Advanced Quantum Mechanics 2023/24

Alessandro Pilloni

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QUANTUM MECHANICS

1. Angular momentum in quantum mechanics and commutation rules (§3.1 [1]). Brief discussion of rotations groups and algebras, $SO(3)$ and $SU(2)$.

2. Rotations and systems of spin 1/2 (§3.2 [1])

3. Eigenvalues and eigenstates of angular momentum (§3.5 [1])

4. Orbital angular momentum and spherical harmonics. Laplacian operator in polar coordinates (§3.6 [1])

5. Parity and spherical harmonics (§4.2 [1])

6. Central potentials. Hydrogen atom (for example from [2]). 3D-harmonic oscillators (briefly from §6.3 [3])

7. Sum of angular momenta and Clebsch-Gordan coefficients (§3.7 [1] and [4])

8. Indistinguishable particles (§6.1-6.3 [1]), helium atom (briefly from §6.4 [1])

9. Pauli Hamiltonian and the Zeeman effect (briefly from §5.3 [1])

FIELD THEORY

1. Reminders of special relativity (§1-7 [5], §1.1-2.4 [6])

2. Classical field theory. Lagrangian and hamiltonian formalisms (chapter 11 [7], §3.1-3.2 [6]).

3. Klein-Gordon equation. Real and complex classical fields. Solutions of homogeneous and inhomogeneous equations (Green functions) (§4.1-4.2 [6])

4. Simmetries and Noether Theorem (§3.4-3.5 [6] and §2.4 [8])

5. Covariance of fields under Poincaré transformations. Energy-momentum and angular momentum tensors (the latter in the spinless case) (§3.6 [6] and §2.4 [8])

6. Quantization of scalar fields. Canonical commutators. Energy and momentum in quantized form. Normal products. Definition of states (Fock space) (§4.3 [6] and §3.1-3.2 [8])

7. Expectation values of fields, time-ordered product and Feynman propagator. Pauli-Jordan function and microcausality (§8.1-8.2 [6] and §3.3-3.4 [8])

8. Dirac equation. Spin and angular momentum conservation (§6-6.1-6.1.1 [6]).

9. Spinors and negative-energy solutions. Resolution of negative energies problem and introduction to antiparticles. v-spinors. Completeness relations and projectors (§6.1.4 [6])

10. Nonrelativistic limit and Pauli Hamiltoniana (§6.1.5 [6]).

11. Covariance of Dirac equation. Adjoint spinors. Dirac Lagrangian (§6.1.2 [6], §4.2 [8])

12. Quantization of Dirac equation. Energy and momentum in quantized form, need for canonical anticommutators. State definition and connection between spin and statistics (§7.1-7.3 [6], §4.3 [8])

13. Expectation values of fermionic fields. Time ordered product. Feynman Propagator for fermions. Pauli-Jordan function and microcausality for fermions (§7.5-7.6 and §8.3 [6] and §4.4 [8])

14. Maxwell equations in covariant form. Lagrangian of electromagnetic field (§5.1 [6]). Problems with the Hamiltonian formalism caused by gauge freedom. Choice of two possible gauges: covariant (Lorentz) gauge and Coulomb Gauge. Quantization in Coulomb gauge. Polarization vectors (§5.5 [6]). Covariant quantization, Gupta-Bleuler prescription (§5.2 [8]).

15. Photon propagator in Lorentz gauge and reduction to the propagator in Coulomb gauge (§5.2 [6] and §5.3 [8]).

NOTE

- Textbooks are just suggested, but feel free to use any book you like. For each point of the program, I am pointing to the books that discusses the topic closest to how I teach it.

Different editions can have substantially different paragraph numbering, I am referring to my personal copy..

- [11] uses the metrics $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$, other books $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. This introduces different signs in the discussion of Poincaré group.
- Both [6] and [8] use the spinor normalization $\bar{u}u = 1$, while in class we use $\bar{u}u = 2m$

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[1] J. J. Sakurai, "Modern Quantum Mechanics."

[2] `https://www.roma1.infn.it/~sciortif/didattica/FISICATEORICA/FISICATEORICA/atomodiidrogeno.pdf` (in Italian).

[3] S. Patrì and M. Testa, "Fondamenti di Meccanica Quantistica," Ed. Nuova Cultura (in Italian).

[4] `https://pdg.lbl.gov/2018/reviews/rpp2018-rev-clebsch-gordan-coefs.pdf`

[5] L.D. Landau, E.M. Lifshitz, "Course of Theoretical Physics Vol. 2: The classical theory of fields."

[6] L. Maiani, O. Benhar, "Relativisic Quantum Mechanics Relativistica: An introduction to relativistic quantum fields," CRC Press.

[7] H. Goldstein, "Classical Mechanics."

[8] F. Mandl, G. Shaw, "Quantum Field Theory," Wiley.

Other suggested books:

[9] S. Weinberg, "Lectures on Quantum Mechanics," Cambridge University Press.

[10] C. Cohen-Tannoudji, B. Diu, F. Laloë, "Lectures on Quantum Mechanics," Vch Pub.

[11] S. Weinberg, "The Quantum Theory of Fields Vol. 1: Foundations," Cambridge University Press.