“Sapienza” Università di Roma – INFN sez. Roma 1

On the spin of the X(3872)

A. Pilloni

TUM, Munich – September 26th, 2012

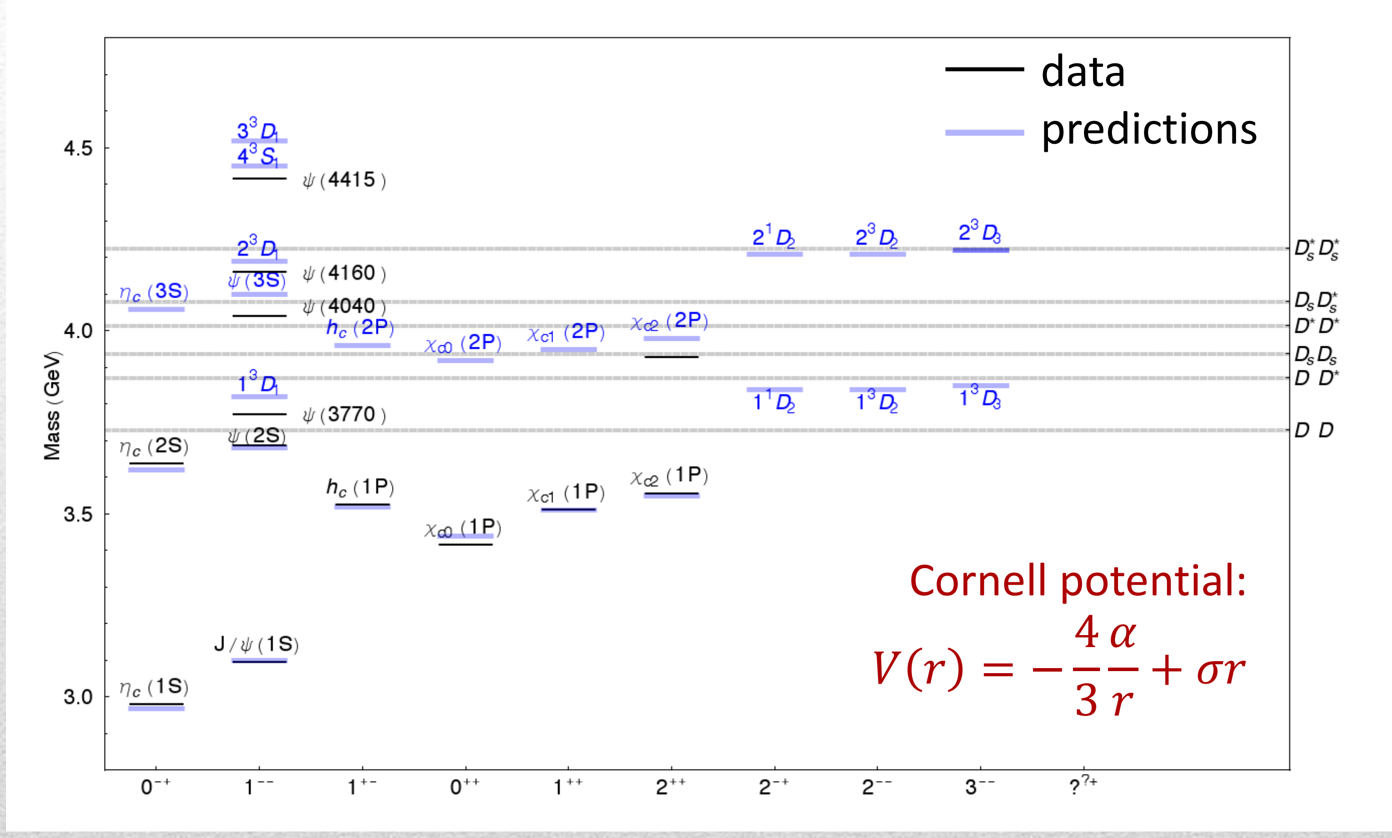
R. Faccini, F. Piccinini, AP, A.D. Polosa

PRD86, 054012 (2012)

Outline

- Exotic states: X Y Z
- Main exotic models
- X(3872): molecule or tetraquark?
- The spin of the X(3872)
- Conclusions

X Y Z



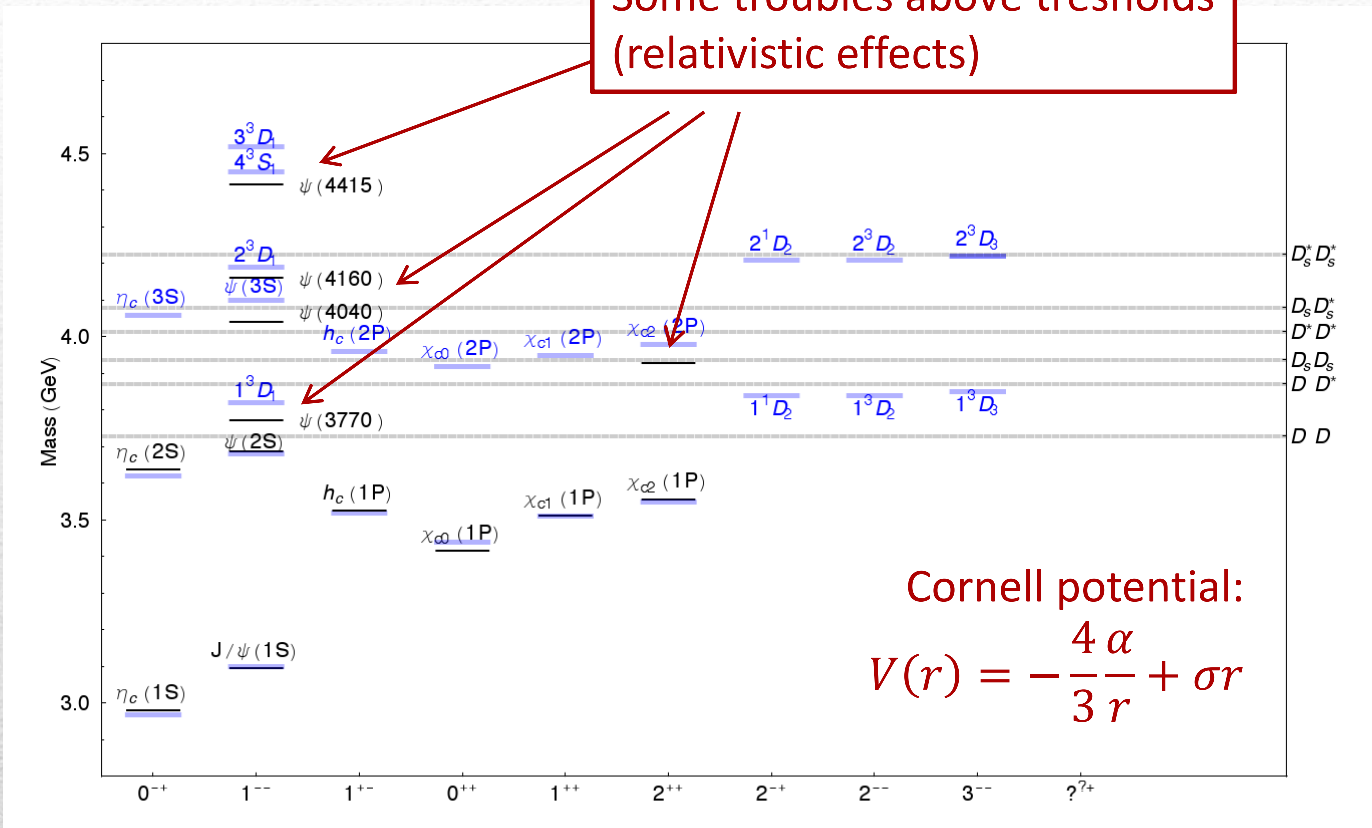
$$V(r) = -\frac{4}{3}\frac{\alpha}{r} + \sigma r$$

C. Sabelli

Before B factories, hidden charm mesons were described as a $c\bar{c}$ system in a non-relativistic potential

X Y Z

Some troubles above tresholds (relativistic effects)



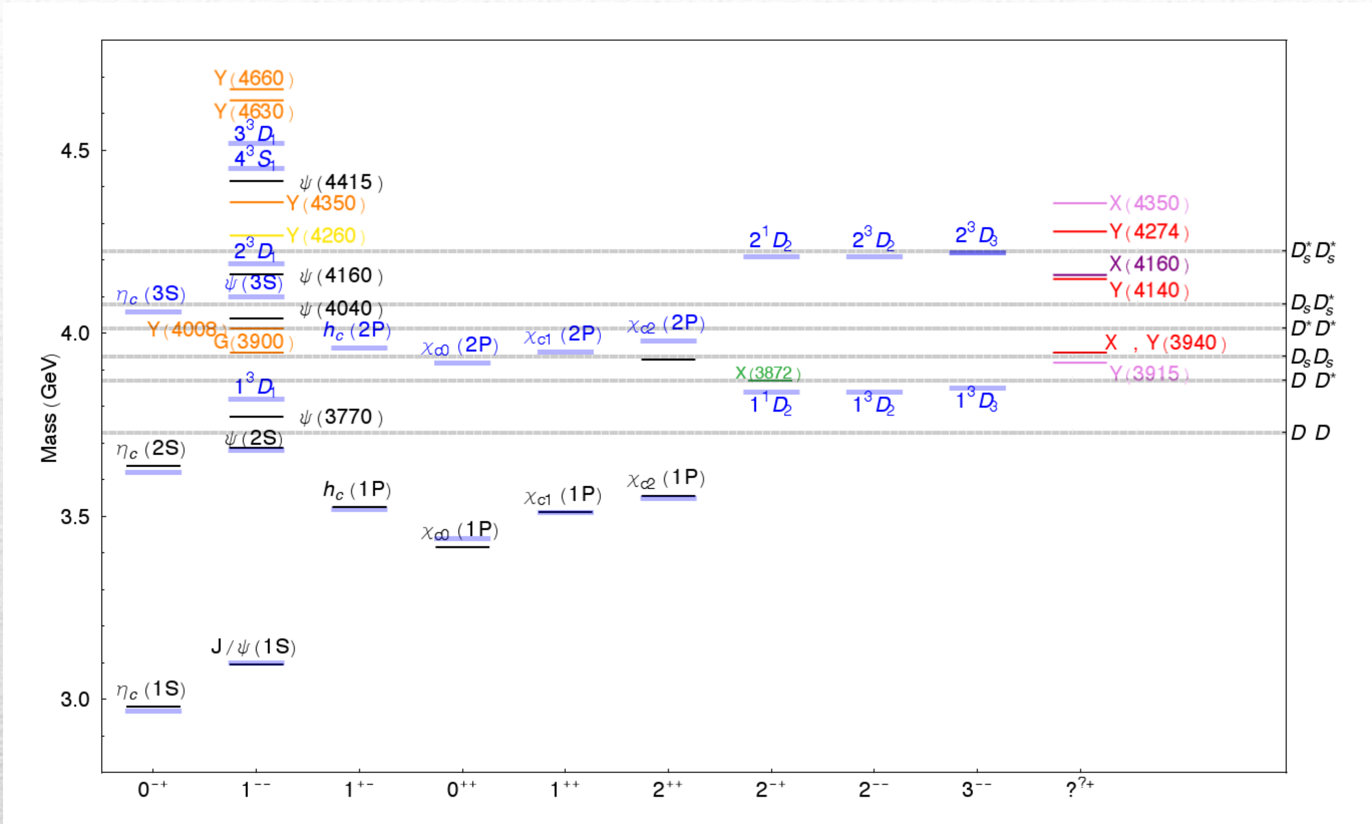
Cornell potential:

$$V(r) = -\frac{4}{3}\frac{\alpha}{r} + \sigma r$$

C. Sabelli

Before B factories, hidden charm mesons were described as a $c\bar{c}$ system in a non-relativistic potential

X Y Z

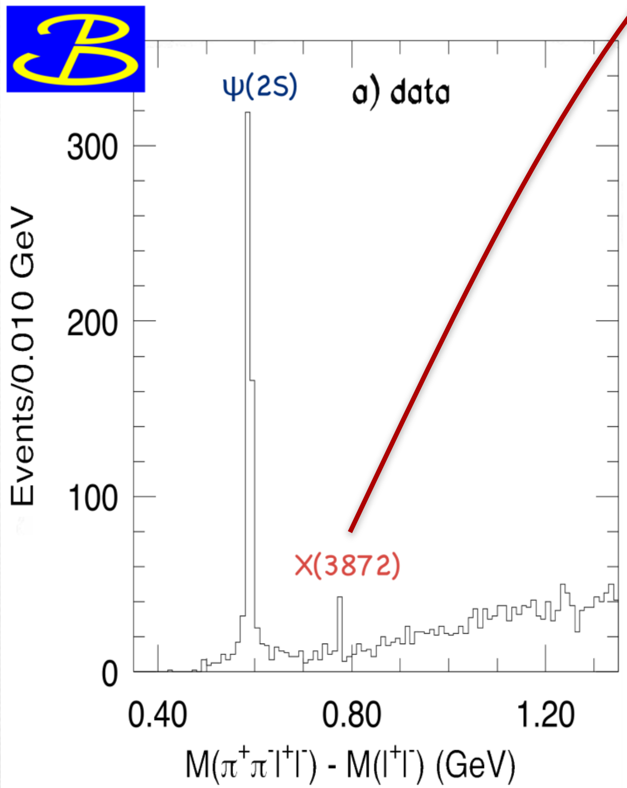


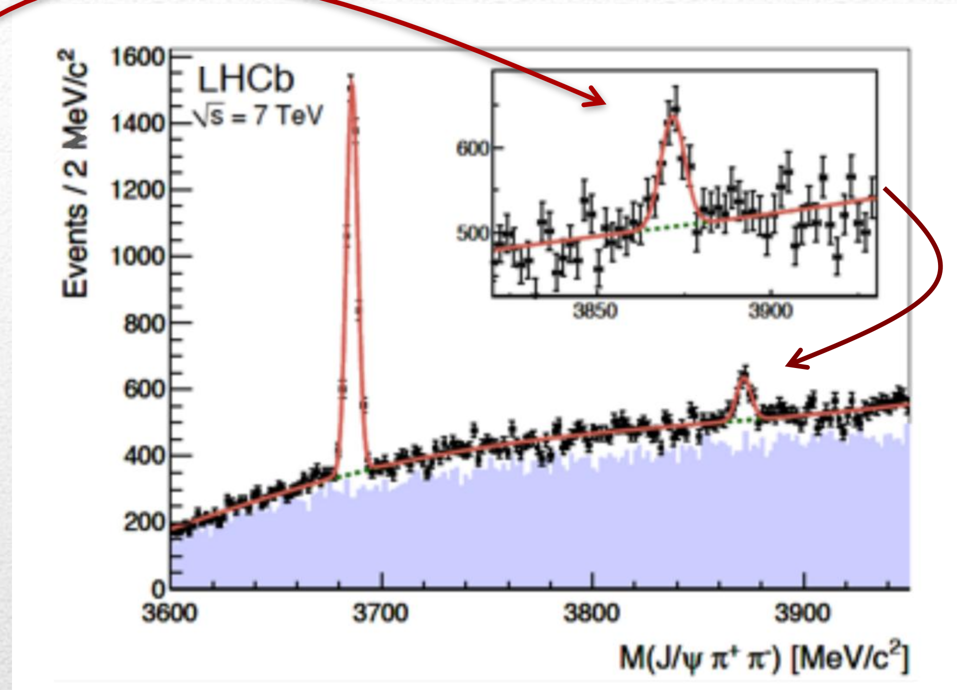
C. Sabelli

A lot of “weird” states appeared

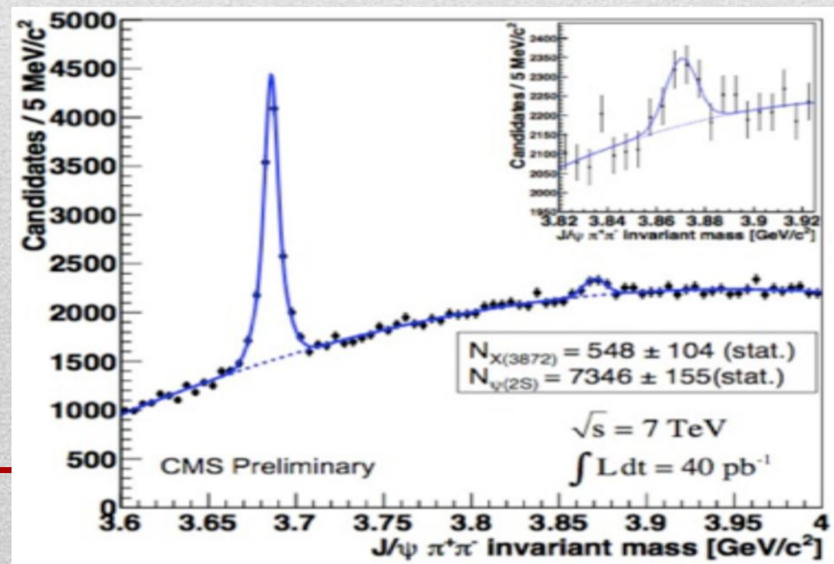
They do not fit in the classic $c\bar{c}$ system

X(3872)

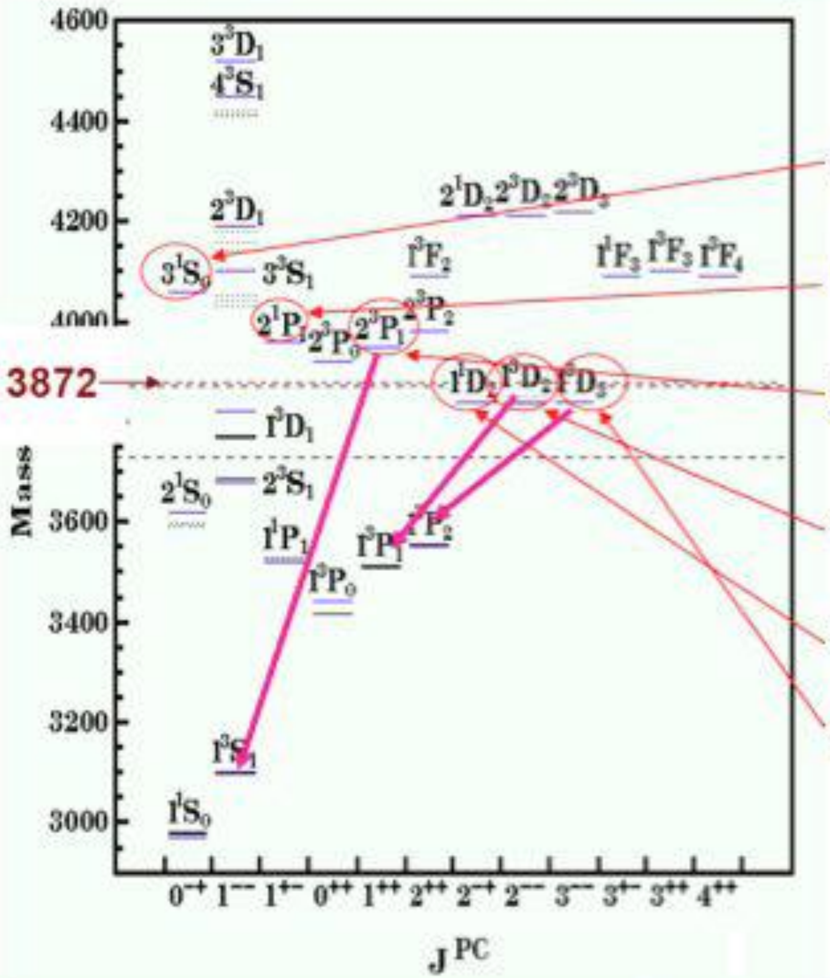




$\Gamma < 1.2$ MeV



$X(3872)$: charmonio?



$\eta_c(3S)$: massa e larghezza troppo piccole

$h_c(2P)$: distribuzioni angolari non compatibili con $J^{PC} = 1^{+-}$

$\chi_{c1}(2P)$: $\mathcal{B}(X \to J/\psi\,\gamma)$ troppo piccolo

ψ_2: $\mathcal{B}(X \to \chi_{c1}\,\gamma)$ troppo piccolo; $m(\pi^+\pi^-)$ non compatibile

η_{c2}: dovrebbe dominare $X \to \eta_c\,\pi^+\,\pi^-$

ψ_3: $\mathcal{B}(X \to \chi_{c2}\,\gamma)$ e $\mathcal{B}(X \to \bar{D}D)$ troppo piccoli

X(3872)

- First exotic charmonium-like state discovered at Belle (2003)
- Too narrow ($\Gamma < 1.2\ \mathrm{MeV}$) for an above-treshold charmonium
- Radiative decay in $J/\psi\ \ \gamma$ too small for charmonium
- Isospin violation: $\frac{\Gamma(X\to J/\psi\ \ \omega)}{\Gamma(X\to J/\psi\ \ \rho)} \sim 0.8 \pm 0.3$ too big
- The mass cannot be predicted as a charmonium excitation (almost equal to $D^0 + D^{0*}$)

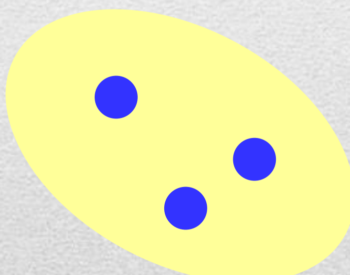
What is that?

(a digression on QCD)

Quarks are the building blocks of matter
Quarks are colored particles: q $\in \mathbf{3_c}$, $\bar{q} \in \bar{\mathbf{3}}_{\boldsymbol{c}}$

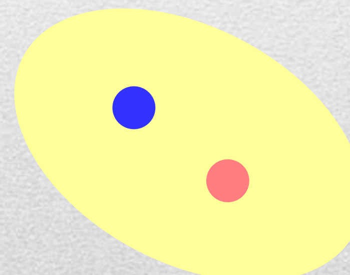
They must arrange in color neutral states

Baryons



$\mathbf{3}_{\boldsymbol{c}} \times \mathbf{3}_{\boldsymbol{c}} \times \mathbf{3}_{\boldsymbol{c}} \in \mathbf{1}_{\boldsymbol{c}}$

Mesons

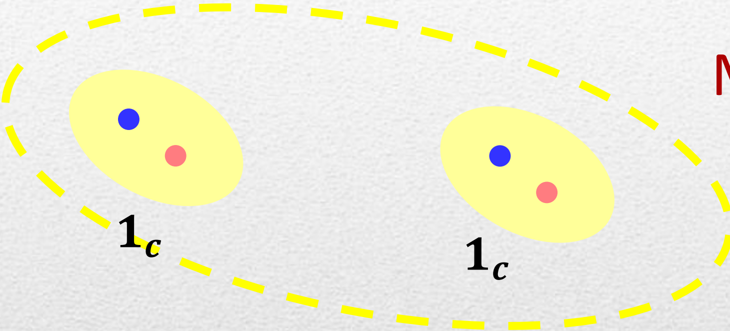


$\mathbf{3}_{\boldsymbol{c}} \times \bar{\mathbf{3}}_{\boldsymbol{c}} \in \mathbf{1}_{\boldsymbol{c}}$

All hadronic matter fits in these two models (up to 2003)

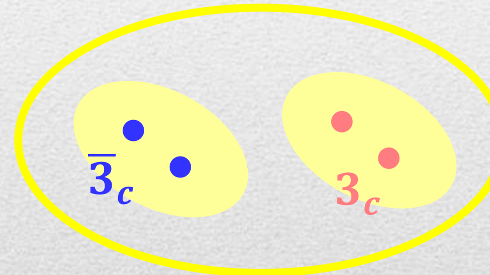
(a digression on QCD)

Can we have other neutral color states?

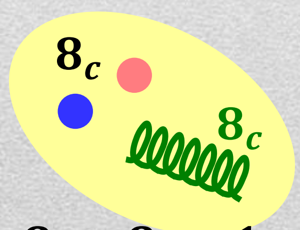


Molecule of hadrons (loosely bound)

Diquark-antidiquark
(tetraquark)

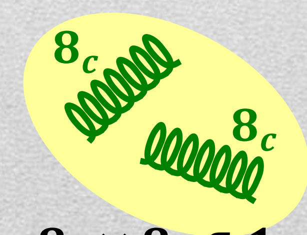


$\mathbf{3}_c \times \bar{\mathbf{3}}_c \in \mathbf{1}_c$



$\mathbf{8}_c \times \mathbf{8}_c \in \mathbf{1}_c$

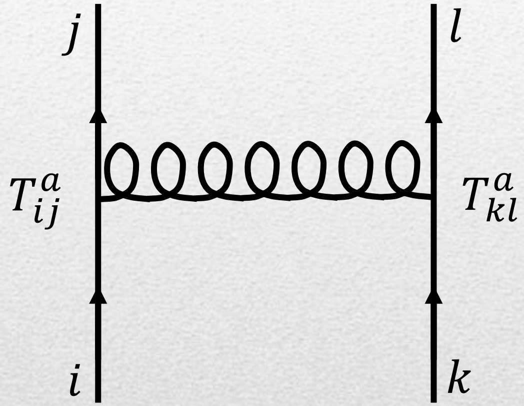
Hybrids and glueballs
(with valence gluons)



$\mathbf{8}_c \times \mathbf{8}_c \in \mathbf{1}_c$

(a digression on QCD)

Attraction and repulsion between electric charges is a matter product of signs.
In QCD it is more complicated than that (matrix tensor products)

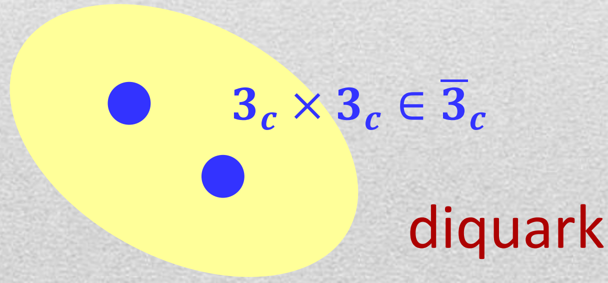


$$T^a_{R_1} \times T^a_{R_2}$$

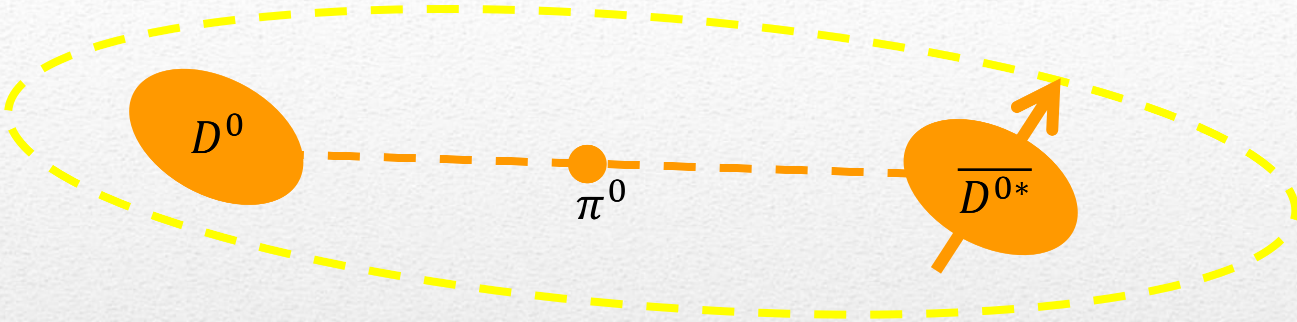
product of representations

The singlet $\mathbf{1}_c$ is an attractive combination

A diquark in $\bar{\mathbf{3}}_c$ is an attractive combination
A diquark is colored, so it can stay into hadrons
but cannot be an asymptotic state
We see diquarks in lattice QCD



X(3872): molecule?



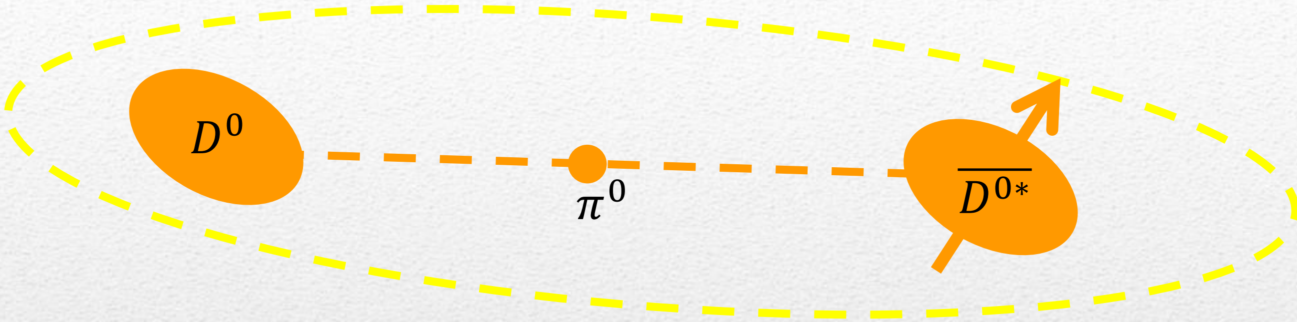
- Molecular state of $\frac{|D^0\overline{D^{0*}}\rangle + |\overline{D^0}D^{0*}\rangle}{\sqrt{2}}$
- Small binding energy: $M_X - M_{D^0} - M_{D^{0*}} \approx (-0.25 \pm 0.40)$ MeV
- Isospin violation because of the threshold D^+D^{*-}
- Two scales:
  - $R \approx 1$ fm radius of the mesons
  - $R \approx 10$ fm radius of the molecule

Analogies with deuteron (but spins!)

1-pion exchange: $V(r) \propto \frac{e^{-m_\pi r}}{r}$

Tornqvist, Z.Phys. C61, 525 (1994)

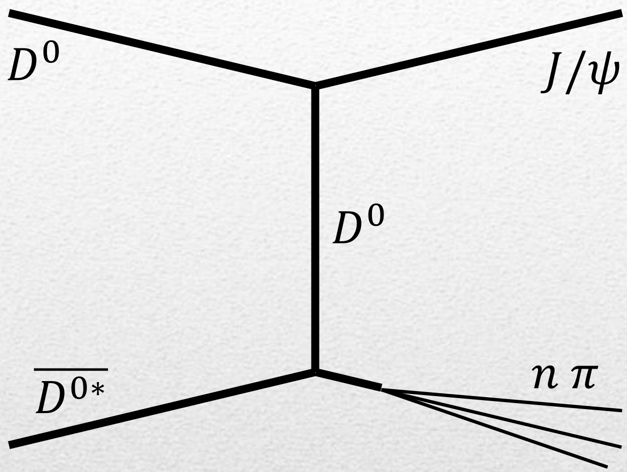
X(3872): molecule?



- Two classes for decay:
  - Long range: $X \to D^0\overline{D^{0*}}$ mesons simply split up
    We would expect $\Gamma_X \approx \Gamma_{D^*} \approx 100\ \mathrm{keV}$
  - Short range: $X \to J/\psi\ \ n\pi$ proportional to $|\psi(0)|^2$

We need a S-wave bound state to have $|\psi(0)|^2 \neq 0$
Also, too little binding energy for a P-wave state:
there should be a long-lived S-wave state

X(3872): molecule?



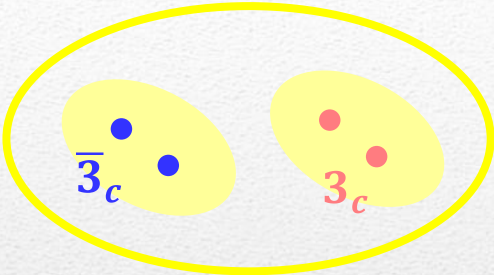
$$R \sim \frac{1}{m_c} \sim 0.2 \text{ fm}$$

Very small radius!

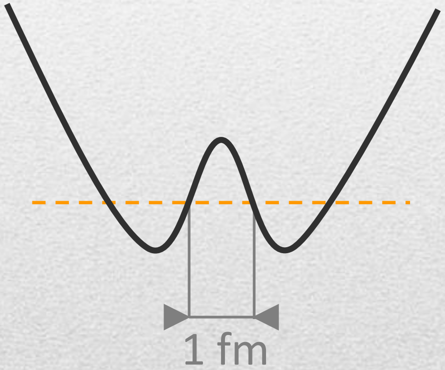
- Short range: $X \to J/\psi\ \ n\pi$ proportional to $|\psi(0)|^2$

We need a S-wave bound state to have $|\psi(0)|^2 \neq 0$
Also, too little binding energy for a P-wave state:
there should be a long-lived S-wave state

X(3872): tetraquark?



$[Cq][\bar{C}\bar{q}]$

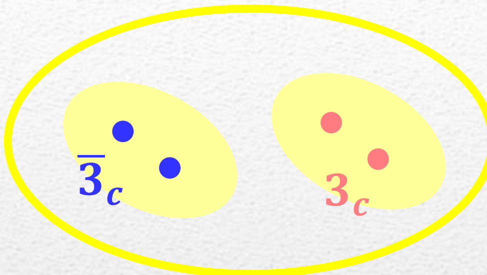


- Large binding energy: non-perturbative effects
- Double well models to describe $X \to J/\psi \ n\pi$
- One scale:
  - $R \sim 1$ fm radius of the meson

Tetraquarks prefer to decay in baryon-antibaryon, but $M_X < M(\Lambda_c \overline{\Lambda_c}) \to$ narrowness

Rossi, Veneziano, NPB123 (1977) 507

X(3872): tetraquark?



$[Cq][\bar{C}\bar{q}]$

We can have both $[cu][\bar{c}\bar{u}]$ and $[cd][\bar{c}\bar{d}]$

Mass eigenstates could be a mixing: big isospin violation

Maiani, Piccinini, Polosa, Riquer, PRD71, 014028 (2005)

String model for P-wave state: Wilczek, hep-ph/0409168

Where are charged partners?

X(3872): résumé

Molecule

- ✓ $M_X = M_{D^0} + M_{D^{0*}}$
- ✓ Isospin violation
- ✓ Large decay into DD^*
- ✗ Too small prompt production cross section in $p\bar{p} \to X + \text{all}$
- ✗ Not possible in P-wave

Tetraquark

- ✓ Isospin violation
- ✓ Narrowness (below $M(\Lambda_c \Lambda_c)$)
- ✓ Models in P-wave
- ✗ Charged partners?

The measure of the spin is no matter of taxonomy, it is important to test exotic models

$J_X = 1$ → S-wave state → **Molecule** and **Tetraquark**

$J_X = 2$ → P-wave state → ~~**Molecule**~~ and **Tetraquark**

The spin of the X(3872)

Unfortunately, there is no agreement on J^{PC} assignment of X(3872)

History

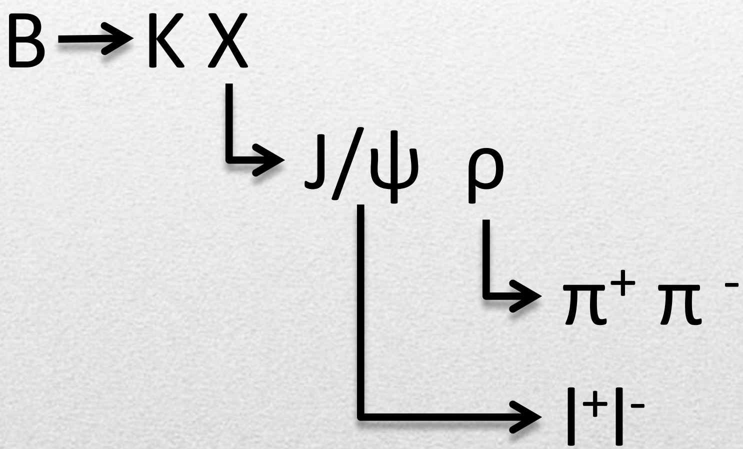
- Belle (2005) estimated $J^{PC} = 1^{++}$
- CDF (2007) ruled out all but $J^{PC} = 1^{++}$ and 2^{-+}
- Babar (2010) prefered $J^{PC} = 2^{-+}$ in 3 π channel
- Belle (2011) both $J^{PC} = 1^{++}$ and 2^{-+}

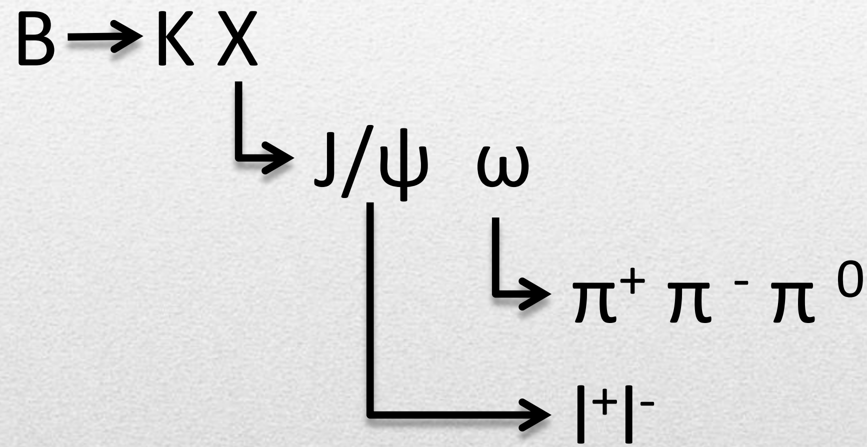
Most of theoretical analyses base on a 1^{++} assignment.

What happens if 2^{-+} ?

The spin of the X(3872)

We explore two channels:



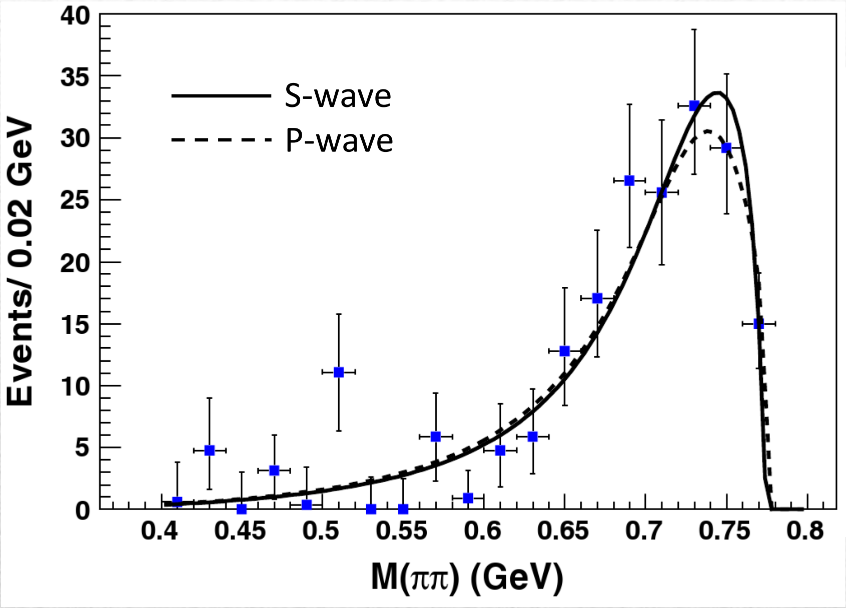


Belle, PRD84, 052004 (2011)
Invariant mass of $\pi^+ \pi^-$
Angular distributions

Babar, PRD82, 011101 (2010)
Invariant mass of $\pi^+ \pi^- \pi^0$

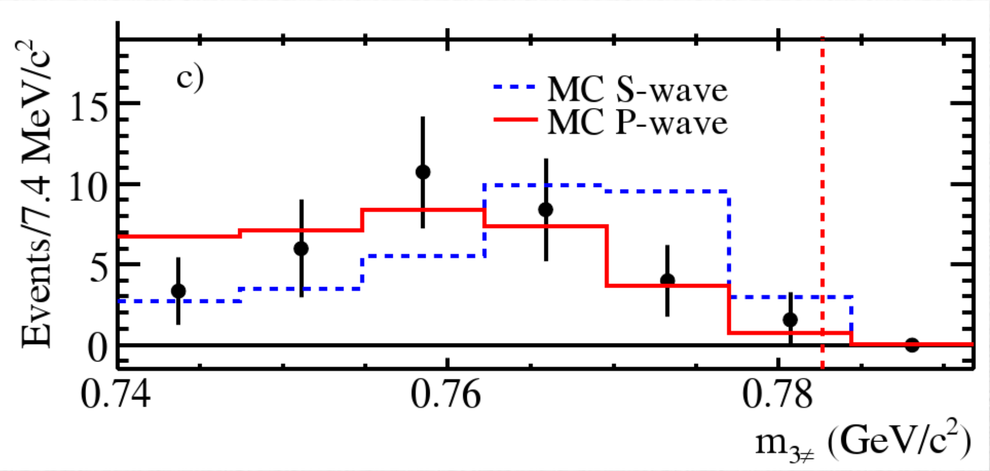
The spin of the X(3872)

Invariant mass distributions



Belle, PRD84, 052004 (2011)

$X \to J/\psi\ \pi^+\pi^-$



Babar, PRD82, 011101 (2010)

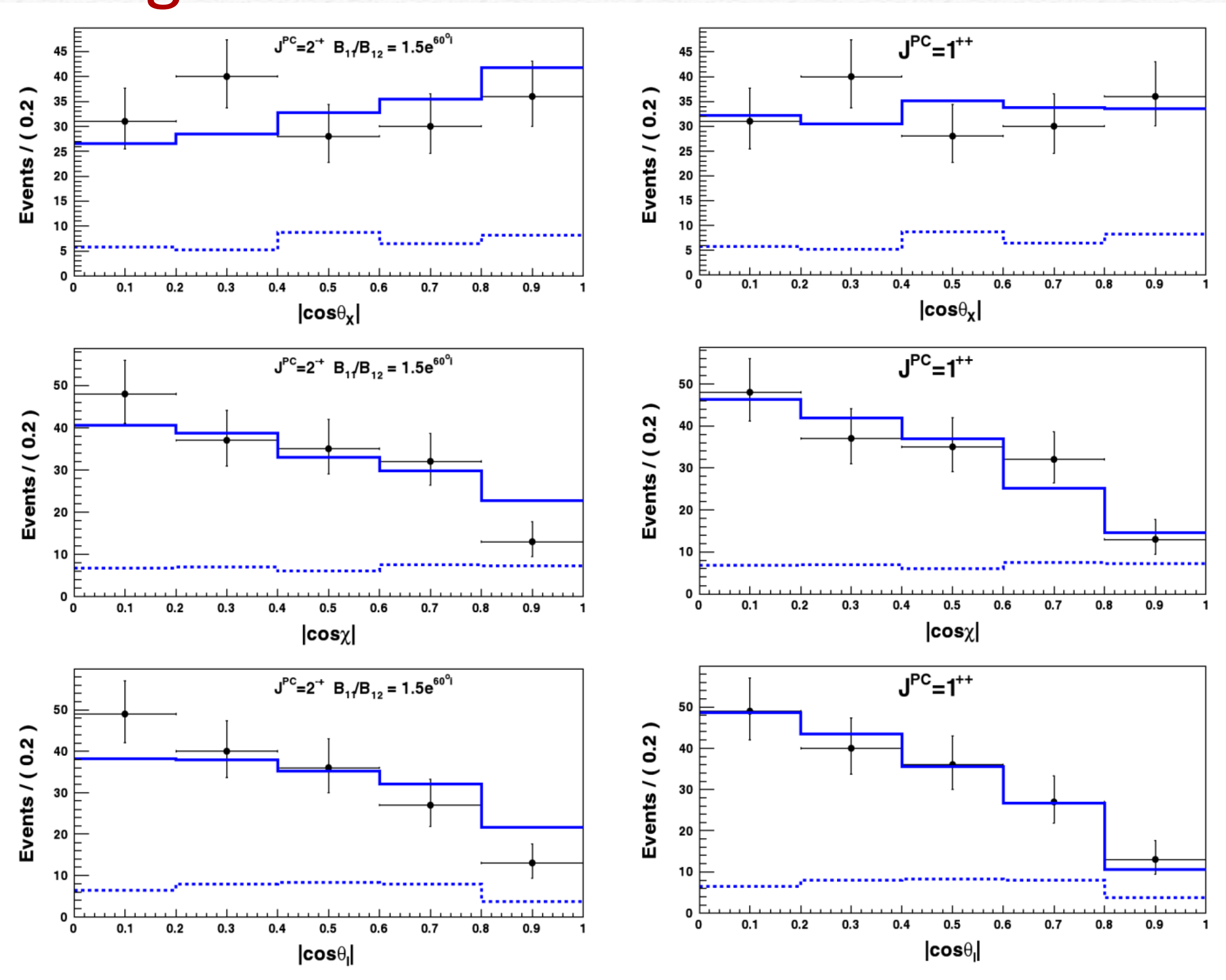
$X \to J/\psi\ \pi^+\pi^-\pi^0$

Experimentalists use Blatt-Weisskopf functions for mass distributions

The spin of the X(3872)

Angular distributions

Belle,
PRD84, 052004 (2011)
$X \to J/\psi\ \pi^+\pi^-$



Non-relativistic
on-shell waves

Exact approach

The imposing of Lorentz, parity and gauge invariance allows us to write the **exact tensorial structure**

If $J_X = 1$

$$\langle \psi(\varepsilon, p)\, V(\eta, q) \mid X(\lambda, P) \rangle = g_{1V}\, \varepsilon^{\mu\nu\rho\sigma}\, \lambda_\mu(P)\, \varepsilon^*_\nu(p)\, \eta^*_\rho(q)\, P_\sigma$$

If $J_X = 2$

$$\langle \psi(\varepsilon, p)\, V(\eta, q) \mid X(\pi, P) \rangle$$

$$= g_{2V}\, \varepsilon^{\mu\nu\rho\sigma}\, \pi_{\alpha\mu}(P) \big(\varepsilon^{*\alpha}(p)\, \eta^*_\sigma(q)\, p_\nu q_\rho - \eta^{*\alpha}(q)\, \varepsilon^*_\sigma(p)\, q_\nu p_\rho \big)$$

$$+ g'_{2V} (p - q)^\alpha \pi_{\alpha\mu}(P)\, \varepsilon^{\mu\nu\rho\sigma}\, \epsilon^*_\rho(p)\, \eta^*_\sigma(q)$$

Faccini, Piccinini, AP, Polosa, PRD86, 054012 (2012)

Exact approach

Our ignorance is in the effective couplings

We parametrize them with **polar form factors**

$$g \to g(k^*) = \frac{g}{(1 + R^2 k^{*2})^n}$$

k^* = decay 3-momentum in X rest frame

Actually this R can be extracted from data as a free fit parameter. We can learn some indications on the model by the size of R

Better results with $n = 1$, but other ns do not alter the analysis

Exact approach

We only simplify matrix elements
of invariant mass distributions with
Narrow Width Approximation

$$\sum_{\text{spin}} |\langle \psi\, n\pi \mid X \rangle|^2 \sim \sum_{\text{spin}} |\langle\, n\pi \mid V \rangle|^2 \frac{1}{|M_{n\pi}^2 - M_V^2 + iM_V\Gamma_V|^2} \frac{1}{3} \sum_{\text{spin}} |\langle \psi\, V \mid X \rangle|^2$$

In practice we neglect the angular correlations between the X and the pions

Good for invariant mass spectra
impossible for angular analysis

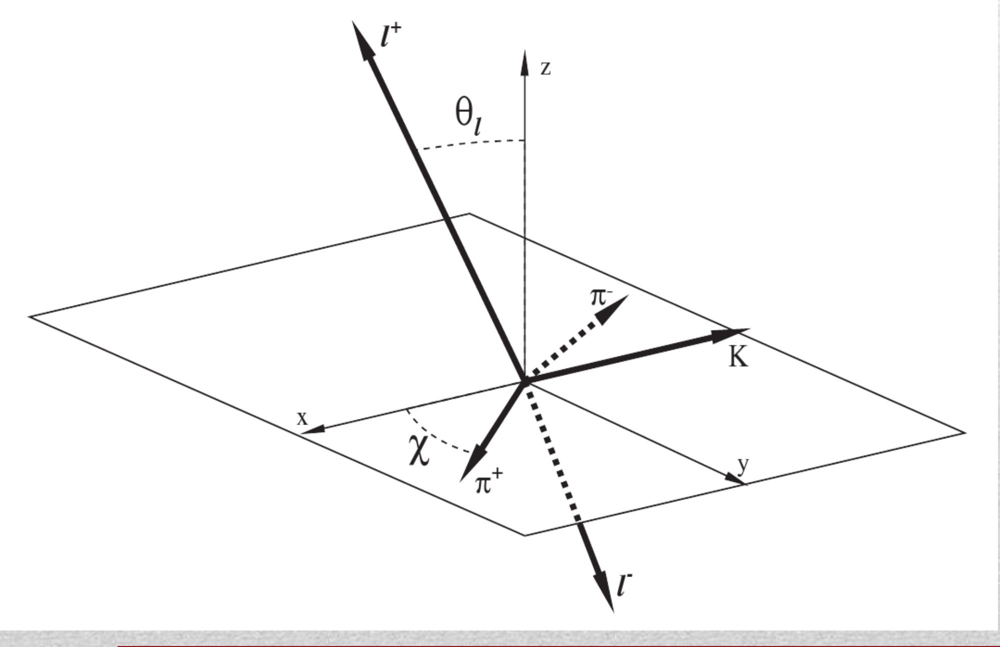
Exact approach

No approximation can be used to study angular distributions

Moreover, the angles used by Belle require

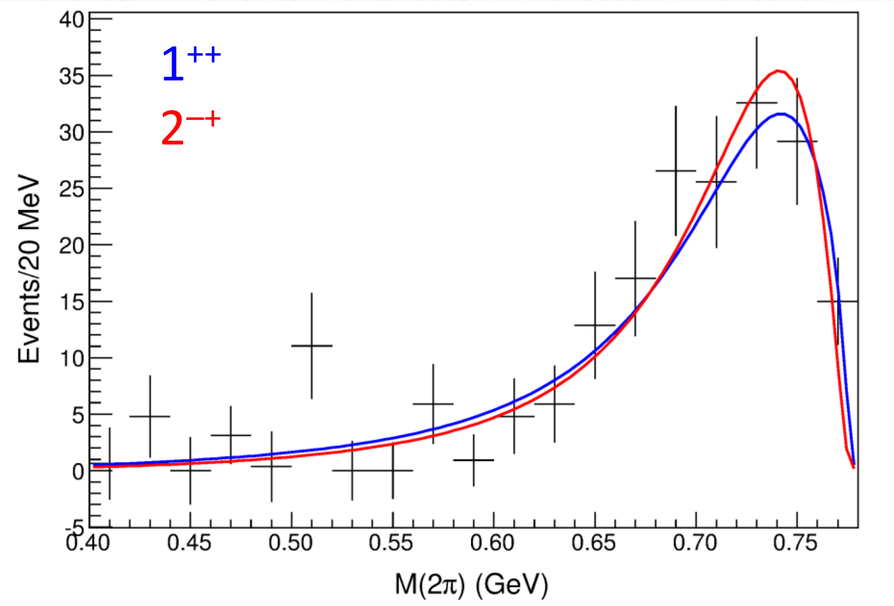
the analysis of the full 5 body decay

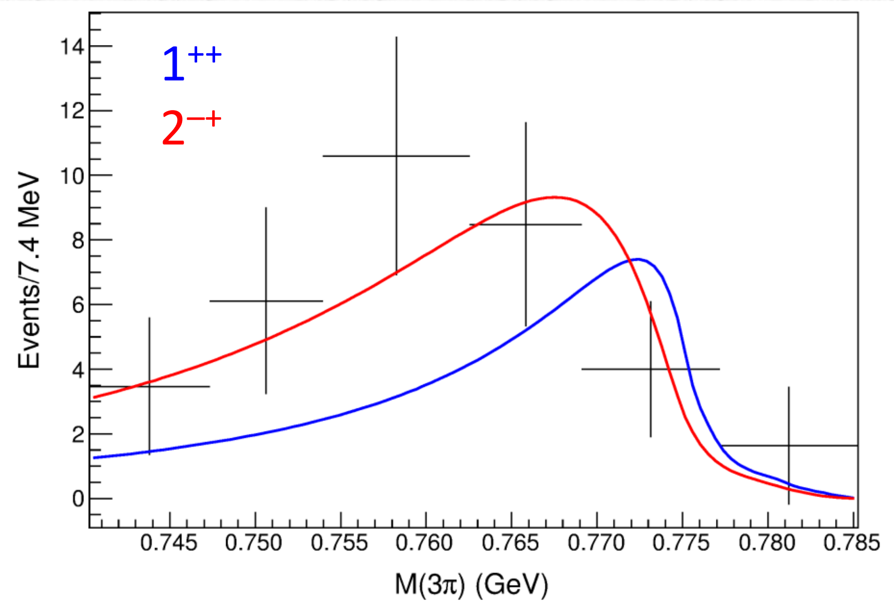
$$B \to X\,K \to J/\psi\,\rho\,K \to l^+ l^-\,\pi^+ \pi^-\,K$$



We use a MC code to take into account the phase space and the huge matrix element (20k lines of code!)

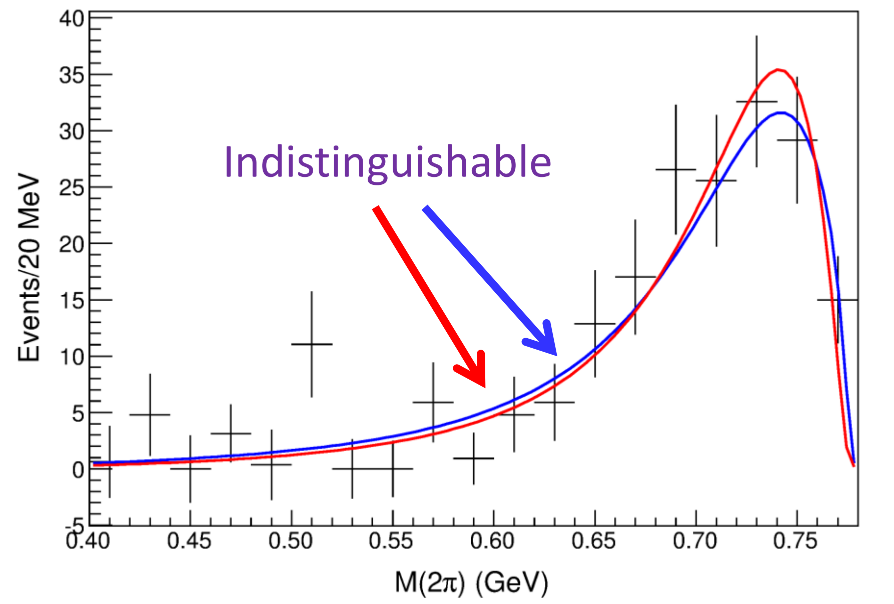
Invariant mass fits

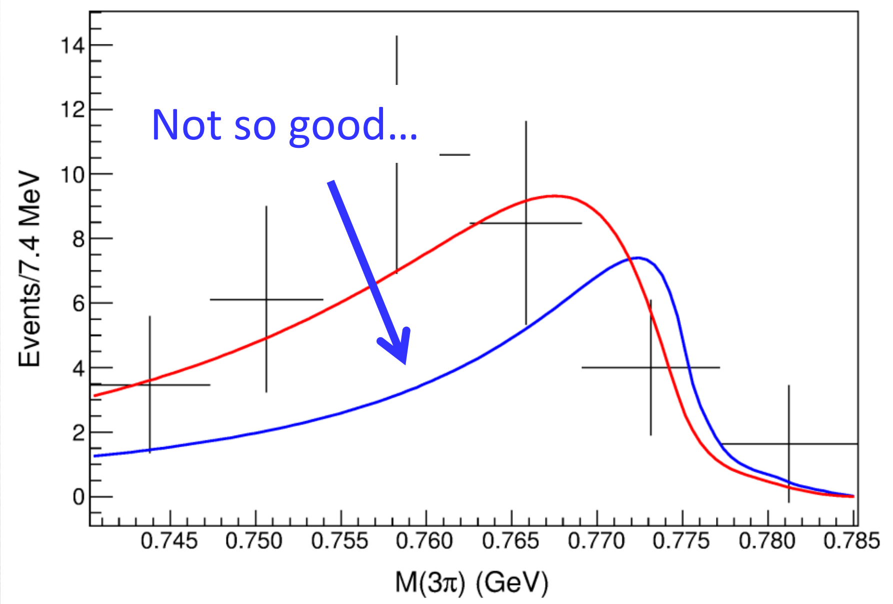




Faccini, Piccinini, AP, Polosa, PRD86, 054012 (2012)

Invariant mass fits

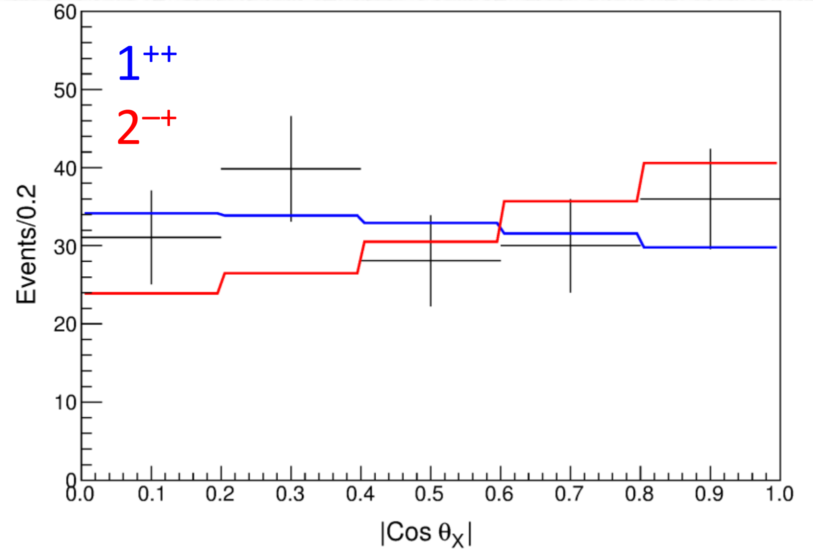


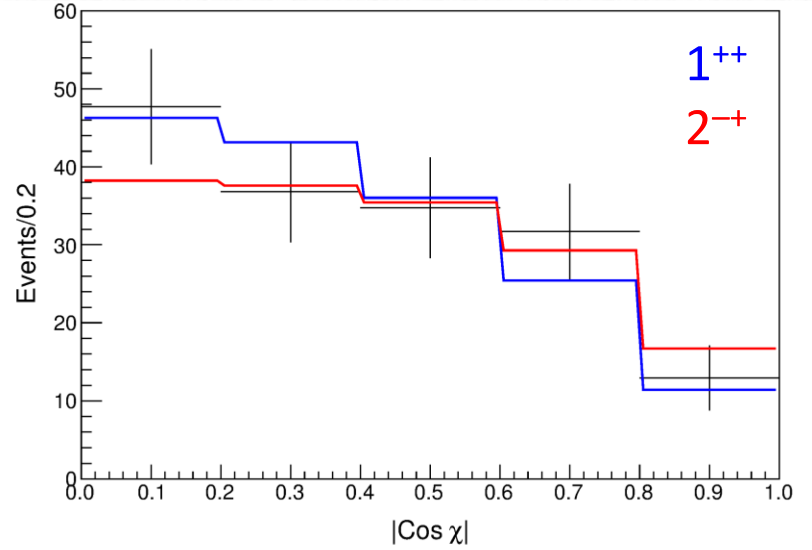


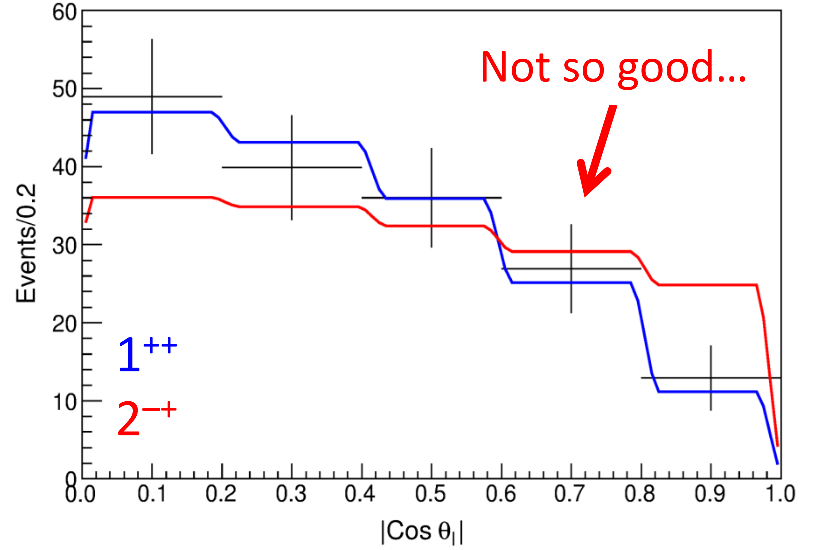
Faccini, Piccinini, AP, Polosa, PRD86, 054012 (2012)

There will be a dilution effect because of the rich useless statistics of the 2π channel

Angular fits







Angular distributions favor 1^{++}

3π mass distribution favors 2^{-+}

Combined fit: results

	1^{++}	2^{-+}
R	$1.6 \pm 0.3\ \mathrm{GeV}^{-1}$	$5.6 \pm 0.8\ \mathrm{GeV}^{-1}$
χ^2/ DOF	31.8/36	37.3/33
$P(\chi^2)$	67%	28%

Both hypotheses fit well BUT
this result is polluted by 2π invariant mass distribution

We want to strengthen the discrimination power

Toy MonteCarlo

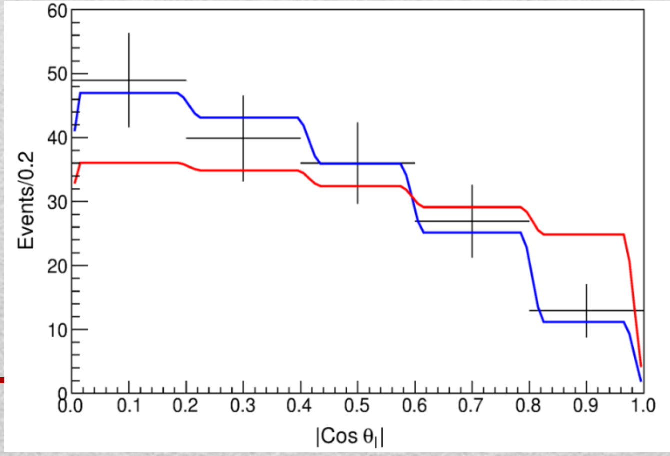
Strategy: with real data we have obtained

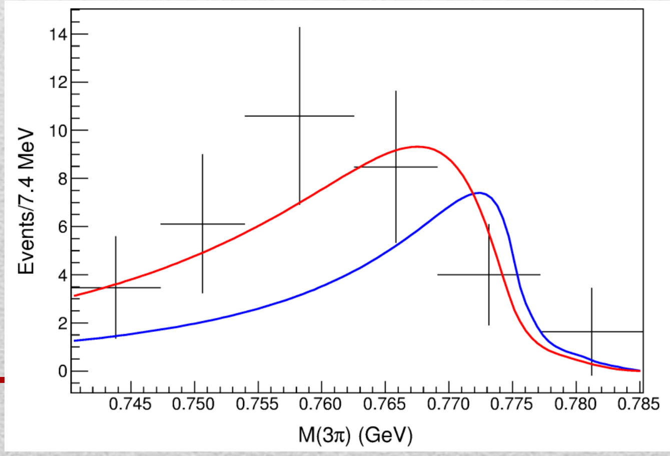
$$\Delta\chi^2 = \chi^2(1^{++}) - \chi^2(2^{-+}) = -5.5$$

If we generate pseudo-data,
how often do we obtain a similar $\Delta\chi^2$?

The insensitive component cancels out

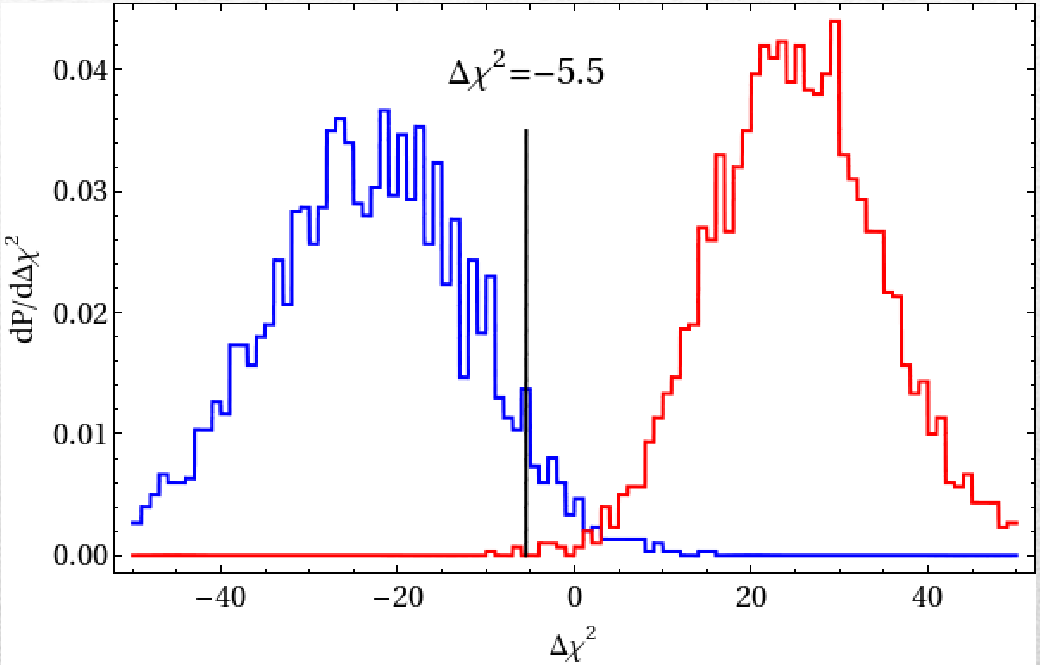
1^{++} ←





→ 2^{-+}

Combined fit



$$\frac{\chi^2(1^{++})}{\text{DOF}} = \frac{31.8}{36}\ (67\%)$$

$$\frac{\chi^2(2^{-+})}{\text{DOF}} = \frac{37.3}{33}\ (28\%)$$

$$\Delta\chi^2 = -5.5$$

$P(1^{++}) \approx 5.5\%$

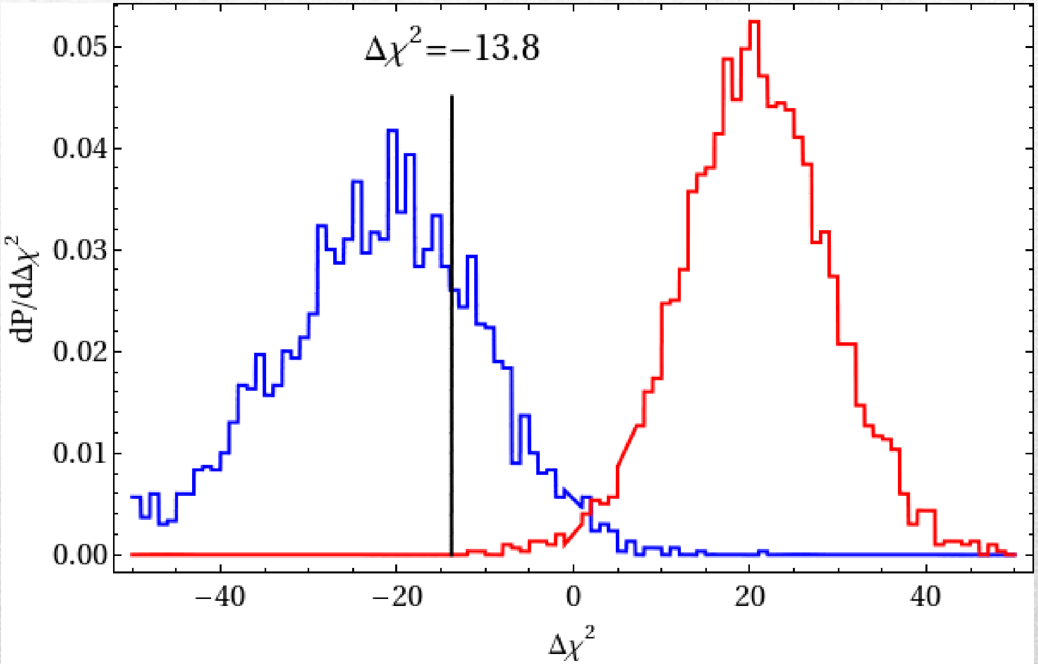
$P(2^{-+}) \approx 0.1\%$

2^{-+} excluded, 1^{++} not but…

Poor compatibility of data

Separate channels

Only 2π channel (angular + mass distributions)



$$\frac{\chi^2(1^{++})}{\text{DOF}} = \frac{20.9}{31}\ (91\%)$$

$$\frac{\chi^2(2^{-+})}{\text{DOF}} = \frac{34.7}{29}\ (21\%)$$

$$\Delta\chi^2 = -13.8$$

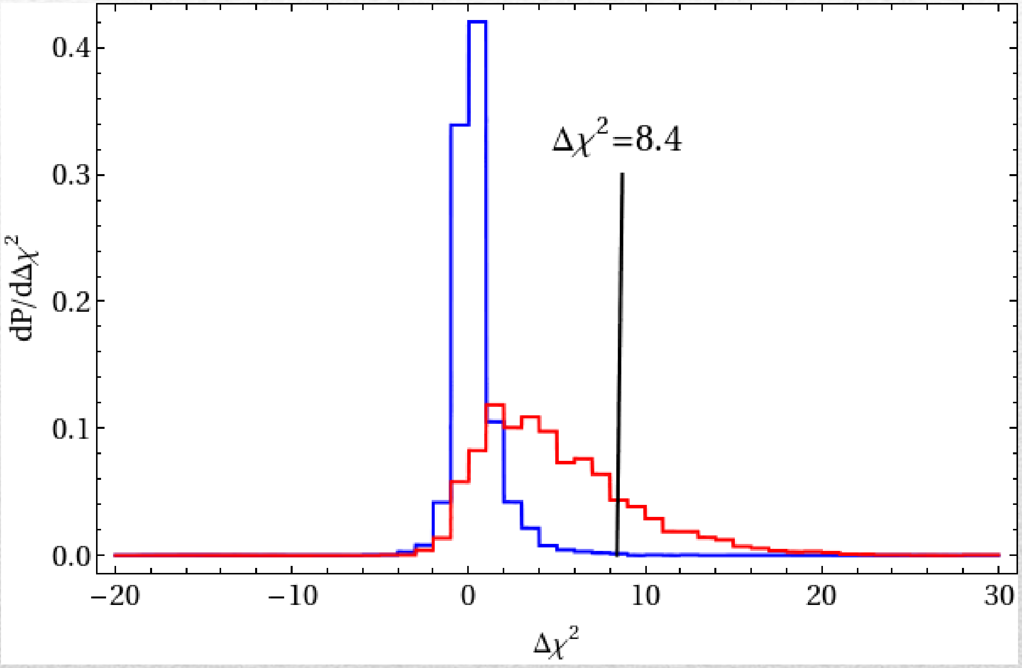
$P(1^{++}) \approx 23\%$

$P(2^{-+}) < 0.1\%$

2^{-+} excluded

Separate channels

Only 3π channel (only mass distributions)



$$\frac{\chi^2(1^{++})}{\text{DOF}} = \frac{9.9}{4}\ (4.2\%)$$

$$\frac{\chi^2(2^{-+})}{\text{DOF}} = \frac{1.5}{3}\ (68\%)$$

$$\Delta\chi^2 = 8.4$$

$$P(1^{++}) \approx 0.1\%$$
$$P(2^{-+}) \approx 81\%$$

1^{++} excluded

Conclusions?

The X(3872) puzzle still has no solution!

3 scenarios:

- 1^{++} confirmed: nothing new…
- 2^{-+} confirmed: the molecule is ruled out,
  open questions for tetraquark: where are charged partners? where is the lighter S-wave state?
- 1^{++} confirmed in 2π and 2^{-+} confirmed in 3π:
  two degenerate states (with different spin), no isospin violation; is this consistent with any existing model?

Conclusions?

The X(3872) puzzle still has no solution!

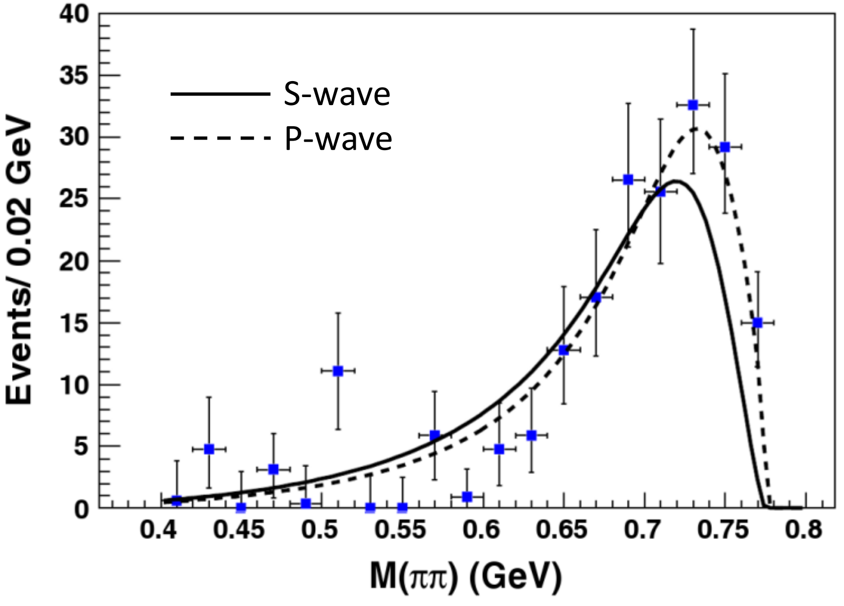
3 scenarios:

Our MC tools will repeat the analysis when new data by Belle and LHCb will be available

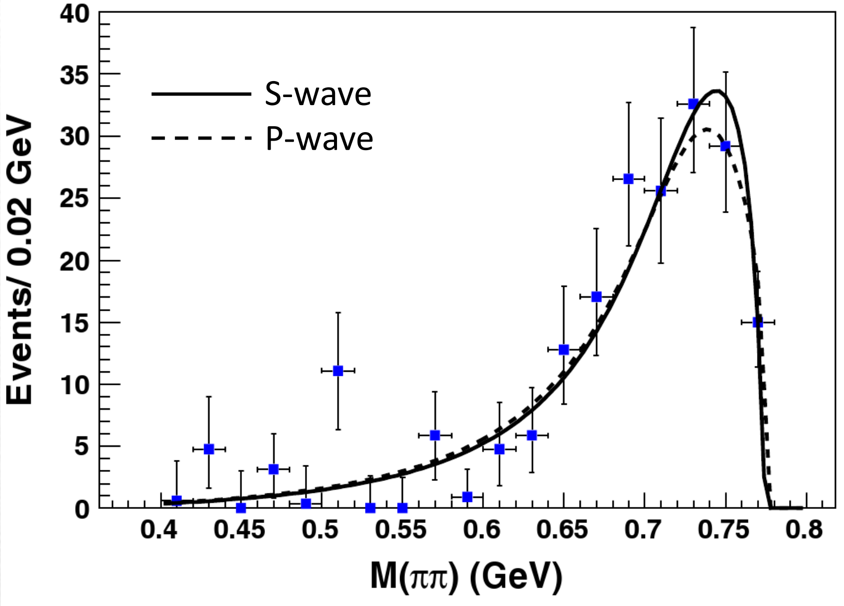
Thank you

BACKUP

ρ-ω mixing



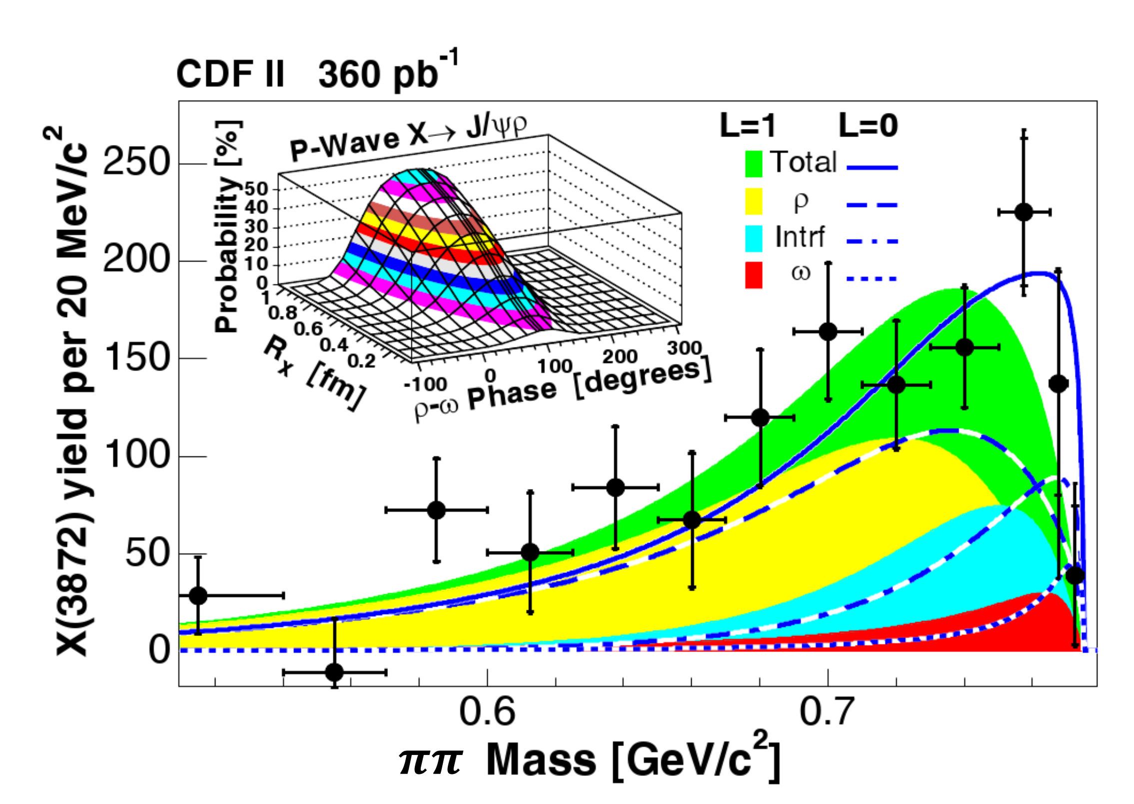
without ρ-ω mixing



with ρ-ω mixing

In particular for the P-wave, we need a big interference term
This can be constrained and ruled out by the 3π channel

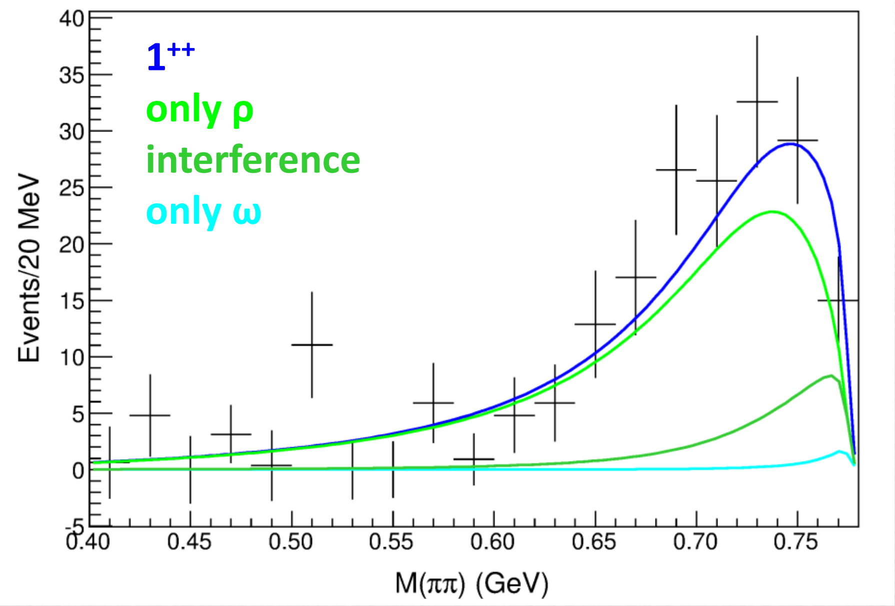
ρ-ω mixing

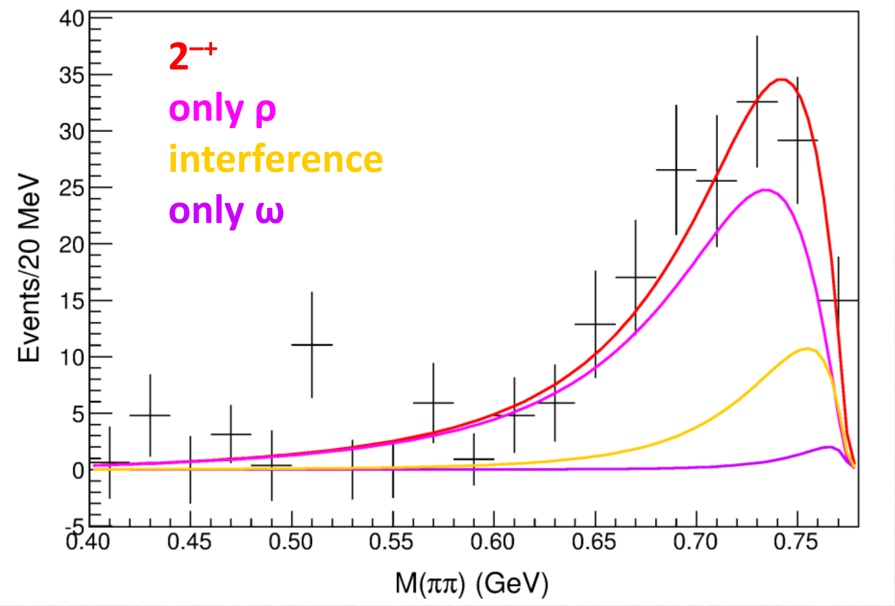


CDF PRL96 (2006) 102002

In particular for the P-wave, we need a big interference term
This can be constrained and ruled out by the 3π channel

ρ-ω mixing





With a polar form factor, the fits are good even without the mixing; we can add it and constrain with the 3π channel

Blatt-Weisskopf

Experimentalists use BW barrier factors to fit invariant mass spectra

$$\frac{dN}{dm_{n\pi}} \propto \left(k^*\right)^{2l+1} f_{lX}^2\left(k^*\right) \left|\frac{\sqrt{m_{n\pi}\Gamma_V}}{m_V^2 - m_{n\pi}^2 - im_V\Gamma_V}\right|^2$$

$$\text{with}\,\Gamma_V = \Gamma_{0V}\left(\frac{q^*(m_{n\pi})}{q^*(m_V)}\right)^3\left(\frac{m_V}{m_{n\pi}}\right)\left(\frac{f_{lV}\left(q^*(m_{n\pi})\right)}{f_{lV}\left(q^*(m_V)\right)}\right)^2$$

BW barrier factors depend on orbital angular momentum of decay products

$f_0\left(k^*\right)=1$ for a S-wave

$f_1\left(k^*\right)=\dfrac{1}{\sqrt{1+R^2k^{*2}}}$ for a P-wave

BW do not depend directly on spin!

Blatt-Weisskopf

BW factors are calculated in nuclear theory

1D model of spin-0 particles (potential well + centrifugal barrier)

Problems:

- Rough model (no spin, only orbital angular momentum)
- Analicity (the square root)
- R cannot be extracted from data, must be fixed:
  - Belle (2010): R = 5 GeV^{-1}: good 2^{-+}
  - Hanhart *et al.* (2011): R = 1 GeV^{-1}: bad 2^{-+}

Form factors

The only assumptions we needed is the form factor:

$$g \to g(k^*) = \frac{g}{(1 + R^2 k^{*2})^n}$$

This form factor is widely used in literature

k^* is the main energy scale in a 2-body decay

- $n = 1/2$ is a BW-like factor,
  but does not allow R to be fitted from data
- $n = 1$ is a standard choice
- $n = 2$ is the Fourier Transform
  of an exponential density

Form factors

The only assumptions we needed is the form factor:

$$g \to g(k^*) = \frac{g}{(1 + R^2 k^{*2})^n}$$

This form factor is widely used in literature

k^* is the main energy scale in a 2-body decay

We do not expect qualitatively different results,
but the larger is n, the smaller is R

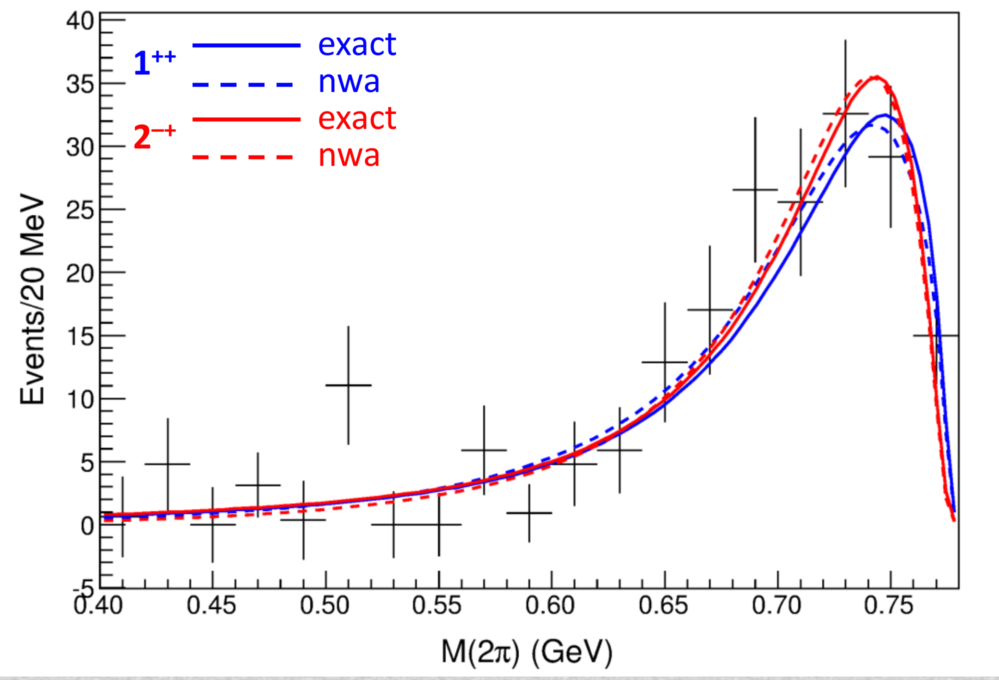
We obtain best results in Toy MC with $n = 1$, so the full analysis has been performed only for this choice.

Narrow width

Is narrow width approximation really good?

$\Gamma_\omega \sim 8$ MeV, very narrow

$\Gamma_\rho \sim 146$ MeV, not so narrow…



We verify *a posteriori* with a MC taking R from the approximated fit

Good, in particular for 2^{-+}

Isospin violation

Molecular picture

The pion-exchange model favors a $I = 0$ combination

$$\frac{\left|D^0\overline{D^{0*}}\right\rangle + \left|D^+D^{-*}\right\rangle}{\sqrt{2}} + c.c.$$

But the D^+D^{-*} threshold is 8 MeV above the X mass, so we expect a $I = 1$ component to suppress the charged contribution.

$$\frac{g_{\psi\rho}}{g_{\psi\omega}} \approx \begin{cases} \dfrac{\sqrt{m_D\Delta}}{m_c} \approx 0.15 & \text{for an S}-\text{wave} \\ \left(\dfrac{\sqrt{m_D\Delta}}{m_c}\right)^3 \approx 10^{-3} & \text{for a P}-\text{wave (excluded)} \end{cases}$$

Hanhart *et al.*, PRD85 (2012) 011501

Isospin violation

Tetraquark picture

At large momentum scales (m_c), the strength of self-energy annihilation diagrams decreases.

Particle masses should be diagonal with quark masses, even for u, d: maximal isospin violation

$$M = \begin{pmatrix} 2m_u + 2m_c & 0 \\ 0 & 2m_d + 2m_c \end{pmatrix} + \delta \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

The two mass eigenstates are splitted by

$$\Delta m = \frac{m_d - m_u}{\cos 2\theta}$$

Rossi, Veneziano, PLB70, 255 (1977)
Maiani, Piccinini, Polosa, Riquer, PRD71, 014028 (2005)

Fit variables

	1^{++}	2^{-+}
r_ρ	0.089 ± 0.006 a.u.	0.69 ± 0.13 a.u.
r_ω	0.0026 ± 0.0003 a.u.	0.030 ± 0.016 a.u.
r_{ang}	1.32 ± 0.04 a.u.	1.03 ± 0.04 a.u.
θ_ρ	-	$(254 \pm 16)^\circ$
φ_ρ	-	$(14 \pm 60)^\circ$
R	$1.6 \pm 0.3\ \text{GeV}^{-1}$	$5.6 \pm 0.8\ \text{GeV}^{-1}$

For 2^{-+}, we have $\begin{cases} g_\xi^1 = r_\xi \cos\theta_\xi \\ g_\xi^2 = r_\xi \sin\theta_\xi\, e^{i\varphi_\xi} \end{cases}$ where $\xi = \rho, \omega, \text{ang}$

But $\theta_{\text{ang}} = \theta_\rho$, $\varphi_{\text{ang}} = \varphi_\rho$ and$\theta_\omega, \varphi_\omega$ are irrelevant

Fit variables

	1^{++}	2^{-+}
r_ρ	0.089 ± 0.006 a.u.	0.69 ± 0.13 a.u.
r_ω	0.0026 ± 0.0003 a.u.	0.030 ± 0.016 a.u.
r_{ang}	1.32 ± 0.04 a.u.	1.03 ± 0.04 a.u.
θ_ρ	-	$(254 \pm 16)^\circ$
φ_ρ	-	$(14 \pm 60)^\circ$
R	$1.6 \pm 0.3\ \mathrm{GeV}^{-1}$	$5.6 \pm 0.8\ \mathrm{GeV}^{-1}$

- $m_{2\pi}$ is sensitive to r_ρ, R
- $m_{3\pi}$ is sensitive to r_ω, R
- Angular distributions are sensitive to $r_{\mathrm{ang}}, \theta_\rho, \varphi_\rho$