# Written test of Advanced Quantum Mechanics

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(Dated: 10/07/2025)

Exam time: 2 hours. You can use the Clebsch-Gordan sheet by PDG.

## EXERCISE 1

A system of two identical particles of mass $m$ and spin 1 is described, in the center-of-mass frame, by the Hamiltonian:

$$H = \frac{p^2}{2\mu} + \frac{1}{2}\mu\omega^2 r^2 + \frac{\omega}{\hbar}\vec{J}^2$$

where $\mu = \frac{m}{2}$ is the reduced mass, $\vec{r} = \vec{r}_1 - \vec{r}_2$, and $\vec{p}$ is the corresponding conjugate momentum. $\vec{J} = \vec{L} + \vec{S}$, with $\vec{L}$ the orbital angular momentum and $\vec{S} = \vec{S}_1 + \vec{S}_2$ the total spin.

Determine the spectrum of eigenvalues and eigenvectors, and find the ones corresponding to the first four energy levels and discuss their degeneracies.

## EXERCISE 2

Consider a particle of mass $m$ and spin 1/2, constrained to move on a unit-radius sphere. The particle is in the state specified by the normalized spinor:

$$|\psi\rangle = A \begin{pmatrix} 3\sin\theta\, e^{-i\phi} \\ 2 \end{pmatrix}$$

where $A$ is the normalization constant.

1. Compute $|A|$. If one measures $\vec{L}^2$ on the state $|\psi\rangle$, what values can be obtained, and with what probabilities?

2. At time $t = 0$, a measurement of $\vec{J}^2$ (with $\vec{J} = \vec{L} + \vec{S}$) gives the result $\frac{3}{4}\hbar^2$. Write the normalized state $|\psi_1\rangle$ the system collapses into after the measurement.

3. The state $|\psi_1\rangle$ evolves with the Hamiltonian:

$$H = \omega(L_z + 2S_z) + \frac{\omega}{2\hbar}L^2$$

Compute the state $|\psi_1(t)\rangle$ at time $t$. Does the state evolve with time?