# Written test of Advanced Quantum Mechanics

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Exam time: 2 hours. You can use the Clebsch-Gordan sheet by PDG.

## EXERCISE 1

We consider a system of two identical particles $A$ and $B$, having mass $m$ and spin 1. The hamiltonian in the center of mass is given by:

$$H = \frac{\boldsymbol{p}^2}{2\mu} + \frac{1}{2}\mu\omega^2\boldsymbol{r}^2 + \frac{\alpha}{\hbar}S_z \tag{1}$$

where $\boldsymbol{p}$ and $\boldsymbol{r}$ are the relative momentum and relative coordinate, and $\mu = m/2$ is the reduced mass. We take $\alpha \ll \hbar\omega$.

1. Calculate the commutator $\left[H, \boldsymbol{J}^2\right]$, where $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, $\boldsymbol{L}$ the orbital angular momentum, and $\boldsymbol{S}$ the total spin.

2. Determine the spectrum (i.e. the eigenstates, and the associated energies and degenerations) up to $E < 3\hbar\omega$.

3. Consider the normalized state:

$$|\psi\rangle = \delta\,|n=0, l=0, l_z=0\rangle\,|s=0, s_z=0\rangle + \gamma\,|n=1, l=1, l_z=0\rangle\,|s=1, s_z=0\rangle \tag{2}$$

   where $\delta$ and $\gamma$ are real numbers. Determine $\delta$ and $\gamma$, assuming that a measurement of $\boldsymbol{J}^2$ gives $6\hbar^2$ with probability 1/3.

**EXERCISE 2**

Consider a particle of spin 1 with Hamiltonian:

$$H = \frac{\boldsymbol{p}^2}{2\mu} + \frac{1}{2}\mu\omega^2\boldsymbol{r}^2 + \frac{\alpha}{\hbar}\boldsymbol{J}^2 \tag{3}$$

with $\alpha \ll \omega$, $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$.

1. Calculate energies and degenerations of the first 4 levels.
2. Consider the state $|\psi\rangle = |n = 1, l = 1, l_z = 0\rangle\,|s = 1, s_z = 1\rangle$. Calculate the time evolution, and the mean values of $L_z$, $\boldsymbol{L}^2$, $S_z$, $\boldsymbol{J}^2$ as a function of time.
3. When the result is time independent, comment about that.