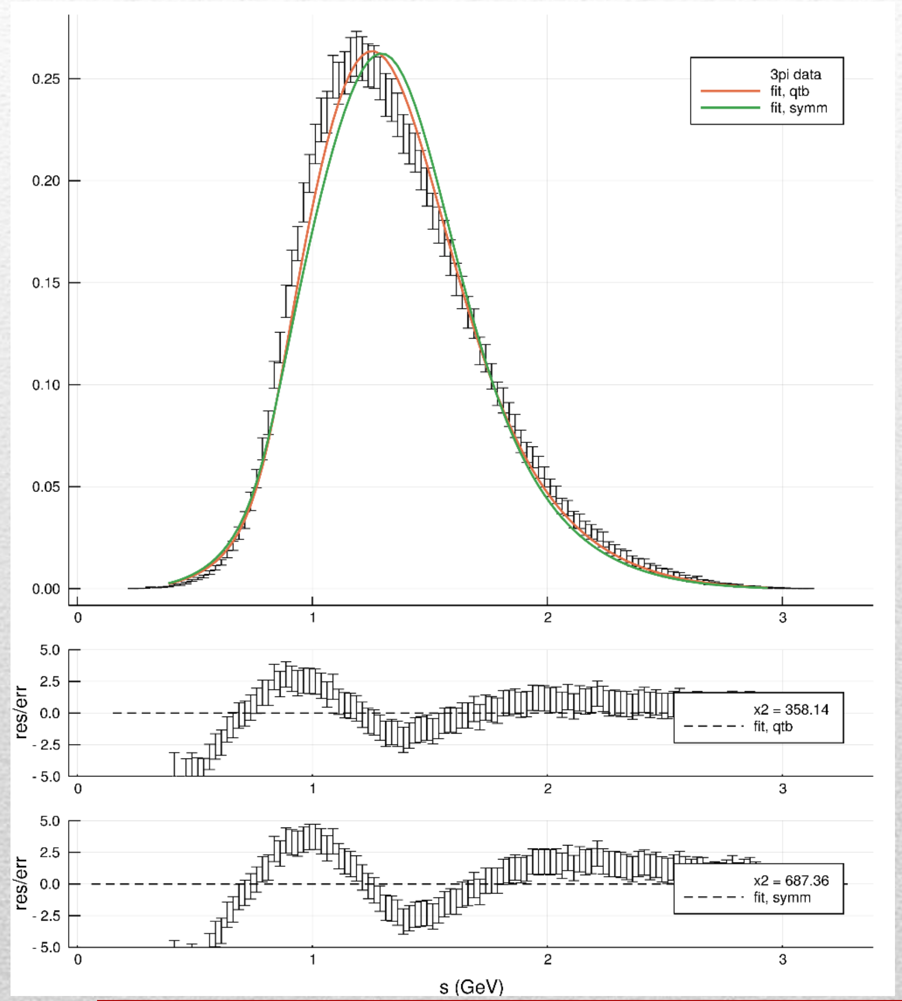
Toy experiments according to the different hypotheses, to estimate the relative rejection of various scenarios

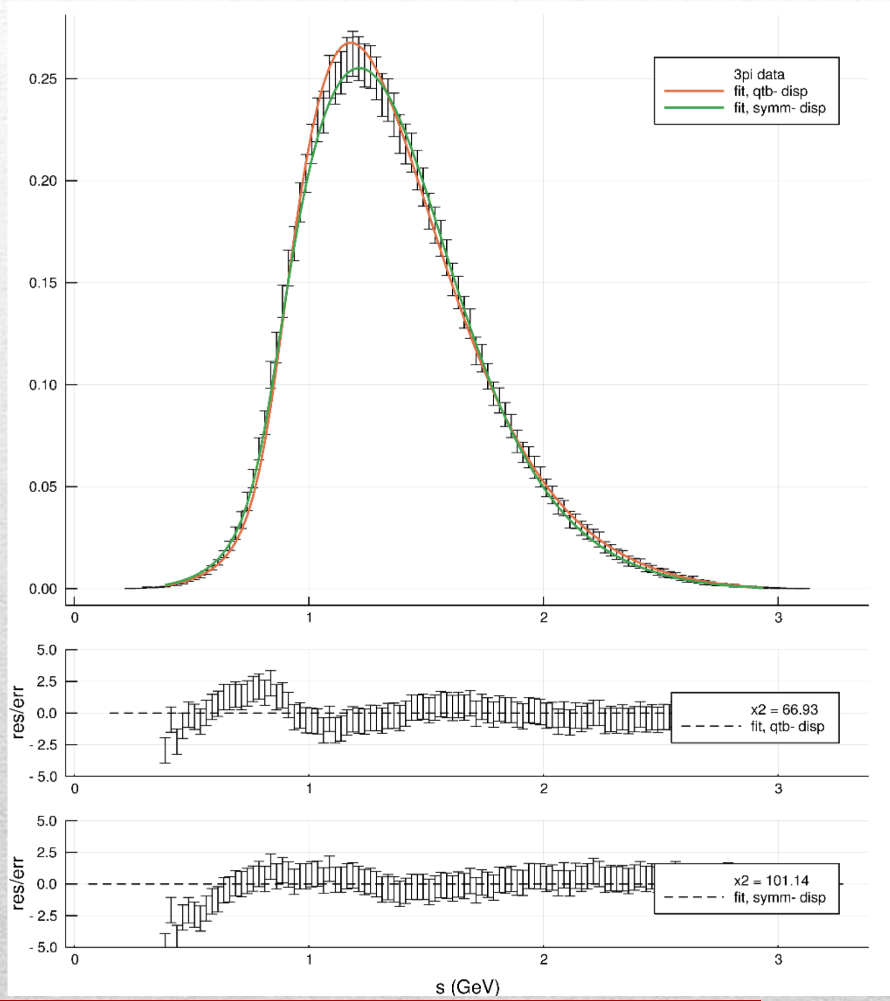
Scenario	III+tr.	IV+tr.	tr.
III	1.5σ (1.5σ)	1.5σ (2.7σ)	"2.4σ" ("1.4σ")
III+tr.	–	1.5σ (3.1σ)	"2.6σ" ("1.3σ")
IV+tr.			"2.1σ" ("0.9σ")

Not conclusive at this stage

The $a_1(1260)$

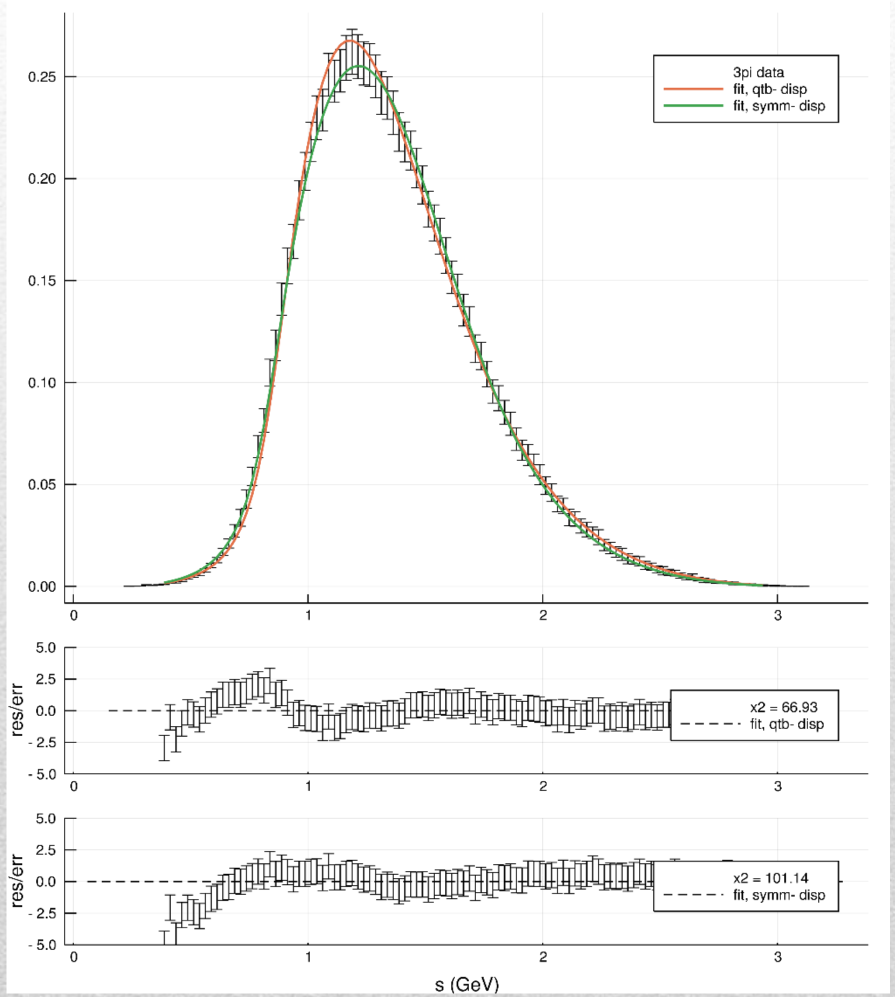
M. Mikhasenko, A. Jackura, AP, *et al.*, in preparation





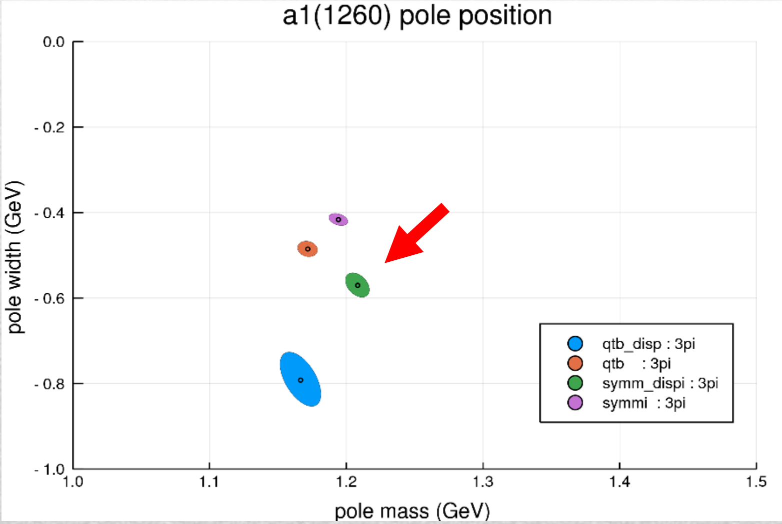
The $a_1(1260)$

M. Mikhasenko, A. Jackura, AP, *et al.*, in preparation



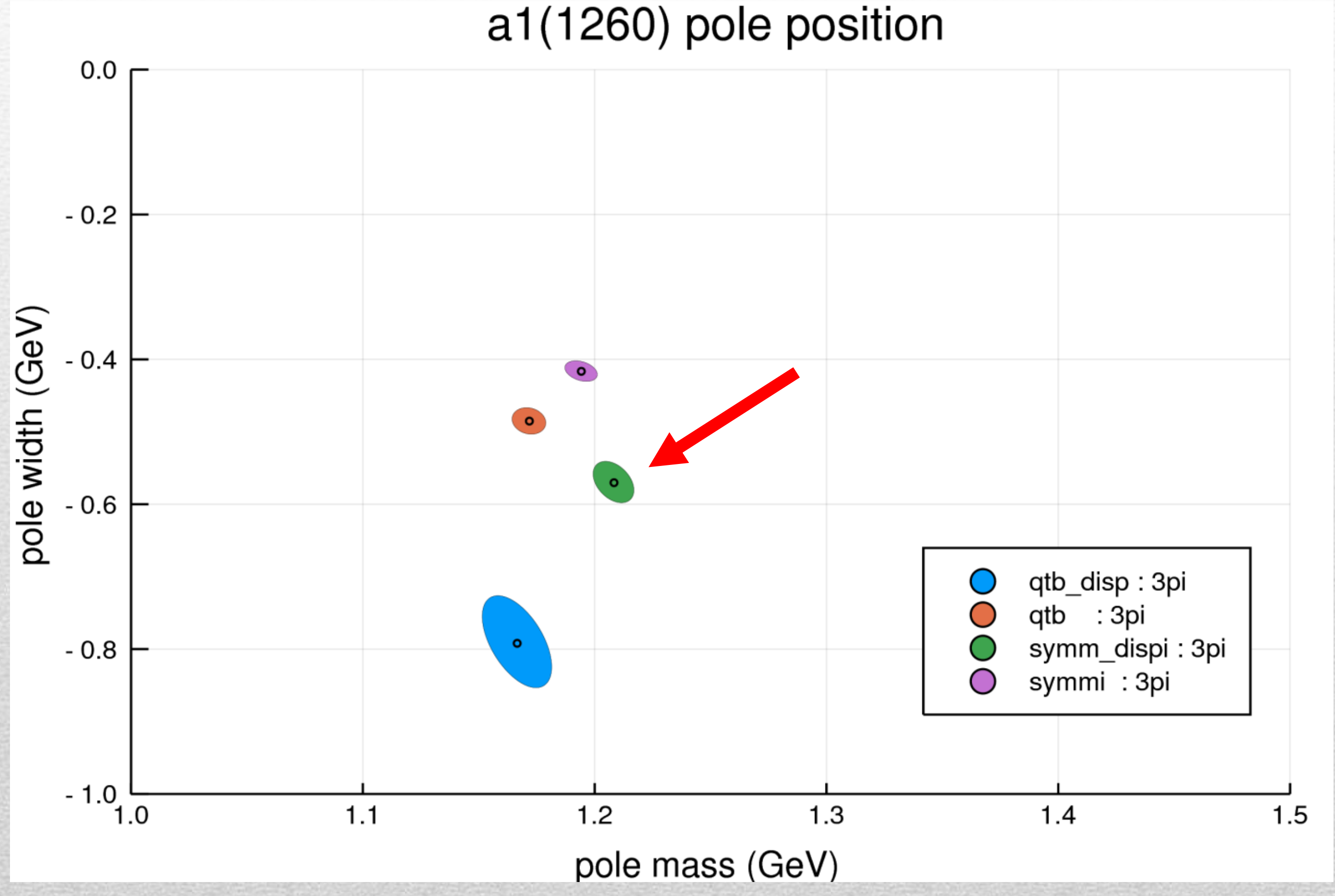
We can use these models to fit $\tau^- \to 2\pi^-\pi^+\,\nu$ and describe the $a_1(1260)$

The dispersed improved model describes better the data at threshold



The $a_1(1260)$

M. Mikhasenko, A. Jackura, AP, *et al.*, in preparation



The $Y(4260)$

A. Amor, C. Fernandez-Ramirez, AP, U. Tamponi, in preparation

$$f(s) = \frac{N(s)}{K^{-1}(s) - \frac{i}{2}\rho_3(s)},$$

$$i\rho_3(s) = \sum_{k=0}^{n-1} a_k\,(s-s_0)^k + \frac{(s-s_0)^n}{\pi}\int_{(2m_\pi+M_\psi)^2}^{\infty} \frac{\rho_2(s')}{(s'-s)}\frac{ds'}{(s'-s_0)^n}$$

$$\rho_2(s') = \int_{4m_\pi^2}^{\left(\sqrt{s'}-M_\psi\right)^2} \frac{ds_{\pi\pi}}{2\pi}\,\frac{\lambda^{1/2}(s',s_{\pi\pi},m_{J/\psi}^2)}{8\pi s'}\,\frac{\lambda^{1/2}(s_{\pi\pi},m_\pi^2,m_\pi^2)}{8\pi s_{\pi\pi}}\,|t_{2\to 2}(s_{\pi\pi})|^2$$

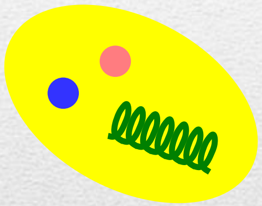
Need model for the Dalitz distribution

Same game,
we start analyzing the single channel
$e^+e^- \to J/\psi\,\pi\pi$ data

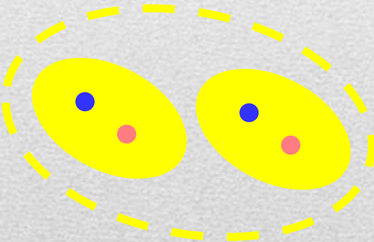
We consider the amplitude in the elastic, quasi two-body approximation

Models

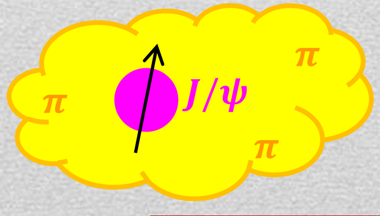
Meson/Baryon+continuum
Ferretti *et al.*, PRC88, 015207
Ferretti *et al.*, PRD90, 094022



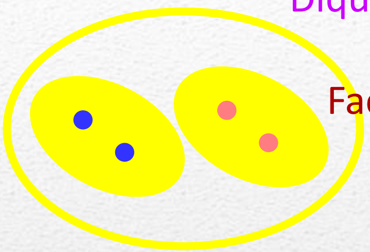
Hybrids/BO tetraquarks
Kou and Pene, PLB631, 164
Braaten, PRL111, 162003
Berwein *et al.*, PRD92, 114019



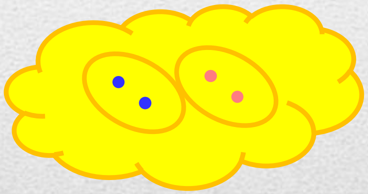
Molecule
Tornqvist, Z.Phys. C61, 525
Braaten and Kusunoki, PRD69 074005
Swanson, Phys.Rept. 429 243-305



Hadroquarkonium
Dubynskiy *et al.*, PLB 666, 344
Dubynskiy and Voloshin, PLB 671, 82
Li and Voloshin, MPLA29, 1450060



Diquark-Antidiquark
Maiani, *et al.* PRD71, 014028
Faccini, AP, *et al.* PRD87, 111102
Maiani, et al. PRD89, 114010
Maiani, et al., PLB778, 247



Hybridized Tetraquaks
Esposito, AP, Polosa
PLB758, 292

Kinematical effects
Szczepaniak, PLB747, 410
Szczepaniak, PLB757, 61
Guo *et al.*, PRD92, 071502
Swanson, IJMPE25, 1642010

Three-Body Unitarity

Imaginary parts of B, τ, S are fixed by unitarity and matching
(for simplicity $v = \lambda$)

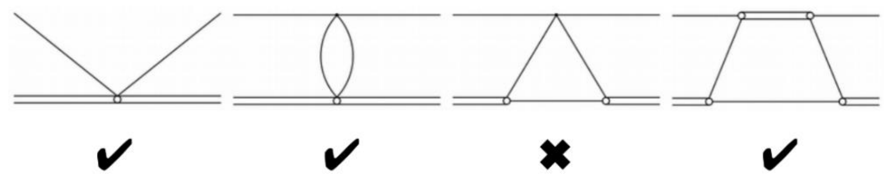
$$\tau(\sigma(k)) = (2\pi)\delta^+(k^2 - m^2)S(\sigma(k))$$

$$-\frac{1}{S(P^2)} = \sigma(k) - M_0^2 - \frac{1}{(2\pi)^3}\int d^3\boldsymbol{\ell}\,\frac{\lambda^2}{2E_\ell(\sigma(k) - 4E_\ell^2 + i\epsilon)}$$

- in the rest-frame of isobar (***Lorentz invariance***!)
- twice subtracted dispersion relation in $\sigma(k)=(P-k)^2$

$$\langle q|B(s)|p\rangle = -\frac{\lambda^2}{2\sqrt{m^2+\mathbf{Q}^2}\left(E_Q - \sqrt{m^2+\mathbf{Q}^2} + i\epsilon\right)}$$

- un-subtracted dispersion relation
- one-π exchange in TOPT
- real contributions can be added to B



The freedom of adding real terms to B allows us to use this solution as a flexible parametrization

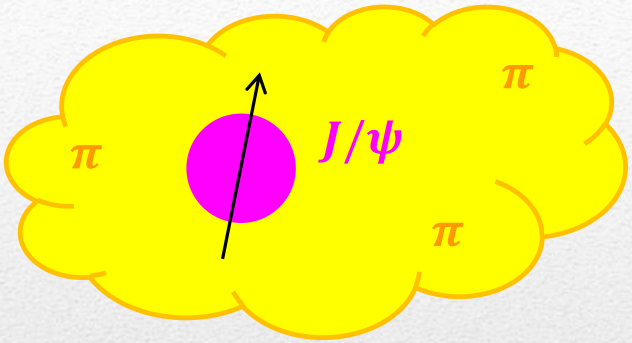
Numerics in progress:

- D. Sadasivan, M. Mai, AP, M. Doring, A. Szczepaniak for the $a_1(1260)$ and $a_1(1420)$

Alternative approach based on N/D:

- A. Jackura, AP et al. (JPAC) for the $X(3872)$
- J.M. Alarcon, E. Passemar, AP, C. Weiss for the nucleon isoscalar vector form factor

Hadro-charmonium



Dubynskiy, Voloshin, PLB 666, 344
Dubynskiy, Voloshin, PLB 671, 82
Li, Voloshin, MPLA29, 1450060

Born in the context of QCD multipole expansion

$$H_{eff} = -\frac{1}{2} a_\psi E_i^a E_i^a$$
$$a_\psi = \langle \psi | (t_c^a - t_{\bar{c}}^a) r_i \, G \, r_i (t_c^a - t_{\bar{c}}^a) | \psi \rangle$$

the chromoelectric field interacts with soft light matter (highly excited light hadrons)

A bound state can occur via Van der Waals-like interactions

Expected to decay into core charmonium + light hadrons,
Decay into open charm exponentially suppressed

Counting rules

Brodsky, Lebed PRD91, 114025

- Exotic states can be produced in threshold regions in e^+e^-, electroproduction, hadronic beam facilities and are best characterized by cross section ratios
- Two examples:

1) $\frac{\sigma(e^+e^-\to Z_c^+\,\pi^-)}{\sigma(e^+e^-\to\mu^+\mu^-)} \propto \frac{1}{s^6}$ as $s\to\infty$

2) $\frac{\sigma(e^+e^-\to Z_c^+(\bar{c}c\bar{d}u)+\pi^-(\bar{u}d))}{\sigma(e^+e^-\to\Lambda_c(cud)+\bar{\Lambda}_c(\bar{c}\,\bar{u}\bar{d}))} \to const$ as $s\to\infty$

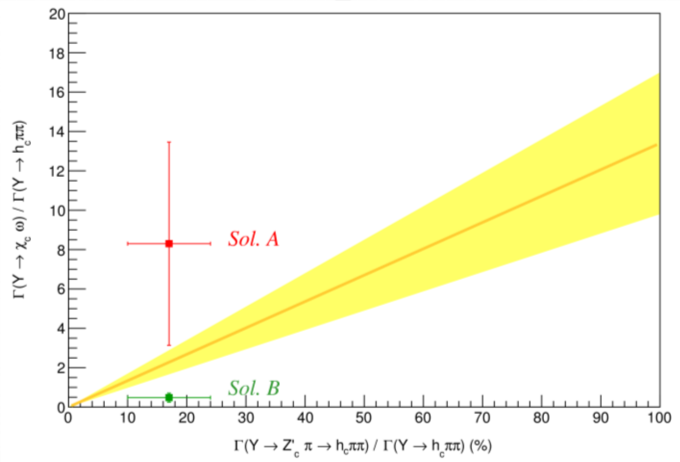
  - Ratio numerically smaller if Z_c behaves like weakly-bound dimeson molecule instead of diquark-antidiquark bound state due to weaker meson color van der Waals forces

Different estimates close to thesholds, and in presence of annihilating $q\,\bar{q}$

Guo, Meissner, Wang, Yang, 1607.04020

Voloshin PRD94, 074042

Tetraquark: the $Y(4220)$



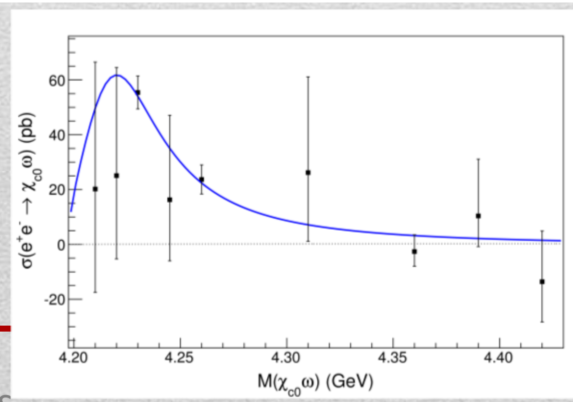
$$\langle \chi_{c0}(p)\,\omega(\eta,q)|Y(\lambda,P)\rangle = g_\chi\,\eta\cdot\lambda,$$
$$\langle Z_c'(\eta,q)\,\pi(p)|Y(\lambda,P)\rangle = g_Z\,\eta\cdot\lambda\frac{P\cdot p}{f_\pi M_Y},$$
$$\langle h_c(\eta,q)\,\sigma(p)|Y(\lambda,P)\rangle = g_h\,\varepsilon_{\mu\nu\rho\sigma}\eta^\mu\lambda^\nu\frac{P^\rho q^\sigma}{P\cdot q},$$
$$\langle \pi(q)\pi(p)|\sigma(P)\rangle = \frac{P^2}{2f_\pi},$$

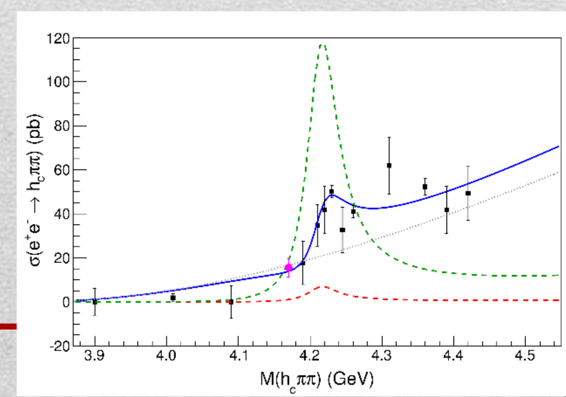
$$\frac{\Gamma\left(Y(4220)\to\chi_{c0}\omega\right)}{\Gamma(Y(4220)\to h_c\pi^+\pi^-)} = (13.4\pm 3.6)\times R_{YZ} = 2.3\pm 1.2.$$
$$\frac{\Gamma\left(Y(4220)\to Z_c'^{\pm}\pi^{\mp}\to h_c\pi^+\pi^-\right)}{\Gamma\left(Y(4220)\to h_c\sigma\to h_c\pi^+\pi^-\right)} = 4.8\pm 3.5,$$

A state apparently breaking HQSS has been observed

Compatible to be the Y_3 state

Faccini, Filaci, Guerrieri, AP, Polosa, PRD 91, 117501





Tetraquark: the b sector

Ali, Maiani, Piccinini, Polosa, Riquer PRD91 017502

$$M(Z_b') - M(Z_b) = 2\kappa_b$$
$$M(Z_c') - M(Z_c) = 2\kappa_c \sim 120 \text{ MeV}$$
$$\kappa_b : \kappa_c = M_c : M_b \sim 0.30$$

$$2\kappa_b \;\sim 36 \text{ MeV, vs. } 45 \text{ MeV (exp.)}$$

$$Z_b = \frac{\alpha \left|1_{q\bar{q}} 0_{b\bar{b}}\right\rangle - \beta \left|0_{q\bar{q}} 1_{b\bar{b}}\right\rangle}{\sqrt{2}}$$
$$Z_b' = \frac{\alpha \left|1_{q\bar{q}} 0_{b\bar{b}}\right\rangle + \beta \left|0_{q\bar{q}} 1_{b\bar{b}}\right\rangle}{\sqrt{2}}$$

Data on $\Upsilon(5S) \to \Upsilon(nS)\pi\pi$ and $\Upsilon(5S) \to \mathrm{h_b}(nP)\pi\pi$ strongly favor $\alpha = \beta$

$Z_c(3900) \to \eta_c \rho$

Esposito, Guerrieri, AP, PLB 746, 194-201

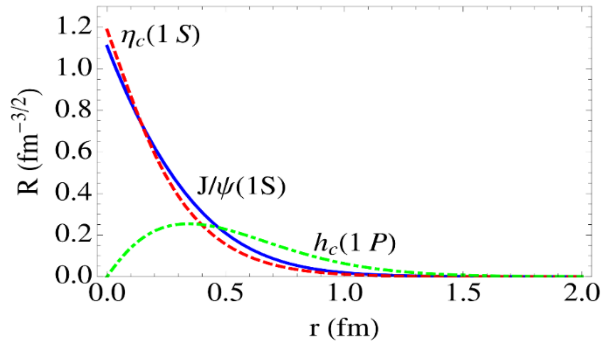
If tetraquark

Kinematics with HQSS, dynamics estimated according to Brodsky et al., PRL113, 112001

$$A = \langle \chi_{c\bar{c}} | \chi_c \otimes \chi_{\bar{c}} \rangle \langle \phi_{c\bar{c}} | \hat{T}_{\perp HQS} | \phi[cq][\bar{c}\bar{q}] \rangle + O\left(\frac{\Lambda_{QCD}}{m_c}\right)$$

Clebsch-Gordan

Uncertainty $\sim 25\%$



Reduced matrix element

- approximated as a constant
- or $\propto \psi_{c\bar{c}}(r_Z)$

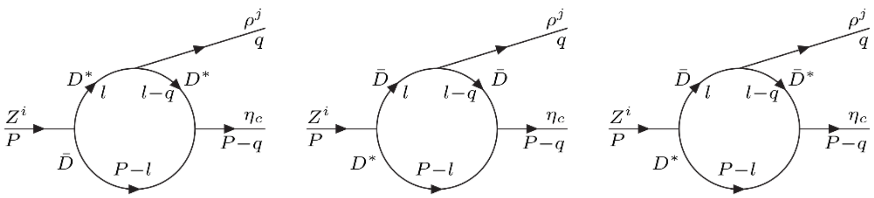
	Kinematics only		Dynamics included	
	type I	type II	type I	type II
$\frac{\mathcal{BR}(Z_c \to \eta_c \rho)}{\mathcal{BR}(Z_c \to J/\psi \pi)}$	$\left(3.3^{+7.9}_{-1.4}\right) \times 10^2$	$0.41^{+0.96}_{-0.17}$	$\left(2.3^{+3.3}_{-1.4}\right) \times 10^2$	$0.27^{+0.40}_{-0.17}$
$\frac{\mathcal{BR}(Z_c' \to \eta_c \rho)}{\mathcal{BR}(Z_c' \to h_c \pi)}$	$\left(1.2^{+2.8}_{-0.5}\right) \times 10^2$		$6.6^{+56.8}_{-5.8}$	

$Z_c(3900) \to \eta_c \rho$

Esposito, Guerrieri, AP, PLB 746, 194-201

If molecule

Non-Relativistic Effective Theory, HQET+NRQCD and Hidden gauge Lagrangian
Uncertainty estimated with power counting at NLO



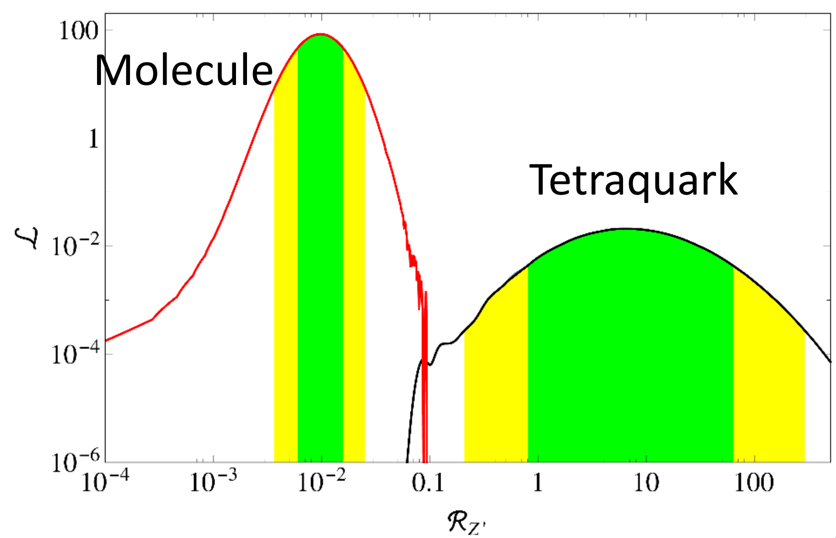
$$\mathcal{L}_{Z_c^{(\prime)}} = \frac{z^{(\prime)}}{2}\left\langle \mathcal{Z}^{(\prime)}_{\mu,ab}\bar{H}_{2b}\gamma^\mu \bar{H}_{1a}\right\rangle + h.c.,$$

$$\mathcal{L}_{c\bar{c}} = \frac{g_2}{2}\left\langle \bar{\Psi} H_{1a} \overleftrightarrow{\not\partial} H_{2a}\right\rangle + \frac{g_1}{2}\left\langle \bar{\chi}_\mu H_{1a}\gamma^\mu H_{2a}\right\rangle + h.c.,$$

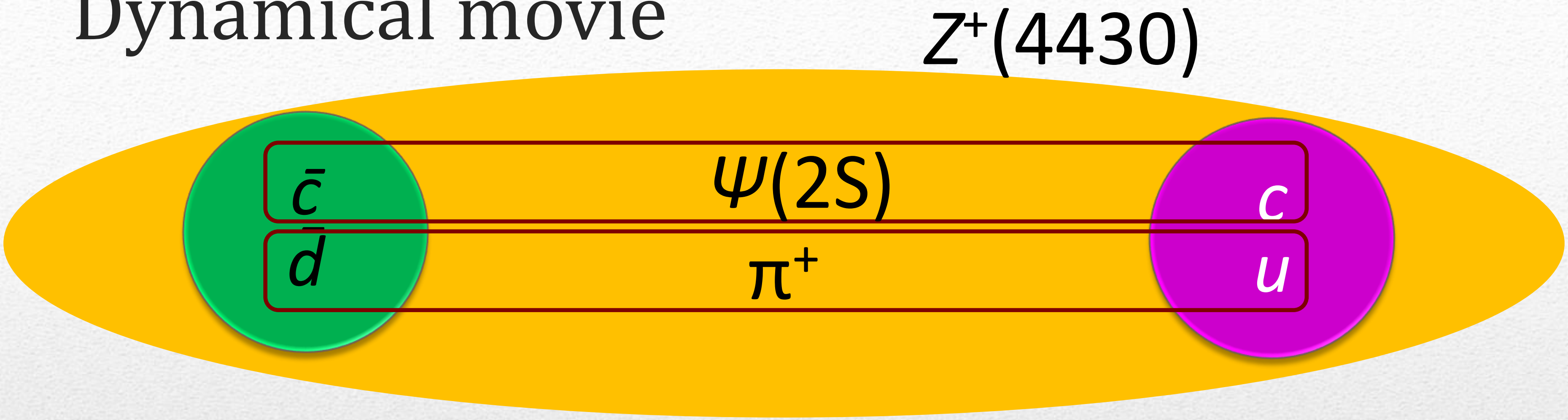
$$\mathcal{L}_{\rho DD^*} = i\beta\left\langle H_{1b} v^\mu \left(\mathcal{V}_\mu - \rho_\mu\right)_{ba}\bar{H}_{1a}\right\rangle + i\lambda\left\langle H_{1b}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ba}\bar{H}_{1a}\right\rangle + h.c.,$$

$$\frac{\mathcal{BR}(Z_c \to \eta_c \rho)}{\mathcal{BR}(Z_c \to J/\psi\, \pi)} = \left(4.6^{+2.5}_{-1.7}\right)\times 10^{-2}\,; \quad \frac{\mathcal{BR}(Z_c^\prime \to \eta_c \rho)}{\mathcal{BR}(Z_c^\prime \to h_c\, \pi)} = \left(1.0^{+0.6}_{-0.4}\right)\times 10^{-2}\,.$$

$$\frac{\mathcal{BR}(Z_c \to h_c\, \pi)}{\mathcal{BR}(Z_c^\prime \to h_c\, \pi)} = 0.34^{+0.21}_{-0.13}\,; \quad \frac{\mathcal{BR}(Z_c \to J/\psi\, \pi)}{\mathcal{BR}(Z_c^\prime \to J/\psi\, \pi)} = 0.35^{+0.49}_{-0.21}$$



Dynamical movie



Brodsky, Hwang, Lebed PRL 113 112001

- Since this is still a $\mathbf{3} \leftrightarrow \bar{\mathbf{3}}$ color interaction, just use the Cornell potential:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + br + \frac{32\pi\alpha_s}{9m_{cq}^2}\left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}\mathbf{S}_{cq}\cdot\mathbf{S}_{\overline{cq}},$$

e.g. Barnes *et al.*, PRD 72, 054026

- Use that the kinetic energy released in $\overline{B}^0 \longrightarrow K^- Z^+(4430)$ converts into potential energy until the diquarks come to rest
- Hadronization most effective at this point (WKB turning point)

$r_Z = 1.16\text{ fm}, \langle r_{\psi(2S)}\rangle = 0.80\text{ fm}, \langle r_{J/\psi}\rangle = 0.39\text{ fm}$

$$\frac{B(Z^+(4430)\to\psi(2S)\pi^+)}{B(Z^+(4430)\to J/\psi\,\pi^+)} \sim 72$$

$(> 10\text{ exp.})$

Towards hybridized tetraquarks

Esposito, AP, Polosa, PLB758, 292

The absence of many of the predicted states might point to the need for selection rules
It is unlikely that the many close-by thresholds play no role whatsoever
All the well assessed 4-quark resonances lie close and above some meson-meson thresholds:
We introduce a mechanism that might provide “dynamical selection rules” to explain the
presence/absence of reso

	Thr.	δ (MeV)	$A\sqrt{\delta}$ (MeV)	Γ (MeV)
$X(3872)$	$\bar{D}^0 D^{*0}$	$0^\dagger$	$0^\dagger$	$0^\dagger$
$Z_c(3900)$	$\bar{D}^0 D^{*+}$	7.8	27.9	27.9
$Z_c'(4020)$	$\bar{D}^{*0} D^{*+}$	6.7	25.9	$24.8^\P$
$X(4140)$	$J/\psi\,\phi$	*a*) 31.6	52.7	28.0
		b) 30.1	54.7	83.0
$Z_b(10610)$	$\bar{B}^0 B^{*+}$	2.7	16.6	18.4
$Z_b'(10650)$	$\bar{B}^{*0} B^{*+}$	1.8	13.4	11.5
$X(5568)$	$B_s^0\,\pi^+$	61.4	78.4	21.9
X_{bs}	$B^+ \bar{K}^0$	5.8	24.1	—

We introduce a mechanism that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data.

Hybridized tetraquarks

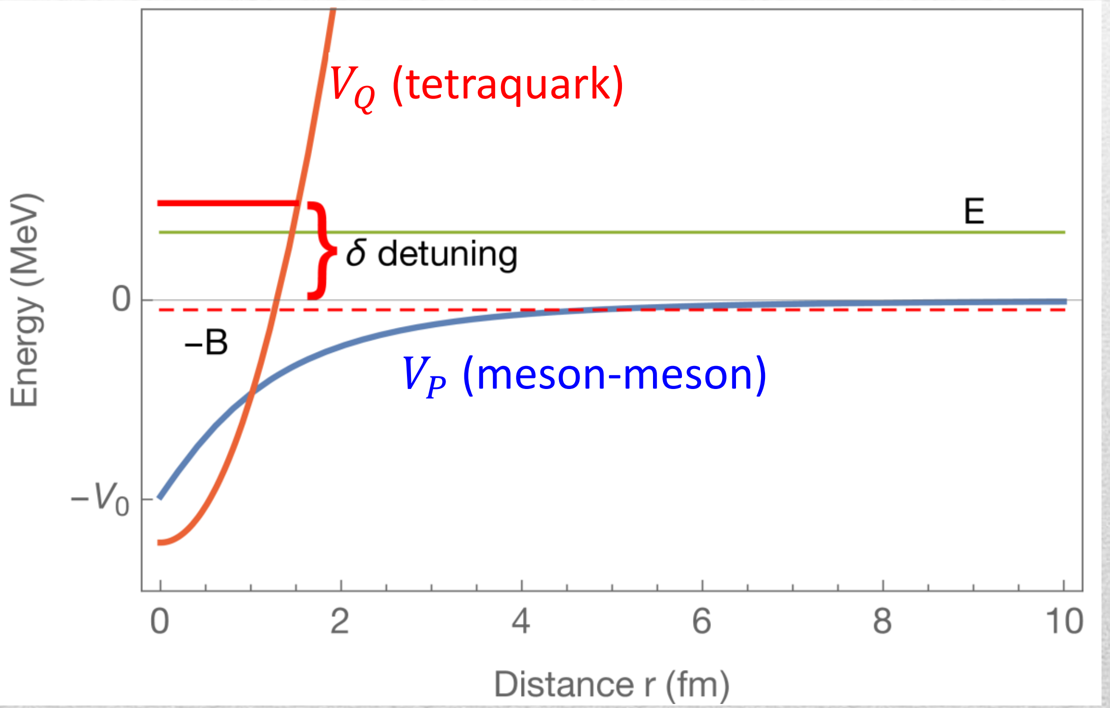
Esposito, AP, Polosa, PLB758, 292

The absence of many of the predicted states might point to the need for selection rules

It is unlikely that the many close-by thresholds play no role whatsoever

All the well assessed 4-quark resonances lie close and above some meson-meson thresholds:

We introduce a mechanism that might provide “dynamical selection rules” to explain the presence/absence of resonances from the experimental data



Let P and Q be orthogonal subspaces of the Hilbert space

$$H = H_{PP} + H_{QQ}$$

We have the (weak) scattering length a_P in the open channel.

We add an off-diagonal H_{QP} which connects the two subspaces

Hybridized tetraquarks

Esposito, AP, Polosa, PLB758, 292

$$\Gamma = -16\pi^3\, \rho\, \Im(T) \sim 16\pi^4\, \rho\, \left|H_{PQ}\right|^2 \delta\left(\frac{p_1^2}{2M} + \frac{p_2^2}{2M} - \delta\right)$$

The expected width is the average over momenta that allow for the existence of a tetraquark $p < \bar{p} = 50 \div 100\text{ MeV}$

$$\Gamma \sim A\sqrt{\delta}$$

We therefore expect to see a level if:

- $\delta > 0$ the state lies above threshold
- $\delta < \frac{\bar{p}^2}{2M}$, only the closest threshold contributes
- The states ψ_Q and ψ_P are orthogonal

$X(3872)^+$ falls below threshold, $M(1^{++}) < M(D^{+*}\bar{D}^0)$

$\delta < 0$, so $a > 0 \to$ Repulsive interaction

No charged partners of the $X(3872)$!

Hybridized tetraquarks

Esposito, AP, Polosa, PLB758, 292

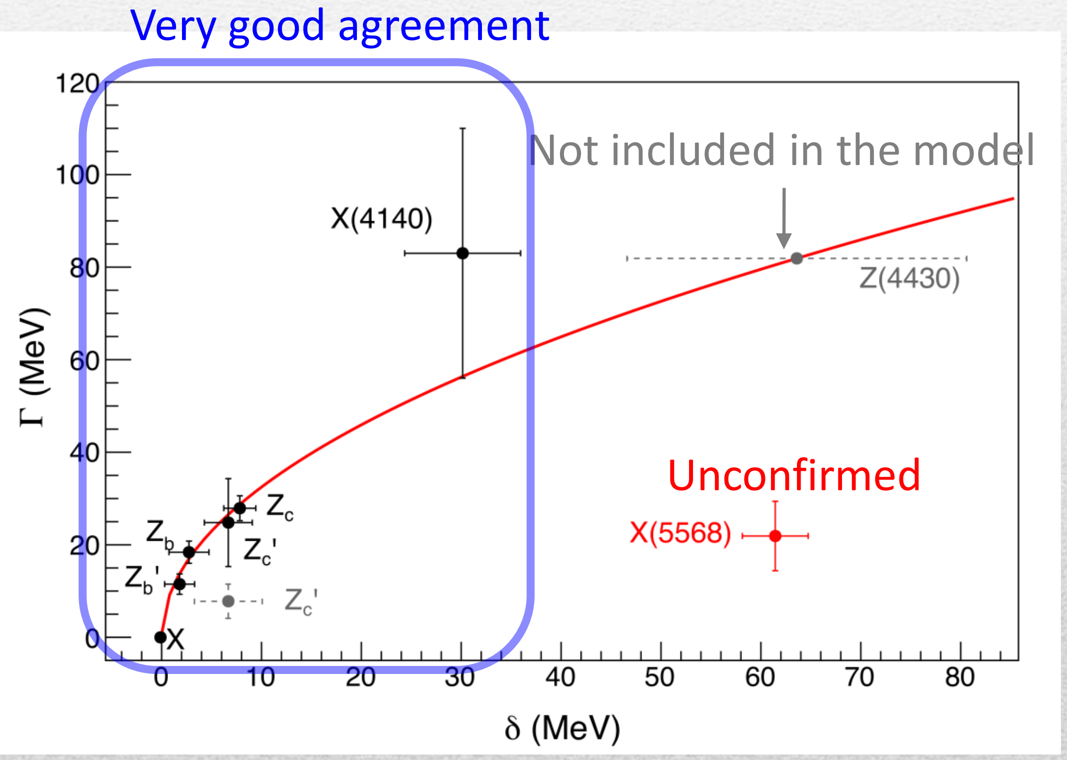
The model works only if no direct transition between closed channel levels can occur
This prevents the straightforward generalization to $L = 1$ and radially excited states (like the Ys or the $Z(4430)$)

In this picture, a $[bu][\bar{s}\bar{d}]$ state with resonance parameters of the $X(5568)$ observed by D0 is not likely

Also, one has to ensure the orthogonality between the two Hilbert subspaces P and Q.
This might affect the estimate for the X(4140)

All the resonances can be fitted with

$$A = (10.3 \pm 1.3)\ \mathrm{MeV}^{1/2}$$
$$\chi^2/\mathrm{DOF} = 1.2/5$$

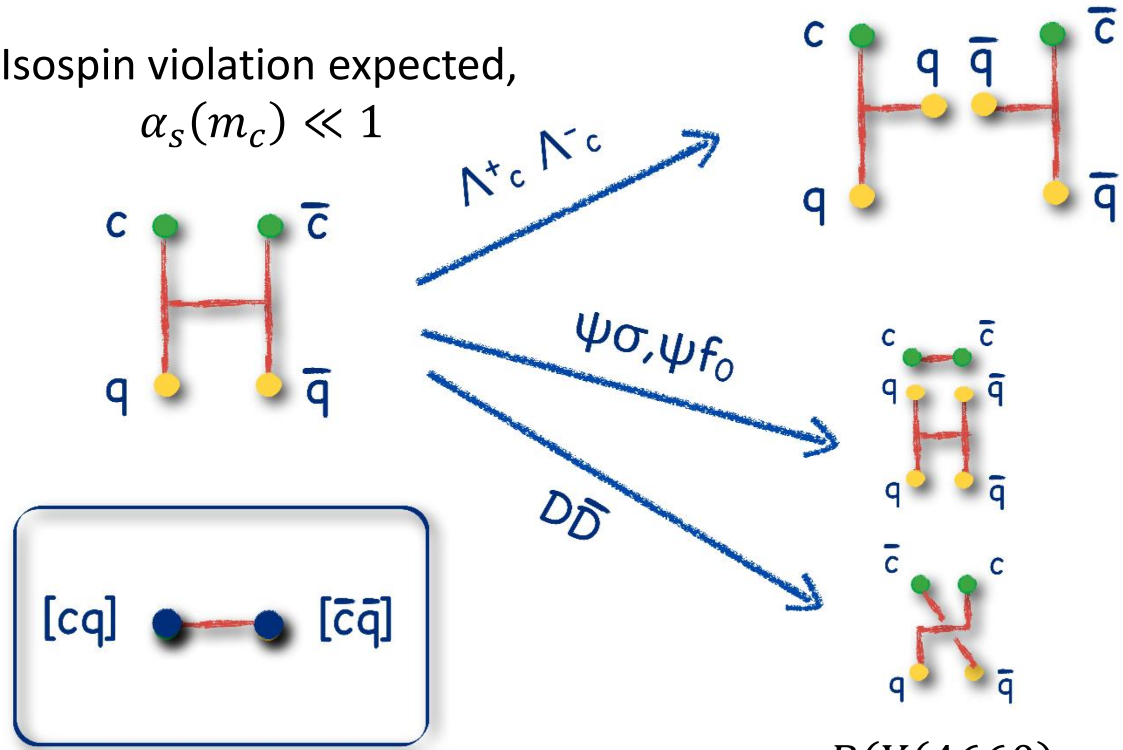


Baryonium

C. Sabelli

a structure $[cq][\bar{c}\bar{q}]$ can explain the dominance of baryon channel

Isospin violation expected,
$\alpha_s(m_c) \ll 1$



Rossi, Veneziano,
NPB 123, 507;
Phys.Rept. 63, 149;
PLB70, 255

$$\frac{B(Y(4660) \to \Lambda_c^+ \Lambda_c^-)}{B(Y(4660) \to \psi(2S)\pi\pi)} = 25 \pm 7$$

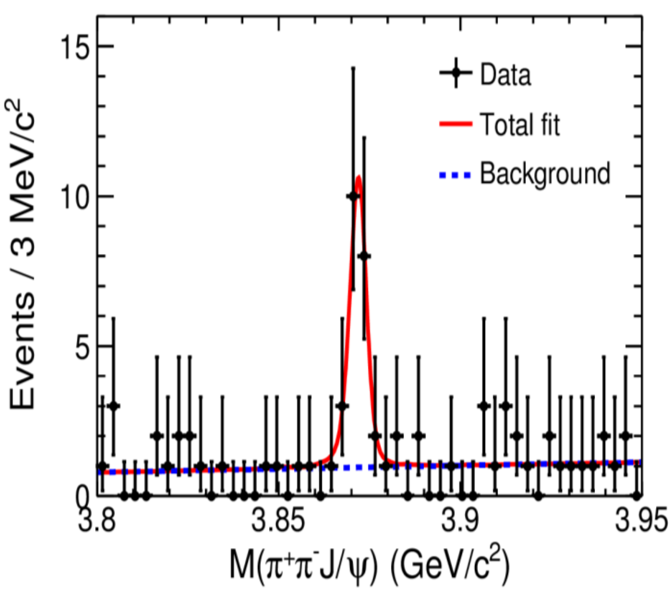
Cotugno, Faccini, Polosa, Sabelli,
PRL 104, 132005

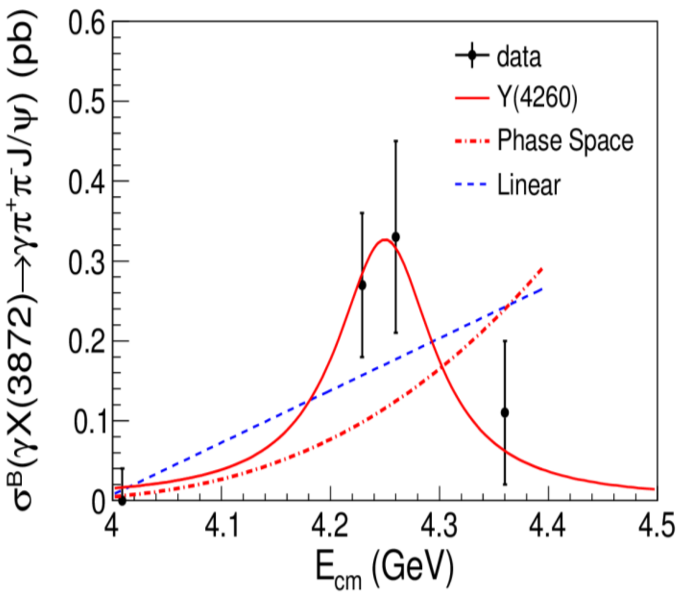
$Y(4260) \to \gamma X(3872)$

M. Ablikim et al., Phys. Rev. Lett. 112 (2014) 092001

F. Piccinini

BESIII: $e^+e^- \to Y(4260) \to X(3872)\gamma$





With $\mathcal{B}[X(3872) \to \pi^+\pi^- J/\psi] = 5\%$

$$\frac{\mathcal{B}[Y(4260) \to \gamma X(3872)]}{\mathcal{B}(Y(4260) \to \pi^+\pi^- J/\psi)} = 0.1$$

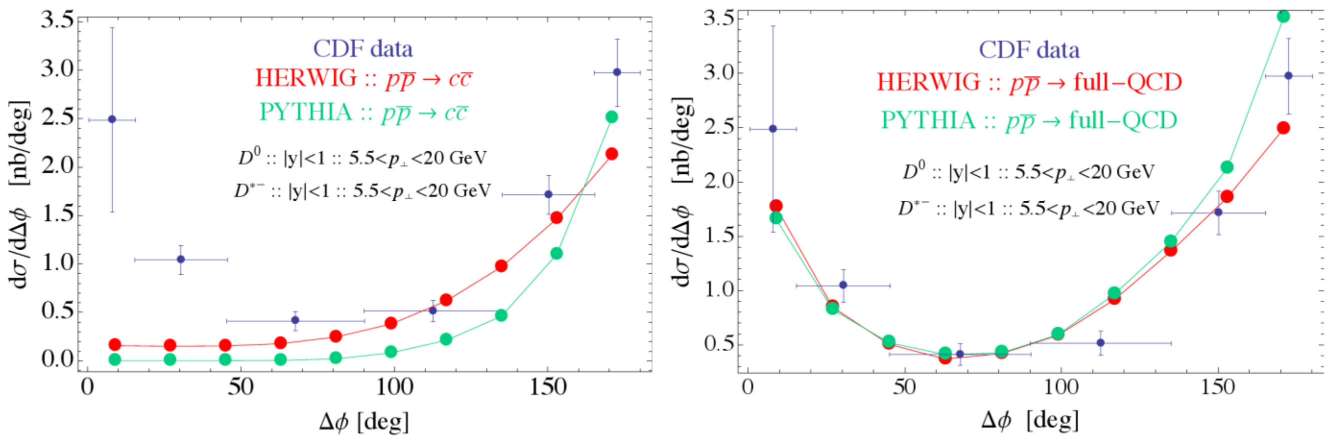
Strong indication that $Y(4260)$ and $X(3872)$ share a similar structure

Tuning of MC

Monte Carlo simulations

A. Esposito

- We compare the $D^0 D^{*-}$ pairs produced as a function of relative azimuthal angle with the results from CDF:



The c-cbar run understimate the low angles (low-k_0) region!

Such distributions of charm mesons are available at Tevatron
No distribution has been published (yet) at LHC

Prompt production of $X(3872)$

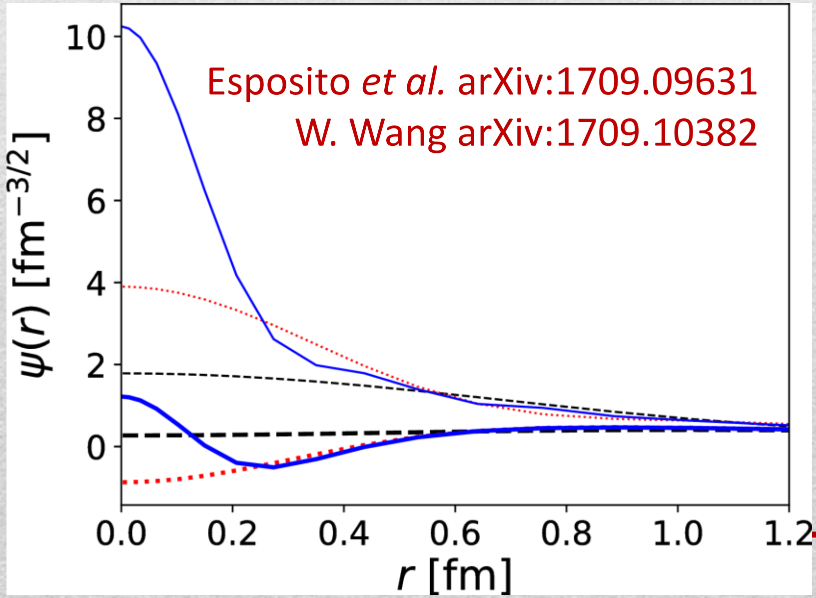
$$\sigma(\bar{p}p \to X) \sim \left|\int d^3\mathbf{k}\langle X|D^0\bar{D}^{*0}(\mathbf{k})\rangle\langle D^0\bar{D}^{*0}(\mathbf{k})|\bar{p}p\rangle\right|^2$$
$$\simeq \left|\int_{\mathcal{R}} d^3\mathbf{k}\langle X|D^0\bar{D}^{*0}(\mathbf{k})\rangle\langle D^0\bar{D}^{*0}(\mathbf{k})|\bar{p}p\rangle\right|^2$$
$$\leq \int_{\mathcal{R}} d^3\mathbf{k}\,|\Psi(\mathbf{k})|^2 \int_{\mathcal{R}} d^3\mathbf{k}\,\left|\langle D^0\bar{D}^{*0}(\mathbf{k})|\bar{p}p\rangle\right|^2$$
$$\leq \int_{\mathcal{R}} d^3\mathbf{k}\,\left|\langle D^0\bar{D}^{*0}(\mathbf{k})|\bar{p}p\rangle\right|^2$$

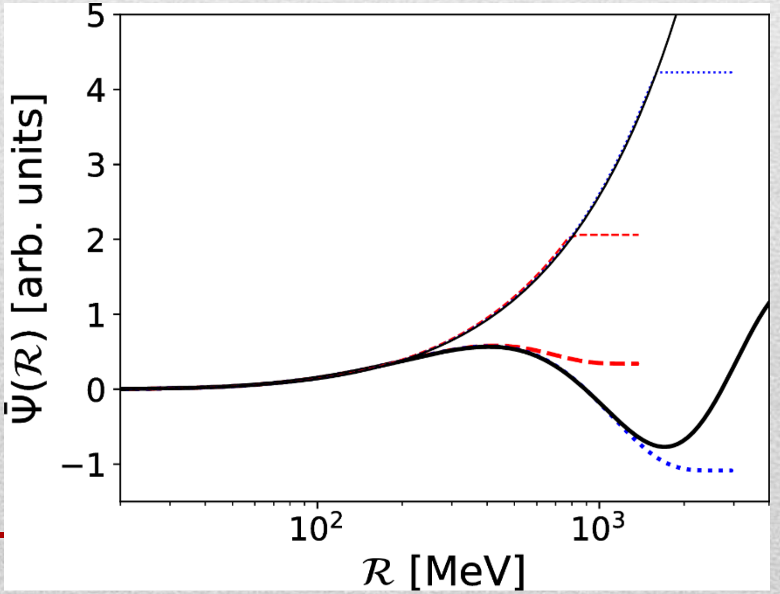
The estimate of the k_{max} has been brought back

Albaladejo *et al.* arXiv:1709.09101

The essence of the argument is that one has to look at the integral of the wave function

$$\int_R d^3\mathbf{k}\,\psi(\mathbf{k})$$





Prompt production of $X(3872)$

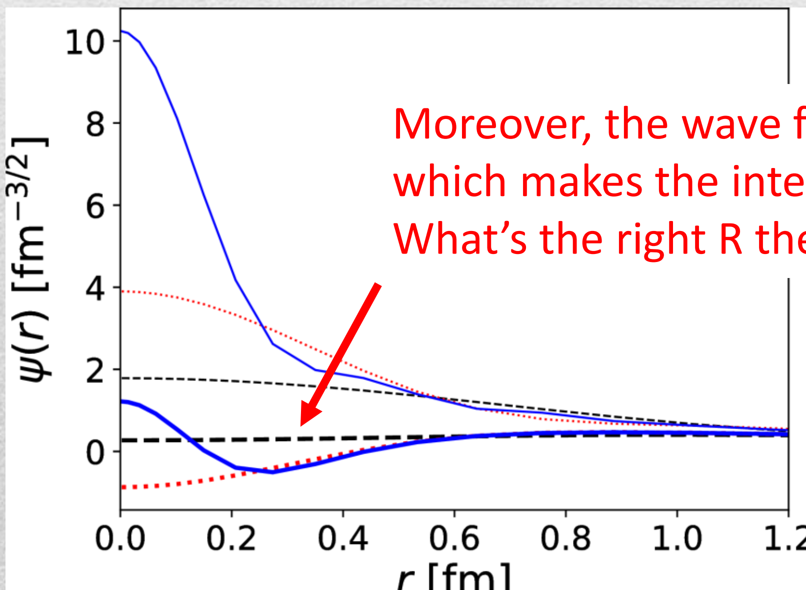
However, the integral of the wave function may not be well defined.
For example, if one considers the wave function in the scattering length approximation,

Esposito *et al.* arXiv:1709.09631

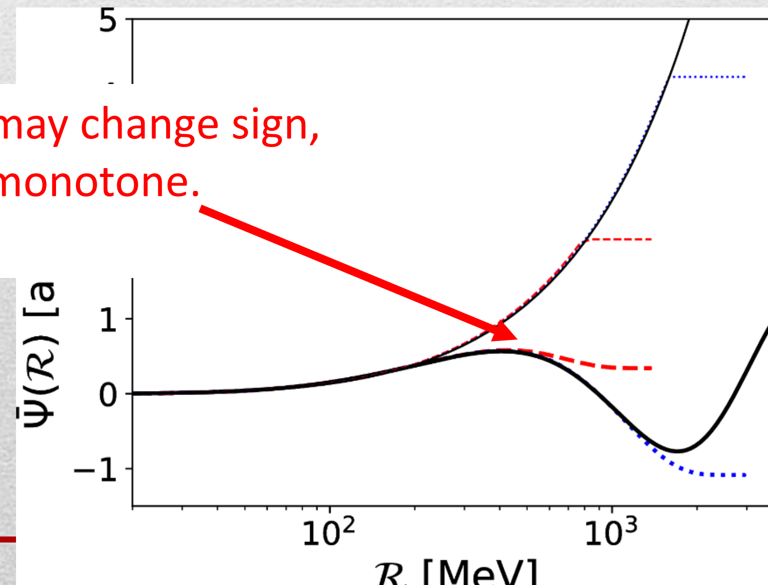
$$\psi(\mathbf{k}) = \frac{1}{\pi}\frac{a^{3/2}}{a^2k^2+1}$$ it's not integrable

A physical value should rather be based on expectation values which involve $|\psi(\mathbf{k})|^2$

For example, an estimate using the virial theorem gives $k \sim 100$ MeV for the deuteron



Moreover, the wave function may change sign,
which makes the integral nonmonotone.
What's the right R then?



Note on $X(3872)$ production at hadron colliders and its molecular structure

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The production of the $X(3872)$ as a hadronic molecule in hadron colliders is clarified. We show that the conclusion of Bignamini *et al.*, Phys. Rev. Lett. **103** (2009) 162001, that the production of the $X(3872)$ at high p_T implies a non-molecular structure, does not hold. In particular, using the well understood properties of the deuteron wave function as an example, we identify the relevant scales in the production process.

The argument is about the value of a nonnormalizable wave function.
Any argument about where the wave function is localized must be calculated for the modulus square

A widespread argument against the interpretation of the $X(3872)$ as a hadronic molecule is its copious production at hadron colliders. Based on the inequality[1]

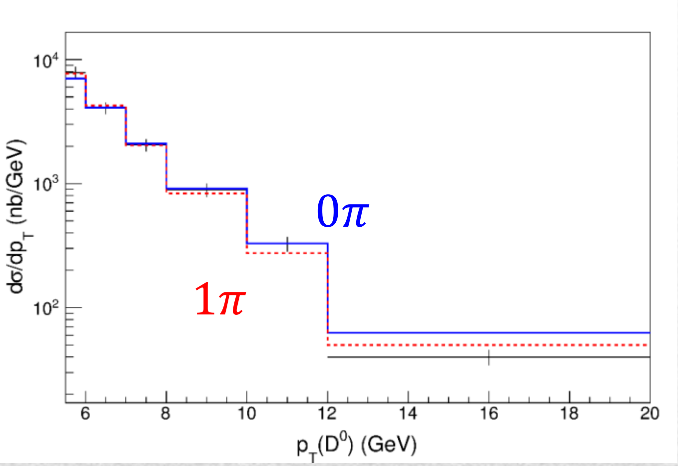
$$\sigma(\bar{p}p \to X) \sim \left| \int d^3\mathbf{k} \langle X | D^0 \bar{D}^{*0}(\mathbf{k}) \rangle \langle D^0 \bar{D}^{*0}(\mathbf{k}) | \bar{p}p \rangle \right|^2$$

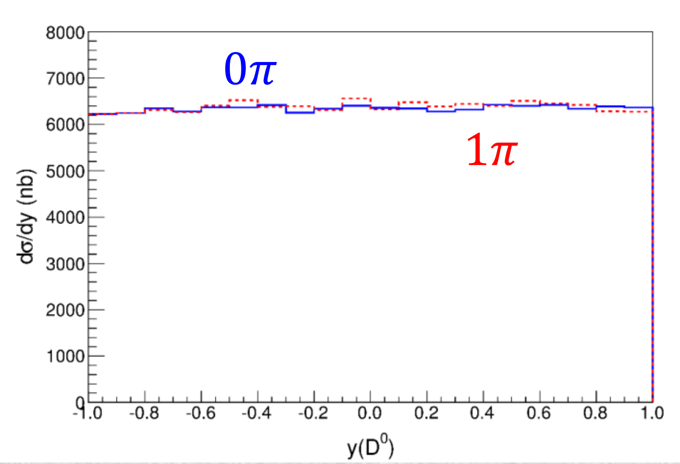
Tuning pions

This picture could spoil existing meson distributions used to tune MC
We verify this is not the case up to an overall K factor

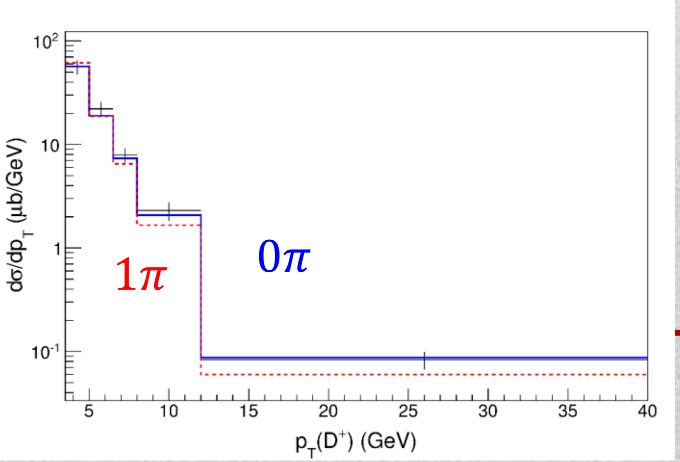
Guerrieri, Piccinini, AP, Polosa, PRD90, 034003

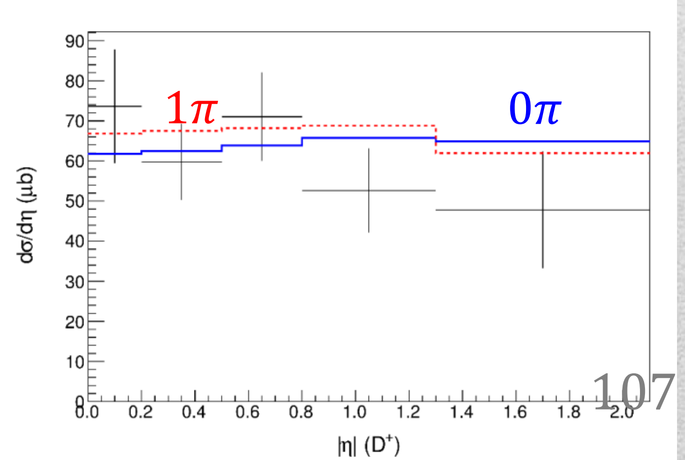
Neither at CDF...





...nor at ATLAS





$Z_c(3900)$

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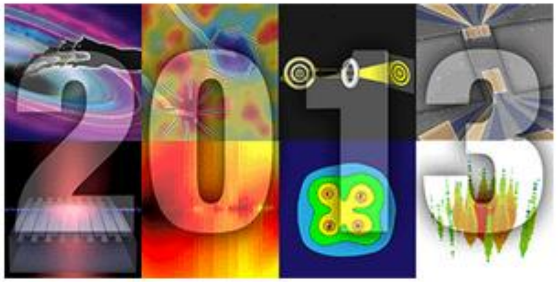
Notes from the Editors: Highlights of the Year

Published December 30, 2013 | Physics **6**, 139 (2013) | DOI: 10.1103/Physics.6.139

***Physics* looks back at the standout stories of 2013.**

As 2013 draws to a close, we look back on the research covered in *Physics* that really made waves in and beyond the physics community. In thinking about which stories to highlight, we considered a combination of factors: popularity on the website, a clear element of surprise or discovery, or signs that the work could lead to better technology. On behalf of the *Physics* staff, we wish everyone an excellent New Year.

– Matteo Rini and Jessica Thomas



Images from popular *Physics* stories in 2013.

Four-Quark Matter

Quarks come in twos and threes—or so nearly every experiment has told us. This summer, the BESIII Collaboration in China and the Belle Collaboration in Japan reported they had sorted through the debris of high-energy electron-positron collisions and seen a mysterious particle that appeared to contain four quarks. Though other explanations for the nature of the particle, dubbed $Z_c(3900)$, are possible, the "tetraquark" interpretation may be gaining traction: BESIII has since seen a series of other particles that appear to contain four quarks.

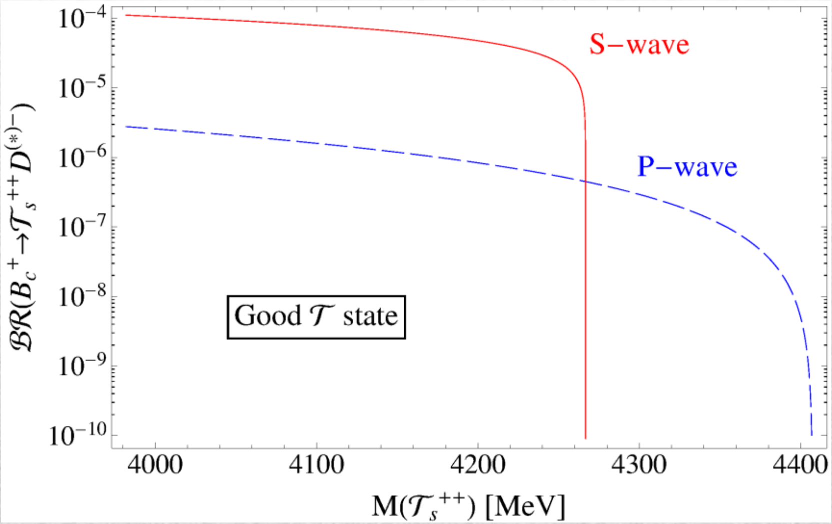
mysterious particle

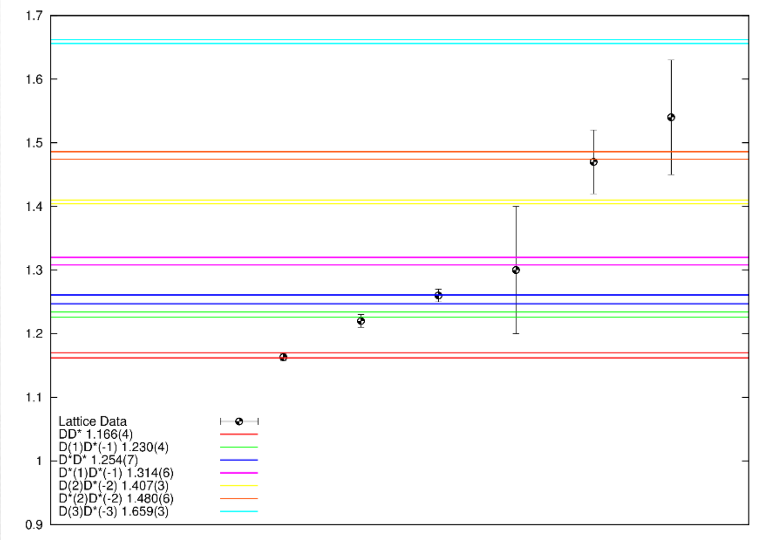
Doubly charmed states

For example, we proposed to look for doubly charmed states, which in tetraquark model are $[cc]_{S=1}[\bar{q}\bar{q}]_{S=0,1}$

These states could be observed in B_c decays @LHC and sought on the lattice

Esposito, Papinutto, AP, Polosa, Tantalo, PRD88 (2013) 054029

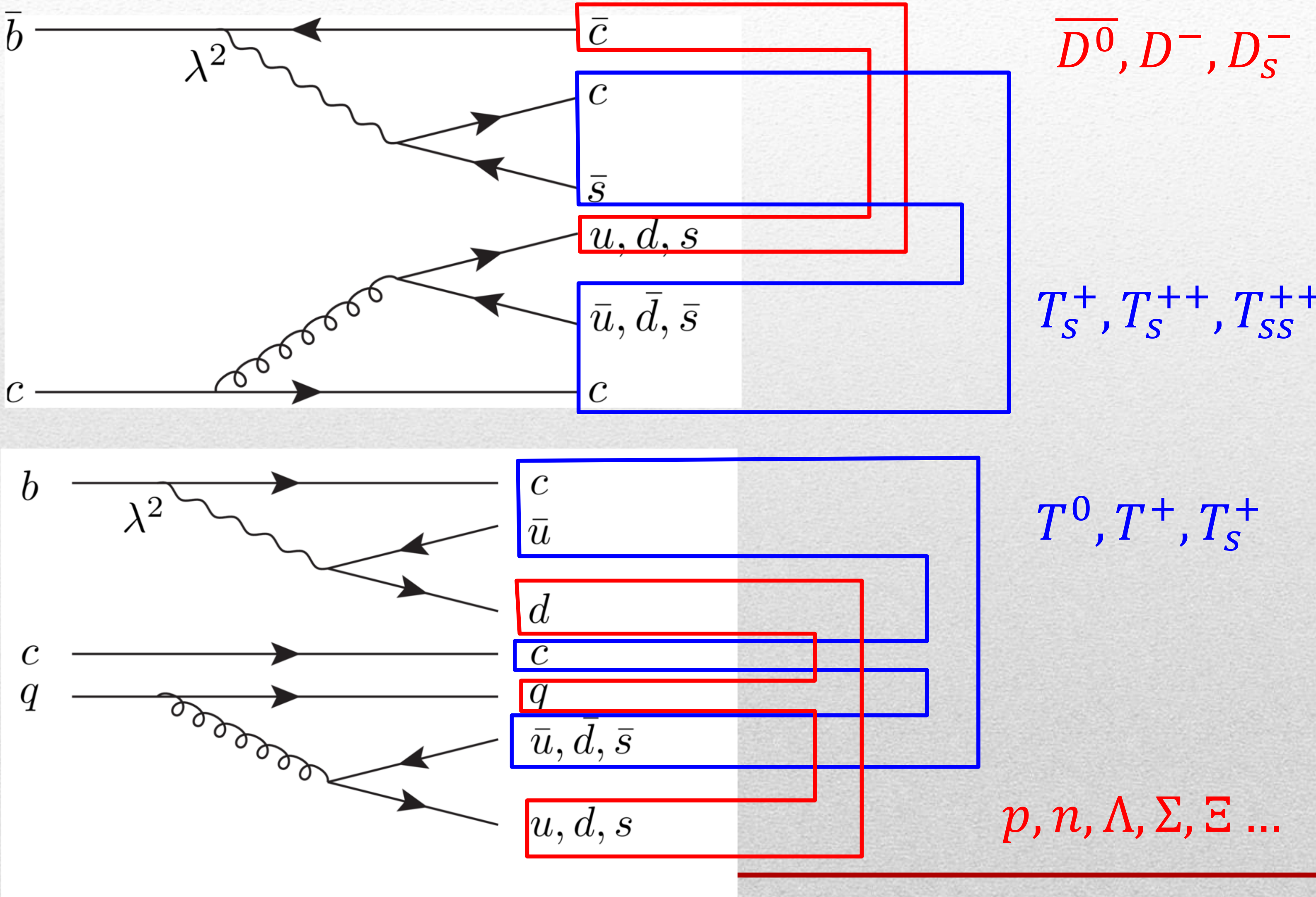




Preliminary results on spectrum for $m_\pi = 490$ MeV, $32^3 \times 64$ lattice, $a = 0.075$ fm

Guerrieri, Papinutto, AP, Polosa, Tantalo, PoS LATTICE2014 106

T states production

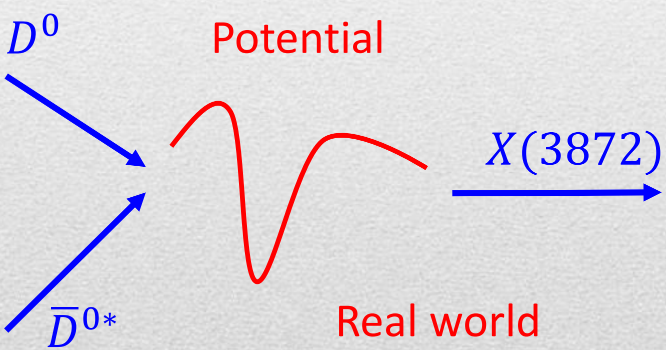


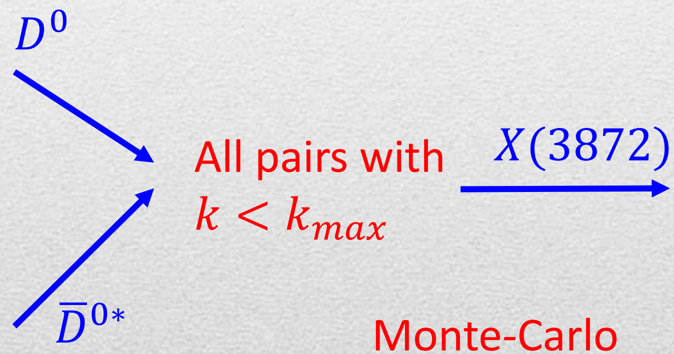
Prompt production of $X(3872)$

$X(3872)$ is the Queen of exotic resonances, the most popular interpretation is a $D^0\bar{D}^{0*}$ molecule (bound state, pole in the 1st Riemann sheet?)

We aim to evaluate prompt production cross section at hadron colliders via Monte-Carlo simulations

Q. What is a molecule in MC? **A.** «Coalescence» model





$$\sigma\big(p\bar{p} \to X(3872)\big) \sim \int d^3k\, |\langle X|D\bar{D}^*\rangle\langle D\bar{D}^*|p\bar{p}\rangle|^2 < \int_{k<k_{max}} d^3k\, |\langle D\bar{D}^*|p\bar{p}\rangle|^2$$

This should provide an upper bound for the cross section

Bignamini, Piccinini, Polosa, Sabelli PRL103 (2009) 162001
Kadastic, Raidan, Strumia PLB683 (2010) 248

Estimating k_{max}

The binding energy is $E_B \approx -0.16 \pm 0.31$ MeV: very small!
In a simple square well model this corresponds to:

$$\sqrt{\langle k^2 \rangle} \approx 50 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 10 \text{ fm}$$

(binding energy reported in Kamal Seth's talk is $E_B \approx -0.013 \pm 0.192$ MeV:

$$\sqrt{\langle k^2 \rangle} \approx 30 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 30 \text{ fm}$$

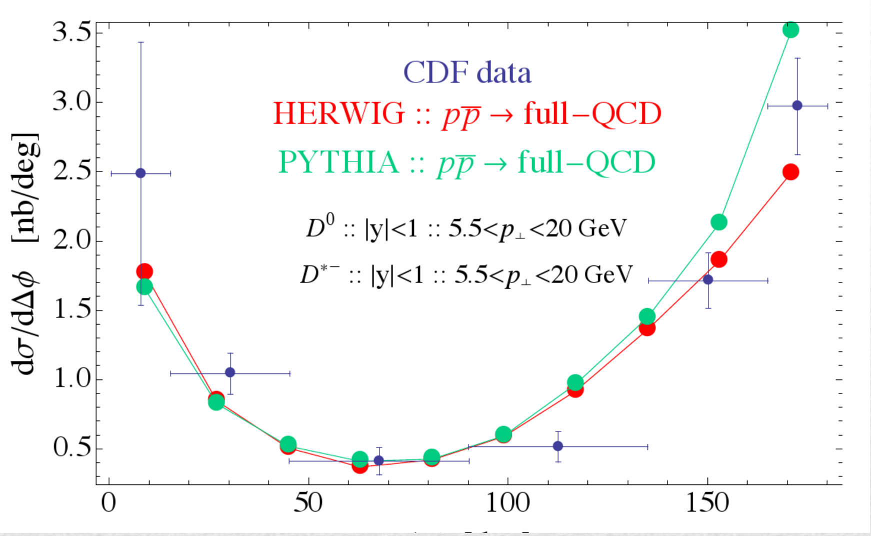
)

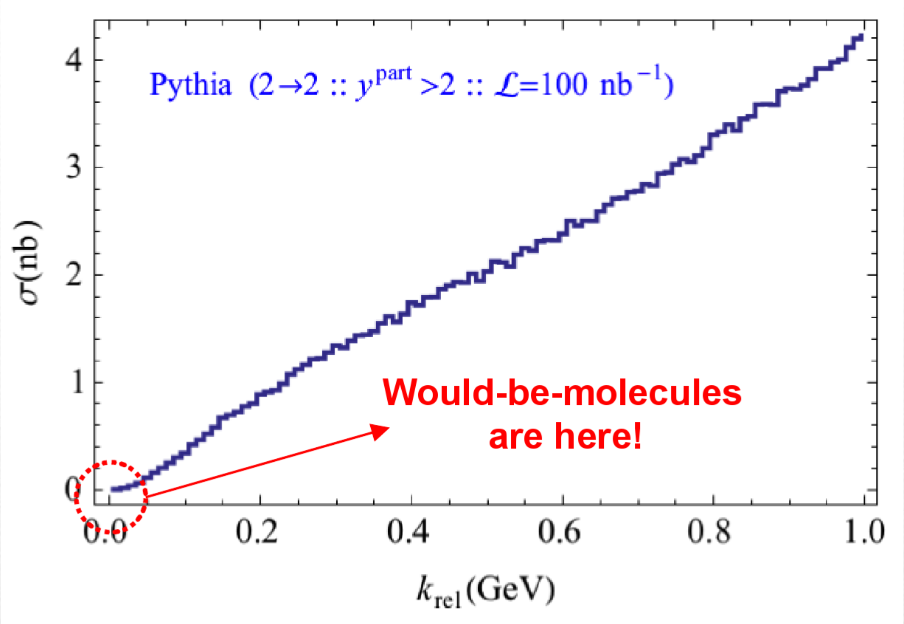
to compare with deuteron: $E_B = -2.2$ MeV

$$\sqrt{\langle k^2 \rangle} \approx 80 \text{ MeV}, \sqrt{\langle r^2 \rangle} \approx 4 \text{ fm}$$

We assume $k_{max} \sim \sqrt{\langle k^2 \rangle} \approx 50 \text{ MeV}$, some other choices are commented later

2009 results





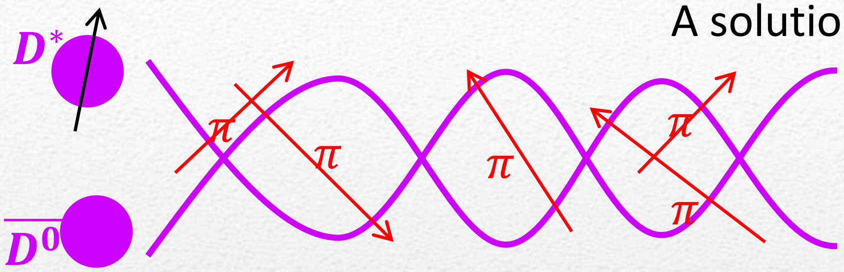
We tune our MC to reproduce CDF distribution of $\frac{d\sigma}{d\Delta\phi}(p\bar{p} \to D^0 D^{*-})$

We get $\sigma(p\bar{p} \to DD^*|k < k_{max}) \approx 0.1$ nb $@\sqrt{s} = 1.96$ TeV

Experimentally $\sigma(p\bar{p} \to X(3872)) \approx 30 - 70$ nb!!!

Bignamini, Piccinini, Polosa, Sabelli PRL103 (2009) 162001

Estimating k_{max}



A solution can be FSI (rescattering of DD^*) , which allow k_{max} to be as large as $5m_\pi \sim 700$ MeV

$\sigma(p\bar{p} \to DD^*|k < k_{max}) \approx 230$ nb

Artoisenet and Braaten, PRD81, 114018

However, the applicability of Watson theorem is challenged by the presence of pions that interfere with DD^* propagation

Bignamini, Grinstein, Piccinini, Polosa, Riquer, Sabelli, PLB684, 228-230

FSI saturate unitarity bound? Influence of pions small?

Artoisenet and Braaten, PRD83, 014019

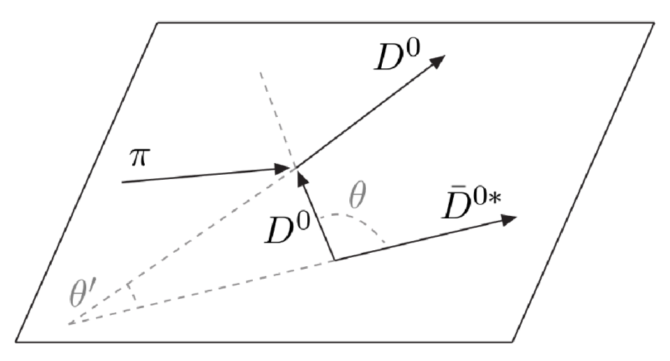
Guo, Meissner, Wang, Yang, JHEP 1405, 138; EPJC74 9, 3063; CTP 61 354
use $E_{max} = M_X + \Gamma_X$ for above-threshold unstable states

With different choices, 2 orders of magnitude uncertainty, limits on predictive power

A new mechanism?

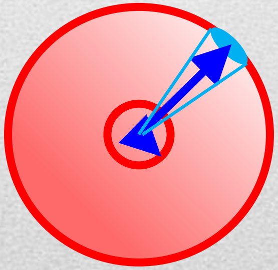
In a more billiard-like point of view, the comoving pions can elastically interact with $D(D^*)$, and slow down the pairs DD^*

Esposito, Piccinini, AP, Polosa, JMP 4, 1569
Guerrieri, Piccinini, AP, Polosa, PRD90, 034003

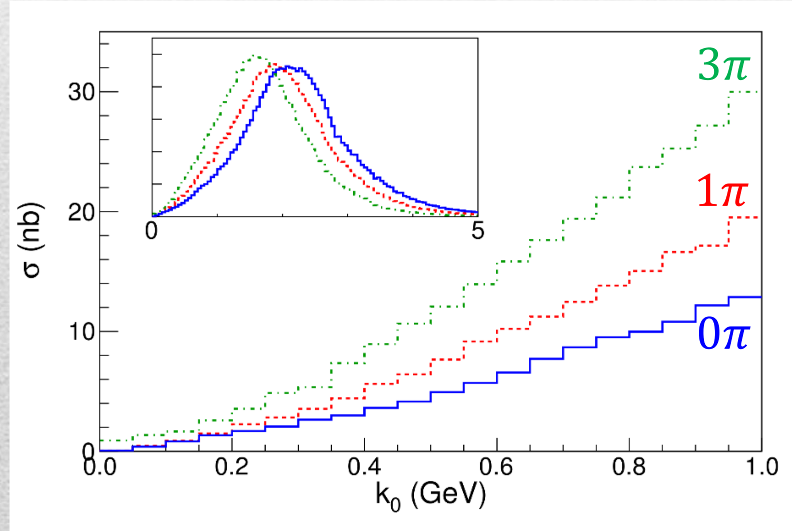


The mechanism also implies: D mesons actually "pushed" inside the potential well (the classical 3-body problem!)

$X(3872)$ is a real, negative energy bound state (stable)
It also explains a small width $\Gamma_X \sim \Gamma_{D^*} \sim 100$ keV



By comparing hadronization times of heavy and light mesons, we estimate up to ~ 3 collisions can occur before the heavy pair to fly apart



We get $\sigma\left(p\bar{p} \to X(3872)\right) \sim 5$ nb, still not sufficient to explain all the experimental cross section

Hybridized tetraquarks – Selection rules

- Consider the down quark part of the $X(3872)$ in the diquarkonium picture:

$$\Psi_{\mathbf{d}} = X_d = [cd]_0[\bar{c}\bar{d}]_1 + [cd]_1[\bar{c}\bar{d}]_0 \sim (D^{*-}D^+ - D^{*+}D^-) + i(\psi \times \rho^0 - \psi \times \omega^0)$$

Fierz rearrangement

- The closest threshold from below is $\Psi_m \sim \bar{D}^0 D^{*0} \longrightarrow \Psi_{\mathbf{d}} \perp \Psi_m$ ✓
- But if we consider the up quark part of the $X(3872)$:

$$\Psi_{\mathbf{d}} = X_u = [cu]_0[\bar{c}\bar{u}]_1 + [cu]_1[\bar{c}\bar{u}]_0 \sim (\bar{D}^{*0}D^0 - D^{*0}\bar{D}^0) - i(\psi \times \rho^0 + \psi \times \omega^0)$$

- But then $\longrightarrow \Psi_{\mathbf{d}} \not\perp \Psi_m$ ✗
- Only X_d is produced via this mechanism $\longrightarrow$ isospin violation; $\longrightarrow$ no hyperfine neutral doublet
- X_b (A) Diquark model predicts $M(X_b) \simeq M(Z_b) \simeq (10607 \pm 2)$ MeV
  (B) The closest orthogonal threshold is $M(B^0 B^{*0}) = (10604.4 \pm 0.3)$ MeV
  (C) This could either be above threshold (very narrow state) or below (no state at all)
  (D) Experimentally the diquark model overpredicts the mass of the X:

$$M(Z_c) - M(X) \simeq 32 \text{ MeV}$$

  (E) We favor the below threshold scenario $\longrightarrow$ no X_b should be seen

A. Esposito

Production of hybridized tetraquarks

Going back to $pp(\bar{p})$ collisions, we can imagine hadronization to produce a state

$$|\psi\rangle = \alpha|[qQ][\bar{q}\bar{Q}]\rangle_C + \beta|(\bar{q}q)(\bar{Q}Q)\rangle_O + \gamma|(\bar{q}Q)(\bar{Q}q)\rangle_O$$

If $\beta, \gamma \gg \alpha$, an initial tetraquark state is not likely to be produced
The open channel mesons fly apart (see MC simulations)

If hybridization mechanism is at work, an open state can resonate in a closed one

α expected to be small in Large N limit, Maiani, Polosa, Riquer JHEP 1606, 160

No prompt production without hybridization mechanism!

Note that only the $X(3872)$ has been observed promptly so far...

...and a narrow $X(4140)$ not compatible with the LHCb one → **needs confirmation**

Tetraquark

Maiani, Piccinini, Polosa, Riquer PRD89 114010

J^{PC}	$cq\,\bar{c}\bar{q}$	$c\bar{c}\,q\bar{q}$	Resonance Assig.	Decays
0^{++}	$\|0,0\rangle$	$1/2\|0,0\rangle + \sqrt{3}/2\|1,1\rangle_0$	$X_0(\sim 3770\text{ MeV})$	$\eta_c, J/\psi$ + light mesons
0^{++}	$\|1,1\rangle_0$	$\sqrt{3}/2\|0,0\rangle - 1/2\|1,1\rangle_0$	$X_0'(\sim 4000\text{ MeV})$	$\eta_c, J/\psi$ + light mesons
1^{++}	$1/\sqrt{2}(\|1,0\rangle + \|0,1\rangle)$	$\|1,1\rangle_1$	$X_1 = X(3872)$	$J/\psi + \rho/\omega,\ DD^*$
1^{+-}	$1/\sqrt{2}(\|1,0\rangle - \|0,1\rangle)$	$1/\sqrt{2}(\|1,0\rangle - \|0,1\rangle)$	$Z = Z(3900)$	$J/\psi + \pi,\ h_c/\eta_c + \pi/\rho$
1^{+-}	$\|1,1\rangle_1$	$1/\sqrt{2}(\|1,0\rangle + \|0,1\rangle)$	$Z' = Z(4020)$	$J/\psi + \pi,\ h_c/\eta_c + \pi/\rho$
2^{++}	$\|1,1\rangle_2$	$\|1,1\rangle_2$	$X_2(\sim 4000\text{ MeV})$	J/ψ + light mesons

$$H_{\text{eff}} = 2m_{\mathcal{Q}} + \frac{B_{\mathcal{Q}}}{2}\boldsymbol{L}^2 - 3\kappa_{cq} + 2a_Y\boldsymbol{L}\cdot\boldsymbol{S} + b_Y\frac{\langle S_{12}\rangle}{4} + \kappa_{cq}\left[2(\boldsymbol{S_q}\cdot\boldsymbol{S_c} + \boldsymbol{S_{\bar{q}}}\cdot\boldsymbol{S_{\bar{c}}}) + 3\right]$$

Ali, *et al.* EPJC78, 1, 29

Maiani, Polosa, Riquer, PLB778, 247-251

Label	$\|S_{\mathcal{Q}}, S_{\bar{\mathcal{Q}}}; S, L\rangle_J$
Y_1	$\|0,0;0,1\rangle_1$
Y_2	$(\|1,0;1,1\rangle_1 + \|0,1;1,1\rangle_1)/\sqrt{2}$
Y_3	$\|1,1;0,1\rangle_1$
Y_4	$\|1,1;2,1\rangle_1$
Y_5	$\|1,1;2,3\rangle_1$

Two different mass scenarios

$$M_1 = 4008 \pm 40^{+114}_{-28},\quad M_2 = 4230 \pm 8,$$
$$M_3 = 4341 \pm 8,\quad M_4 = 4643 \pm 9.$$

$$M_1 = 4219.6 \pm 3.3 \pm 5.1,\quad M_2 = 4333.2 \pm 19.9,$$
$$M_3 = 4391.5 \pm 6.3,\quad M_4 = 4643 \pm 9,$$

Prediction for a high Y_5

$$M_5 = \begin{cases} 6539\text{ MeV} & \text{SI(c1)} \\ 6589\text{ MeV} & \text{SI(c2)} \\ 6862\text{ MeV} & \text{SII(c1)} \\ 6899\text{ MeV} & \text{SII(c2)} \end{cases}$$