Written test of Advanced Quantum Mechanics

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Exam time: 2 hours. You can use the Clebsch-Gordan sheet by PDG.

EXERCISE 1

A particle of mass m and spin 1/2 moves in 3D space according to the following Hamiltonian:

$$H = \frac{\boldsymbol{p}^2}{2m} + \frac{1}{2}m\omega^2\boldsymbol{r}^2 + \frac{\alpha}{\hbar^2}\left(\boldsymbol{L}\cdot\boldsymbol{S} + \hbar J_z\right) \tag{1}$$

with $0 < \alpha \ll \hbar\omega$. We consider the following states:

$$|\psi_1\rangle = |1\ 1\ -1\rangle\,|+\rangle\,, \qquad |\psi_2\rangle = |1\ 1\ 1\rangle\,|+\rangle\,, \qquad |\psi_3\rangle = |0\ 0\ 0\rangle\,|+\rangle\,, \tag{2}$$

using the notation $|n\ \ell\ m\rangle\left|s = \frac{1}{2}\ s_z = \pm\right\rangle$.

1. Discuss whether the three states are eigenstates of the Hamiltonian or not, and why.
2. Calculate the time evolution for the state $|\psi_1\rangle$.
3. For the same state, calculate the probability as a function of time for a measurement of S_x to be $\hbar/2$.
   Hint: $|\pm\rangle = \frac{1}{\sqrt{2}}\left(|+\rangle_x \pm |-\rangle_x\right)$.

EXERCISE 2

Two identical particles of spin 1/2 are indicated with A and B and are vinculated to a spherical surface of unit radius, with the following Hamiltonian:

$$H = \frac{\epsilon}{\hbar^2}\left(\boldsymbol{L}_A^2 + \boldsymbol{L}_B^2 + \boldsymbol{S}_A\cdot\boldsymbol{S}_B\right), \tag{3}$$

with $\epsilon > 0$. The system is not studied in the center of mass frame.

1. Determine the spectrum and degeneracies for energies up to $E < \frac{5}{2}\epsilon$