Written test of Advanced Quantum Mechanics

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Exam time: 2 hours. You can use the Clebsch-Gordan sheet by PDG.

EXERCISE 1

A particle of mass m and spin 1 moves in 3D space according to the following Hamiltonian:

$$H = \frac{\boldsymbol{p}^2}{2m} + \frac{1}{2}m\omega^2\boldsymbol{r}^2 + \frac{\epsilon\omega}{\hbar}\left(L_z + 2S_z\right) \tag{1}$$

with $\epsilon \ll 1$. At $t = 0$ the particle is in a state $|\psi\rangle$ such that:

(i) A measurement of energy gives $E < 3\hbar\omega$;

(ii) $\langle\psi| S_z |\psi\rangle = -\hbar$;

(iii) A measurement of L_z never gives zero.

Then:

1. Determine the most general $|\psi\rangle$.
2. Pick the state satisfying $\langle\psi| L_x^2 |\psi\rangle = 0$.
3. With the latter, calculate the possible results of a measurement of $\boldsymbol{J}^2$ and the respective probabilities.

EXERCISE 2

Two identical particles of spin 1/2 are vinculated to a spherical surface of radius R. In the center of mass frame, the dynamics are given by the following Hamiltonian:

$$H = \frac{\vec{L}^2}{2mR^2} + \alpha\,\vec{L}\cdot\vec{S} \tag{2}$$

where $0 < \alpha \ll 1/mR^2$.

1. Calculate the first four energy levels and the respective degenerations.

2. Calculate the mean values of L_z e L_x on the first excited state. Does the result depend on time?